DC

JAMHUURIYADDA DIMOQRAADIGA SOOMAALIYA

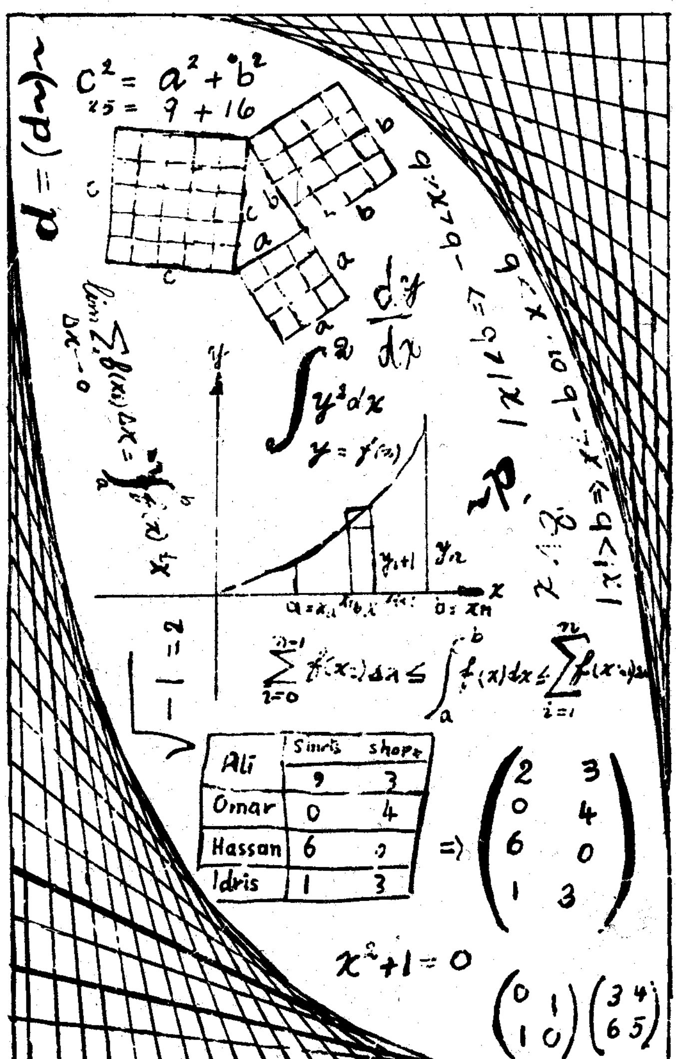
WASAARADDA WAXBARASHADDA IYO BARBAARINTA

XAFIISKA MANAAHIJTA

XISAAB

Fasalka Saddexaad

ee Dugsiga Sare



XISAAB

Fasalka Saddexaad

3

ee Dugsiga Sare

WASAARADDA WAXBARASHADA IYO BARBAARINTA
XAFIISKA MANAAHIJTA

Waxa lagu daabacay
Wakaaladda Madbacadda Qaranka
Xamar 1979

HORDHAC

Buuggan waxa loogu talagalay Xisaabta ardayda ku jirta Fasalka Saddexaad ee Dugsiga Sare, waxaana uu ka kooban yahay lix cutub.

Xafiiska Manahijta wuxuu u mahadnaqayaa guddiga xisaabyahannada ah ee qortay buuggan oo kala ah Xasan Daahir Obsiiye, Xuseen Max'd (Xannaan), Ax'd Saciid Diiriye, Muusa Cabdi Cilmi, Cali Iid Ibraahim, Axmed Geedi Maxamuud, M.E. Bullaleh iyo Maxamed Aw Daahir Cabdi (Gallan) oo isku dubbariday. Waxa kale oo Xafiiskani u mahadnaqayaa Jaallayaashi sawirrada sameeyay iyo kuwii garaacay.

Waxa mahad gaar ah leh Madbacadda Qaranka oo suuragelisay soo bixidda buuggan.

Maamulaha Xafiiska Manahijta

Cabdi Timir Cali

CUTUBKA KOWAAD

Xiriir iyo Fansaar

CUTUBKA LABAAD

Joometeriga Saafan

CUTUBKA SADDEXAAD

Tirignoometeri

CUTUKA AFARAAD

Leebab

CUTUBKA SHANAAD

Taxaneyaal

CUTUBKA LIXAAD

Joometeri

CUTUB 1

XIRIIR IYO FANSAAR

1. LAMMAANE HORSAN:

Marka aan qorno lammaane tirooyin ah, marmarka qaarkood sida loo kala horaysiiyaa micno ma le, oo sida aan doonno baan u kala horaysiin karnaa, marmarka qaarkoodna sida loo kala horaysiiya micno weyn bay ku fadhidaa. Matalan, waxaan qori karnaa {4,3} ama {3,4} labada tiro ee 4 iyo 3 kolba kaan doonno baan horaysiin karnaa, ulajeedadeeniina isbeddali mayso. Mar kasta waxan helaynaa ururka kutirsaneyaashiisu ay yihiin 3 iyo 4. Haddaba, goormay sida loo kala horaysiiyo lammaane tirooyin ah ay micno aad ku fadhidaa? Inta aanan ka jawaabin su'aasha bal tusaalahan u fiirso. Haddii tirooyinka 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 ay u taagan yihiin Gobollada Hargeysa, Togdheer, Sanaag, Bari, Nugaal, Mudug, Galguduud, Hiraan Shabeelle Dhexe, Xamar, Shabeelle-Hoose, Bay, Gedo, Bakool iyo Jubbada Hoose, sida ay u kala horeeyaan, isla markaa lammaanaha (5,8) uu u taagan yahay gobolka shanaad o ah Nugaal baa 8 barood keenay tartankii dhex marayay gobollada, (8,5) na u taagan tahay gobolka sideedaad oo ah Hiraan baa 5 barood keenay, markaa waxa innoo muuqda in sida loo kala horraysiiyo tirooyinka ay micna aad ah ku fadhido, oo haddii si kale loo kala horraysiiyo micnihii isbeddelayo.

Lammaane tirooyin, sida (5,8) waxa la yiraa lammaane horsan. Haddii sida loo kala horraysiiyo ay micna weyn ku fadhido. Lammaanaha horsan waxa loo qoraa sidan: (1,2), (5,8), (a,b) (2,a), (x,y) iwm., labada tiro ee lammaanuhu ka kooban yahay mid walba waxa la yiraa

Xubin Lammaane Horsan, ka hore waxa la yiraa xubinta hore, ka danbana xubinta danbe.

Labo lammaane oo horsani, waxay isle'eg yihiin haddii xubnahooda hore isle'eg yihiin, kuwooda danbana isle'eg yihiin, matalan: (2,5) iyo $\left\{\frac{12}{6}, \frac{15}{3}\right\}$ way isle'eg yihiin, waayo $2 = \frac{12}{6}$ isla markaa $3 = \frac{15}{3}$ laakiin (2,3) iyo (4,8) isma le'eka waayo $2 \neq 4$ isla markaa $3 \neq 8$; sidaas oo kale (4,7) iyo (6,7) isma le'eka waayo $4 \neq 6$. Ma isle'eg yihiin labadani lammaane ee horsani, (2,3) iyo (3,2)? Maya, waayo $2 \neq 3$ isla markaa $3 \neq 2$. Haddii xubnaha hore ama xubnaha danbe ee labo lammaane ee horsani ayna isle'ekeyn, markaa labada lammaane ee horsani isma le'eka.

2. TARANKA KAARTIS:

Haddii B iyo T ay yihiin ururro, taranka kaartis oo loo qoro $(B \times T)$ waa ururka lammaane kasta (x, y) ee X tahay kutirsane B, isla markaana Y tahay kutirsane T. $(B \times T)$ waxa loo akhriyaa «B laanqayr T».

Tusaale 1:

Haddii $B = \{1, 2, 3\}$, $T = \{m, n\}$

$B \times T = \{(1,m), (1,n), (2,m), (2,n), (3,m), (3,n)\}$

Bal u fiirso $T \times B$:

$T \times B = \{(m,1), (m,2), (m,3), (n,1), (n,2), (n,3)\}$

Tusaalaha kor ku qorani wuxuu inoo sheegayaa in $B \times T$ ay ku jiraan kutirsanayaal sida (1,b) oo kale ah, laakiin $T \times B$ waxa ku jira kutirsanayaal sida (b,1) oo kale ah, markaa mar haddii $(1.b) \neq (b.1)$ sidii aan kor ku sheegnay, $B \times T \neq T \times B$, haddii B iyo T ayna isle'keyn.

Tusaale 2:

Haddii $D = \{1\}$; $R = \{0.1\}$

$D \times R = \{(1.0). (1.1)\}$

$R \times D = \{(0.1). (1.1)\}$

Tusaale 3:

$S = \{1, 2, 3, \cdots, n\}$

$M = \{b_1, b_2, \cdots. b_m\}$

Markaa:

$S \times M = \{(1, b_1). (1, b_2)\cdots, (1, b_m). (2. b_1), (2, b_2)\cdots, (2 b_m)\cdots. (n. b_1), (n, b_2)\cdots, (n, b_m)\}$

Tusaalaha 4:

Haddii $B = \{1, 2, 3\}$

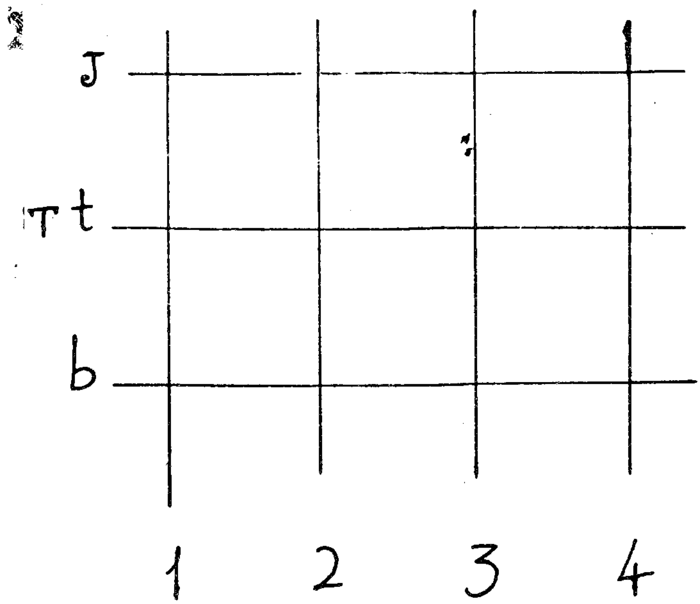
$B \times B = \{(1.1). (1.2), (1, 3), (2, 1), (2. 2), (2,3), (3, 1) (3, 2) (3, 3)\}$

Ogow:

Haddii $B \times T = \phi$ markaa $B = \phi$ ama $T = \phi$ ama B iyo T ba waa ururro madhan. Garaaf ahaan, taranka kaartis ee labo urur B iyo T waa ururka baraha isgoyska u ah xarriiqyada taagan ee u taagan ku tirsaneyaasha B iyo xarriiqda jiifa ee u taagan ku tirsaneyaasha T.

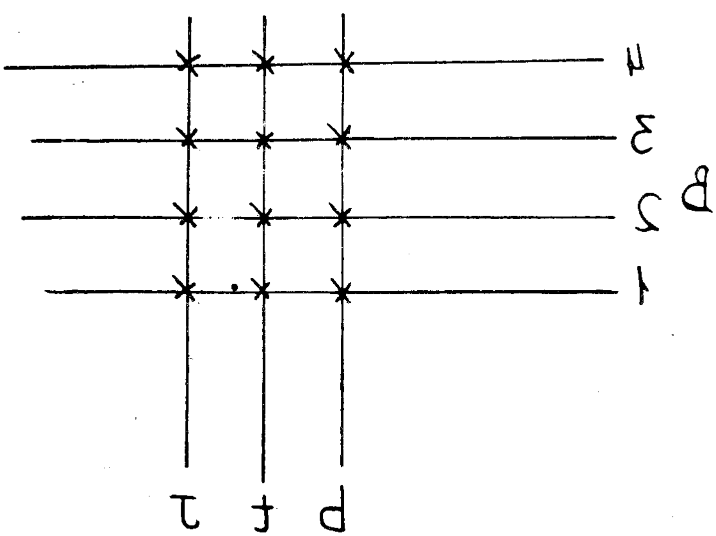
Tusaale 5:

Haddii $B = \{1, 2, 3, 4\}$; $T = \{b, t, j\}$



SH. 1: B × T

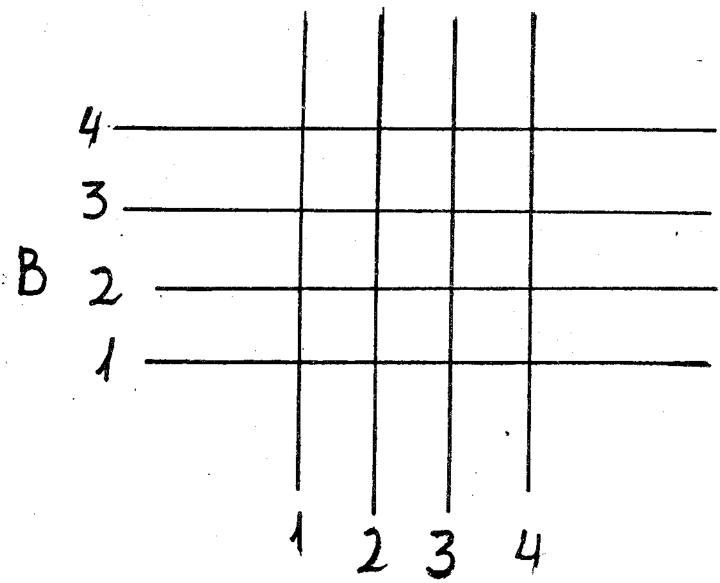
Sadaqa baraha ahi waa garaafka B × T.



SH. 2: T × B

Garaafka T × B

Shaxanka hoos ku yaali waa garaafka $B \times B$, waxayna u egtahay sadaq baro ah oo labajibbaarane ah.

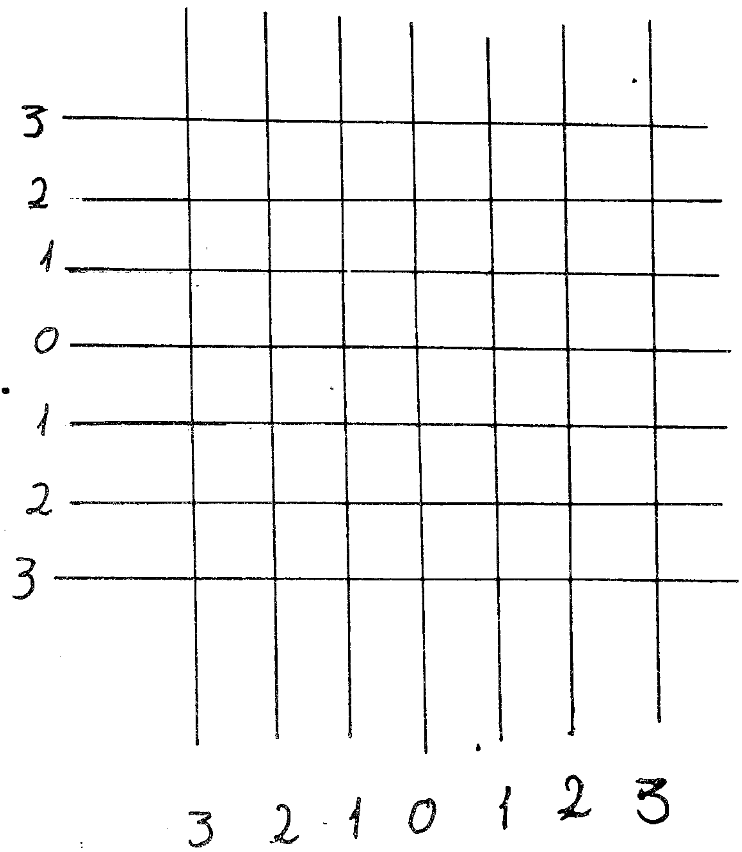


SH: 3 $B \times B$

Tusaale 6:

Samee garaafka $N \times N$ haddii:

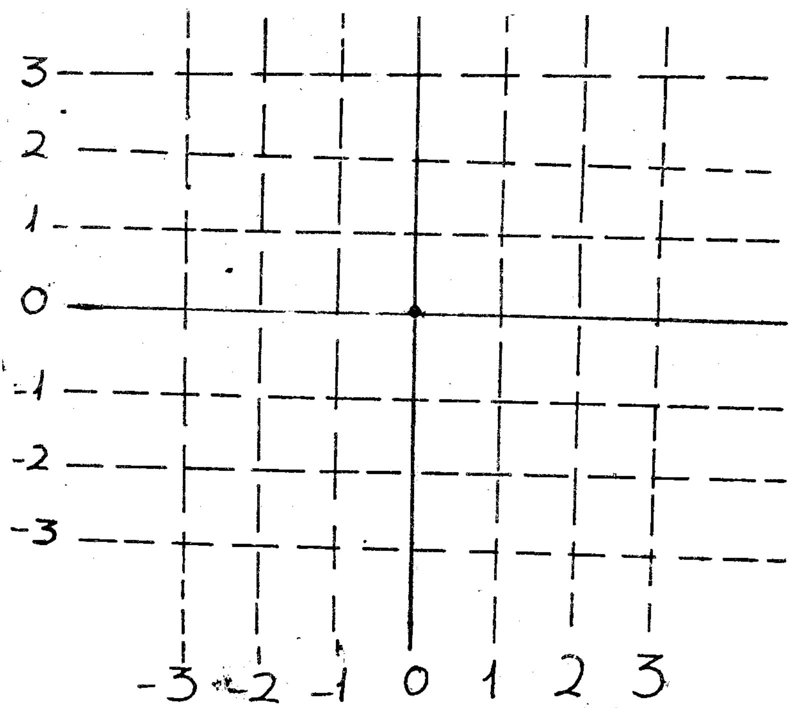
$N = \{-3, -2, -1, 0, 1, 2, 3\}$



SH. 4

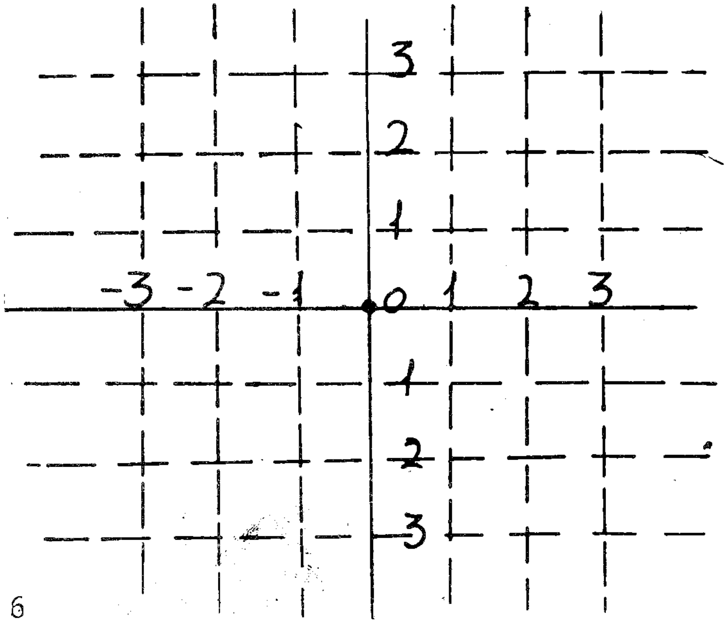
Garaafka N × N.

Shaxanka 4aad haddii labada xarriiqood ka jiifa iyo ka taagan ee mid walba eber u taagan yahay aan ugu hor samayno, oo kuwa kalena aan xarriiqyo googo'an ka dhigno waxaan helaynaa shaxanka hoos ku yaal.



SH. 5

Xarriiq kasta waxay u taagan tahay tiro, ka soo qaad xarriiqda taagan ee 2 ku hoos qoran tahay. Xarriiqdaasi waxay u taagan tahay 2; tirada 2 meeshii aan doonno baan xarriiqda kaga qori karnaa. Markaa, haddii tiro kasta oo axrriiq u taagan aan ku qorno meesha xarriiqdaasi iyo xarriiqda eber u taagani ay iska gooyaan waxan helaynaa shaxanka hoos ku yaal. Ogow, eber waxan ku qoraynaa meesha xarriiqaha eber u taagani ay iska gooyaan.



SH. 6

Shaxanka ɓaad u fiirso. Maxaa ka dhexeeya isaga iyo habdhiska kulanka laydi?

l a y l i

1. Haddii $B = \{1, 2, 3, 4\}$; $T = \{3, 5, 6\}$;
$J = \{0, 2, 3, 4, 5\}$; $D = \{0\}$

Raadi taranka kaartis, dabadeedna samee garaaf-kiisa.

b) $B \times T$
t) $T \times J$
j) $B \times D$
x) $B \times J$
kh) $B \times B$
d) $D \times D$
r) $T \times T$
s) $J \times J$
sh) $T \times B$
dh) $T \times D$

2. Haddii $T = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$. Raadi taranka $T \times T$ dabadeedna samee garaafkiisa.

3. Raadi kutirsaneyaasha $B \times T$ haddii B iyo T lagu siiyo

b)	$B = \{0, 1, 2\}$	$T = \{1, 2\}$
t)	$B = \{b, t\}$	$T = \{j, x\}$
j)	$B = \{L, m, n, d\}$	$T = \{1, 2\}$
x)	$B = \{-1, 0, 1\}$	$T = \{-3, -2, -1, 0, 1, 2, 3\}$
kh)	$B = \{3\}$	$T = \{4\}$

4. Haddii $B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ Samee garaafka $B \times B$, dabadeedna calaamadee baraha u taagan:

$(-4, 3)$ $(4, 3)$ $(3, 4)$, $(0, 0)$, $(1, 3)$

5. **Xiriir:**

Haddii B iyo T ay yihiin ururro aan madhneyn, hormo kasta oo taranka kaartis $B \times T$ waa xiriir min B ilaa T ah, taasi waxay la mid tahay, haddii r tahay xiriir min B ilaa T ah, markaa $r \in B \times T = \{(x, y) \mid x \in B, y \in T\}$.

Tusaale 1:

Haddii $B = \{1, 2, 3, 4\}$ $T = \{b, t, j\}$

$r_1 = \{(1, b), (2, t), (3, j)$

$r_2 = (1, b), (4, j)$

Markaa r_1 iyo r_2 labaduba waa xiriiryo min B ilaa T ah.

Tusaale 2:

Haddii $T = \{6, 7, 8\}$ $j = \{1, 2, 3, 4\}$

$S_1 = \{(6, 1), (6, 2), (6, 3) (7, 3)\}$

$S_2 = \{(6, 2), (7, 2), (8, 3)\}$

$S_3 = \{(7, 3), (8, 4), (8, 1), (8, 2)\}$

$S_4 = \{(6, 1), (7, 3), (8, 5)\}$

Markaa S_1, S_2 iyo S_3 waa xiriiryo min T ilaa J ah, laakiin S_4 maaha xiriir min R ilaa J ah, waayo waxa jira lammaane horsan (8,5) oo S_4 oo aan ahayn kutirsane $T \times J$ waayo 5 maaha kutirsane J.

Waxan arkayna in xiriir min B ilaa T ahi yahay urur lammaaneyaal horsan oo xubinta hore ee lammaanaha horsani tahay kutirsane B, xubinta danbena tahay kutirsane T.

HORAAD IYO DANBEED:

Ururka dhammaan xubnaha hore ee lammaaneyaal horsan ee xiriir waxa la yiraa **Horaadka xiriirka.** Ururka dhammaan xubnaha danbena waxa la yiraa **Danbeedka** xiriirka.

Tusaale 1:

Haddii r tahay xiriir min B ilaa T ah:

$B = \{b, t, j, x, kh\}$

Markaa:

$T = \{1, 2, 3, 4, 5, 6\}$, $r_1 = \{(b, 2), (t, 2), (t, 1), (j, 3)$
$(x, 5)\}$

Horaadka r_1 oo loo qoro $H(r_1)$ waa ururka $\{b, t, j, x\}$ danbeedka r_1 oo loo qoro $D(r_1)$ waa ururka $\{1, 2, 3, 5\}$.

Tusaale 2:

Horaadka r_1 oo loo qoro $H(r_1)$ waa ururka $\{b, t, j, x\}$. $(3, 2)$ $(4, 5)\}$, markaa horaadka r, $H(r) = \{1, 2, 3, 4\}$ isla markaa danbeedka r, $D(r) = \{1, 2, 3, 5\}$.

Tusaale 3:

Haddii f ay tahay xiriirka $(1, 1), (2, 1), (3, 1) \cdots (n, 1)$ markaa horaadka f, $H(f) = 1, 2, 3, \cdots, n$, danbeedka f, $D(f) = 1$.

Xiriir oo isku aaddan ah:

Ka soo qaad in B iyo T ay yihiin ururro aan madhnayn $B = \{1, 2, 3, 4\}$, $T = \{b, t, j\}$. Kana soo qaad in:

$$r_1 = \{(1, b), (1, t), (3, b)\}$$

$$r_2 = \{(2, b), (1, t), (2, t)\}$$

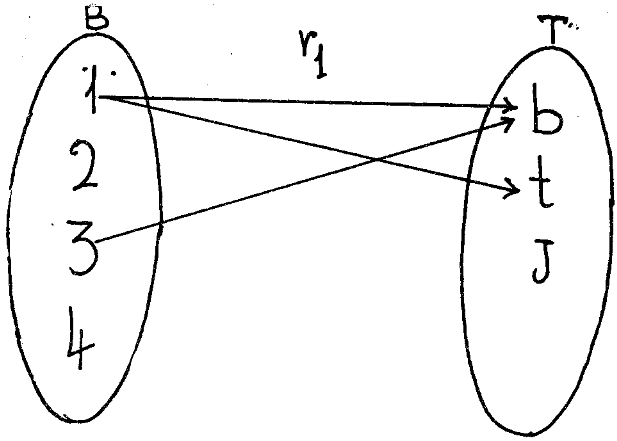
$$r_3 = \{(1, b), (2, b), (3, b)\}$$

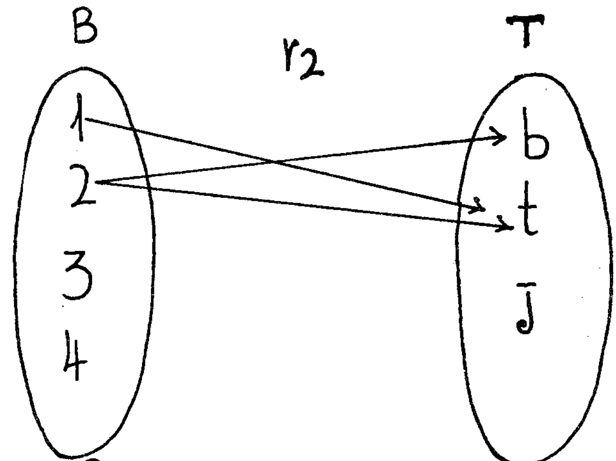
$$r_4 = \{(1, b), (1, t), (1, j)\}$$

$$r_5 = \{(4, b), (3, t)\}$$

$$r_6 = \{(1, b), (2, t), (3, j)\}$$

Shaxanka hoos ku yaal waxay u taagan yihiin xiriiryada kor ku yaal.


B
r1
T
1
2
3
4
b
t
J


B
r2
T
1
2
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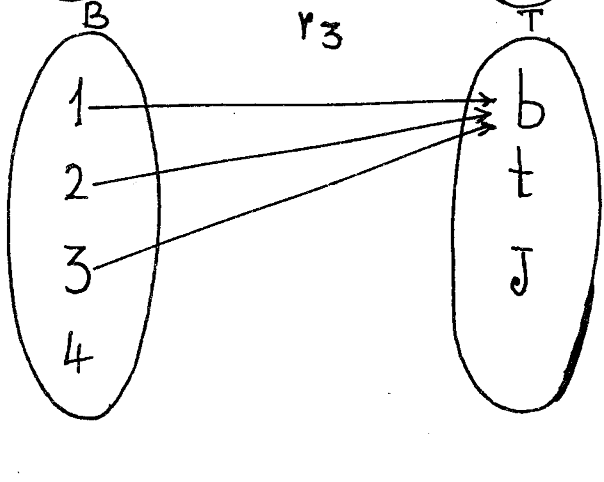
4
b
t
J


B
r3
T
1
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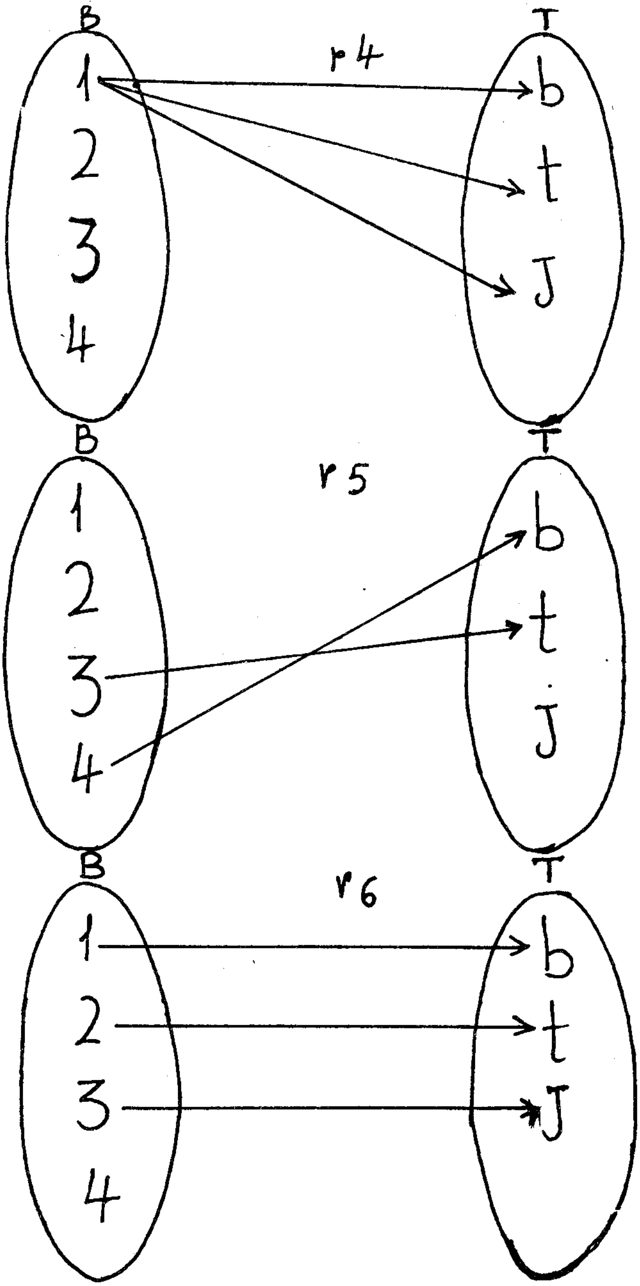
4
b
t
J


B
T
r4
1
2
3
4
b
t
J
B
T
r5
1
2
3
4
b
t
J
B
T
r6
1
2
3
4
b
t
J

Leebabku ku tirsaneyaasha B ayay ku aaddiyaan kuwaa T, hadda xiriir waxan u qeexi karnaa sida soo socota: xiriirka min B ilaa T ahi waa xeerka ku aaddiya kutirsaneyaasha B kuwa T.

Si alla sidii kutirsaneyaasha B aan ugu aaddinno kuwa T, waxan helaynaa xiriir min B ilaa T ah, u fiirso in horaadka xiriir kasta oo ah min B ilaa T ah, uu hormo u yahay B, isla markaa in danbeedka xiriirkaasi uu hormo u yahay T.

Guud ahaan, haddii S tahay xiriir min W ilaa Y ah, horaadka S wuxuu hormo u yahay W, danbeedka S-na wuxuu hormo u yahay Y, taasoo ah $H(x) \leqslant W$ isla markaa $D(S) \leq y$.

Haddii r tahay xiriir min B ilaa B ah, r waxa la yiraa xiriir B. Haddii $B = \{l, m, n, w\}$ isla markaa $r = \{(l, l), (l, m), (m, n), (w, n)\}$ markaa r waa xiriir B. In kasta oo hormo kasta oo taranka kaartis, $B \times T$ ay tahay xiriir min B ilaa T ah, haddana waxa jira xiriiryo gaar ah oo leh xeer sheegaya sida ay isugu aaddan yihiin kutirsaneyaasha danbeedka iyo kuwa horaadku. Matalan, haddii $B = \{1, 2, 3, 4, 5\}$ isla markaa «m» tahay xiriir min B ilaa B ah (xiriir B), oo xubnahiisa hore iyo kuwiisa danbe isle'eg yihiin, m wuxuu noqon karaa xiriiryada hoos ku yaal:

$$m_1 = \{(1, 1)\}, m_2 = \{(1, 1), (2, 2), (3, 3)\}$$
$$m_3 = \{(3, 3), (5, 5)\} \text{ ama } m_4 = \{(1, 1),$$
$$(2, 2) (3, 3) (4, 4) (5, 5)\}.$$

Xiriiryada aan soo sheegnay, ka ugu danbeeya ama m_4 waxa loo qori karaa $m_4 = \{(x, y) | x \in B, y \in B$ isla markaa $y = x\}$, waxana loo akhriyaa ururka lammaane kasta oo horsan (x, y) ee x tahay kutirsane B, y-na tahay kutirsane B, isla markaa y le'eg tahay x.

Tusaale:

Haddii $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$
$r_1 = \{(x, y) \mid x, y \in B$ isla markaa $y = 2x\}$

Markaa kutirsaneyaasha r_1 waxa loo tixi karaa sida soo socota: $r_1 = \{(1, 2), (2, 4)\ (3, 6), (4, 8), (5, 10)\ (6, 12)\}$

Tusaale 2:

Haddii $B = \{1, 2, 3, 4\}$ $r_2 = \{(x, y) \mid x, y \in B, y > x)\}$ markaa, kutirsaneyaasha r_2 waa kuwa soo socda:

$r_2 = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

Tusaale 3:

Haddii $B = \{1, 2, 3, 4, 5, 6, 7\}$

$r_3 = \{(x, y) \mid x, y \in B, y = 5\}$

markaa, kutirsaneyaasha r_3 waxay noqonayaan kuwa hoos ku qoran:

$r_4 = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)\}$

Tusaale 4:

Haddii $T = \{-2, -1, 0, 1, 2\}$

$r_4 = \{(x, y) \mid x, y \in T, y \geqslant x\}$ tax kutirsaneyaasha r_4.

Furfuris:

$r_4 = \{(-2, -2), (-2, -1), (-2, 0), (-2, 1), (-2, 2), (-1, -1), (-1, 0), (-1, 1), (-1, 2), (0, 0), (0, 1), (0, 1), (0, 2), (1, 1), (1, 2), (2, 2)\}$

Tusaale 4:

Haddii $J = \{0, 1, 2, 3\}$

$r_5 = \{(x, y) \mid x, y \in J, x > 1, y < 2\}$ tax kutirsaneyaasha r_5. Sheeg horaadka iyo danbeedka r_5.

Furfuris:

$r_5 = \{(2, 1), (2, 0), (3, 1), (3, 0)\}$

Horaadka r_5, $H(r_5) = \{2, 3\}$

Danbeedka r_5, $D(r_5) = \{0, 1\}$

Tusaale 5:

Haddii $M = \{1, 2, 3, \cdots 17\}$

$r_6 = \{(x, y) \mid x, y \in M, y = x^2\}$

$r_6 = \{(x, y) \mid x, y \in M, y = x^2\}$ tax kutirsaneyaasha r_6, isla markaa sheeg horaadka iyo danbeedka r_6.

Furfuris:

$r_6 = \{(1, 1), (2, 4), (3, 9), (4, 16)\}$

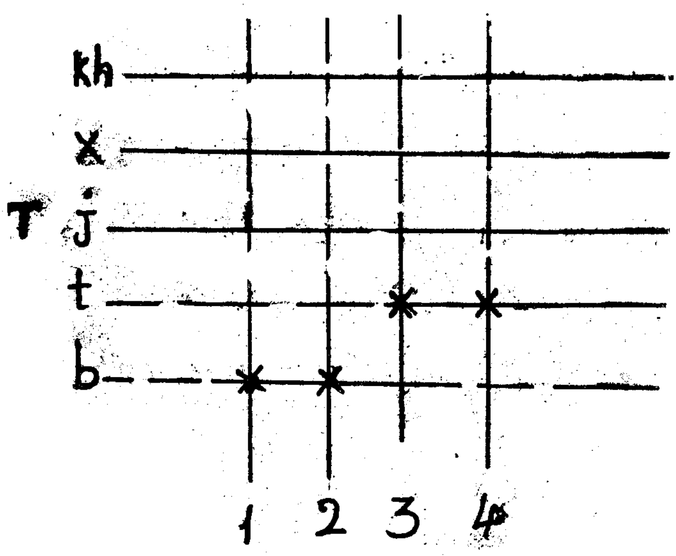
Horaadka r_6, $H(r_6) = \{1, 2, 3, 4\}$

Danbeedka r_6, $D(r_6) = \{1, 4, 9, 16\}$

5. GARAAFKA XIRIIR:

Haddii $B = \{1, 2, 3, 4\}$; $T = \{b, t, j, x, kh\}$

$r = \{(1, b), (2, b), (3, t)\}$, sidee baad u samayn lahayd garaafka r? Marka hore samee garaafka taranka kaartis, $B \times T$, dabadeedna calaamadee baraha ku beegan kutirsaneyaasha xiriirka r.



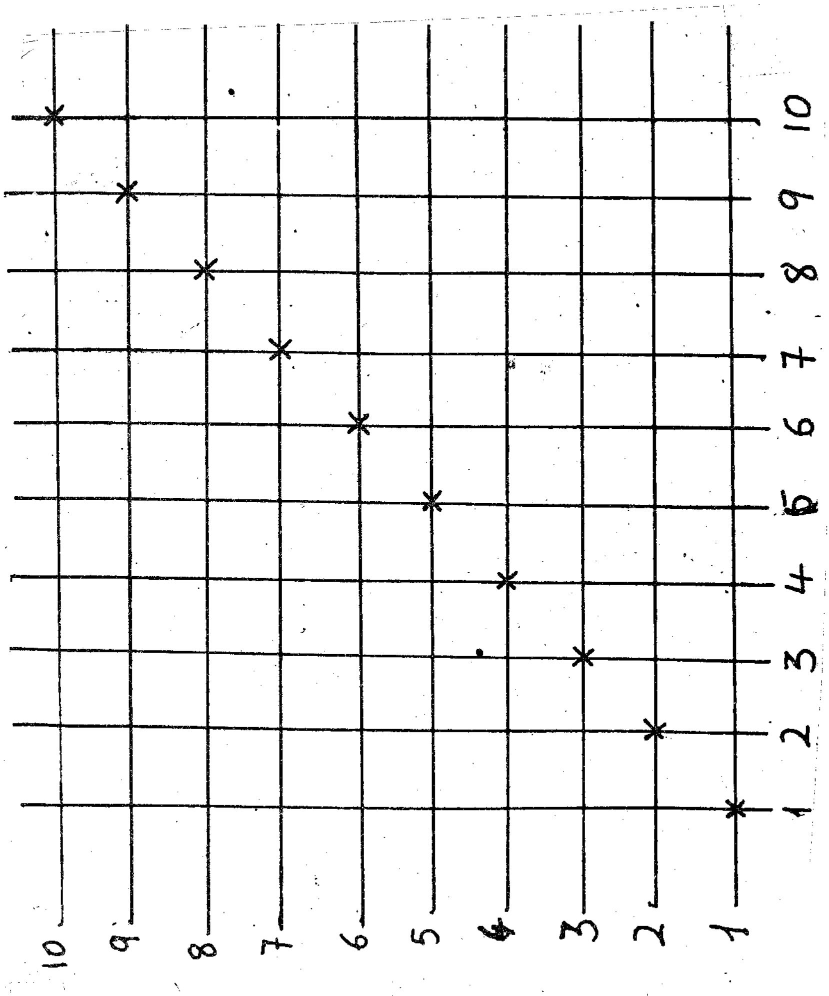
SH. 3

Baraha calaamadaysani waxay u taagan yihiin garaafka r. Ogow in baraha garaafka ee r u taagani ka mid yihiin baraha u taagan $B \times T$, markaa, waxan arkaynaa in r tahay hormo $B \times T$.

Tusaale 1:

Samee garaafka $r_1 = \{(x, y) \mid x, y \in B, x = y\}$, haddii $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ marka u horaysa tax kutirsaneyaasha r_1.

$r_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7)$
$(8, 8), (9, 9), (10, 10)\}$



SH. 9:

Baraha calaamada lahi waa garaafka r_1.

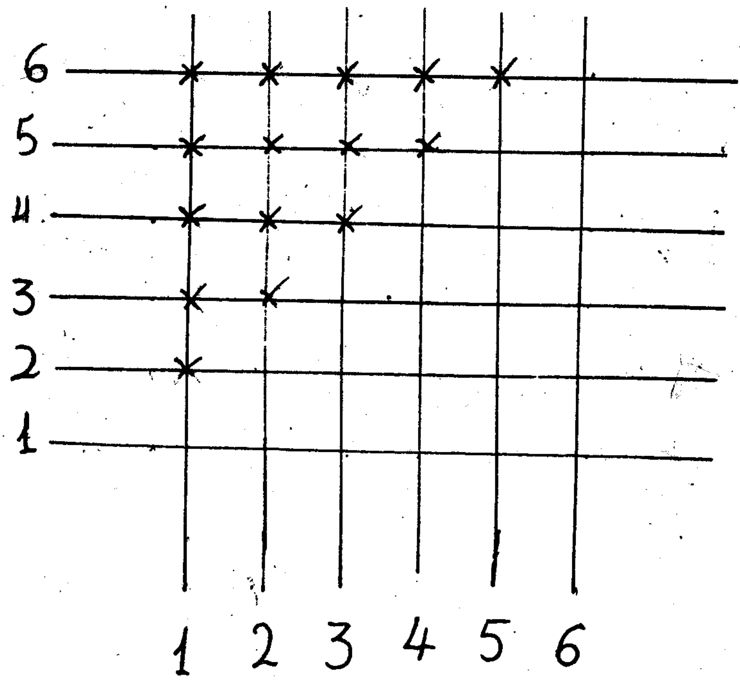
Tusaale 2:

Haddii $T = \{1, 2, 3, 4, 5, 6\}$

$r_3 = \{(x, y) \mid x, y \in T.\ y > x\}$, taswiir garaafka r_3.

$r_3 = \{(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),$
$(2,6),(3,4),(3,5),(3,6),(4,5),(4,6),(5,6)\}$

Baraha calaamadaysani waa garaafka r_3.



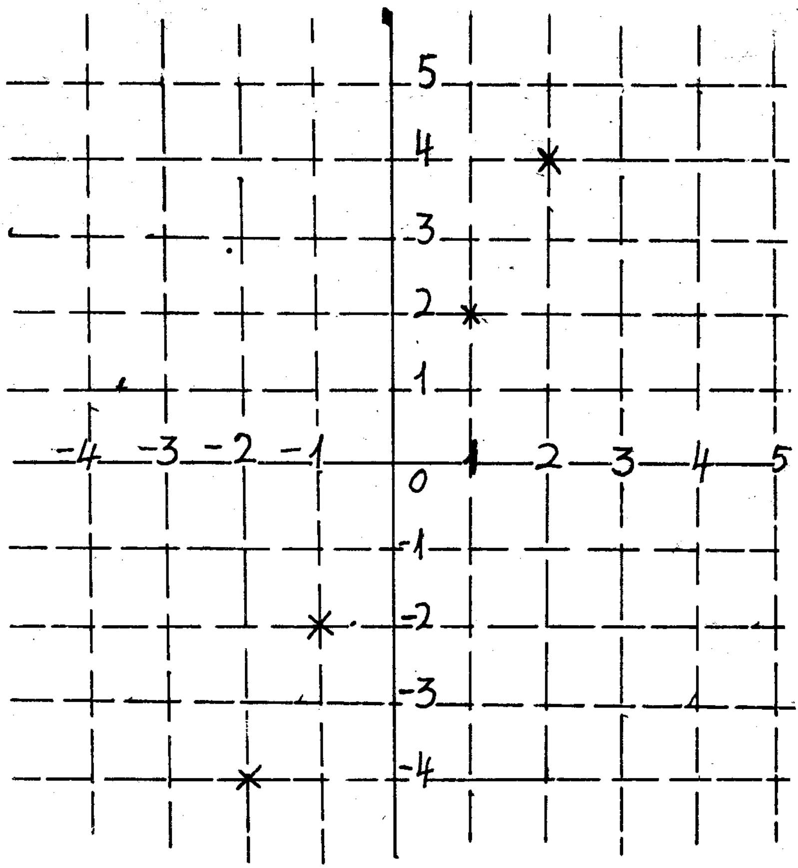
SH. 10:

Tusaale 3:

Haddii $M = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

$F = \{(x, y)\ x, y \in m, y = 2x\}$

Samee garaafka F?



Baraha calaamadaysani waa garaafka F.

F = {(– 2, – 4), (– 1, – 2), (0, 0), (1, 2), (2, 4)}

6. ISWEYDAAR XIRIIR:

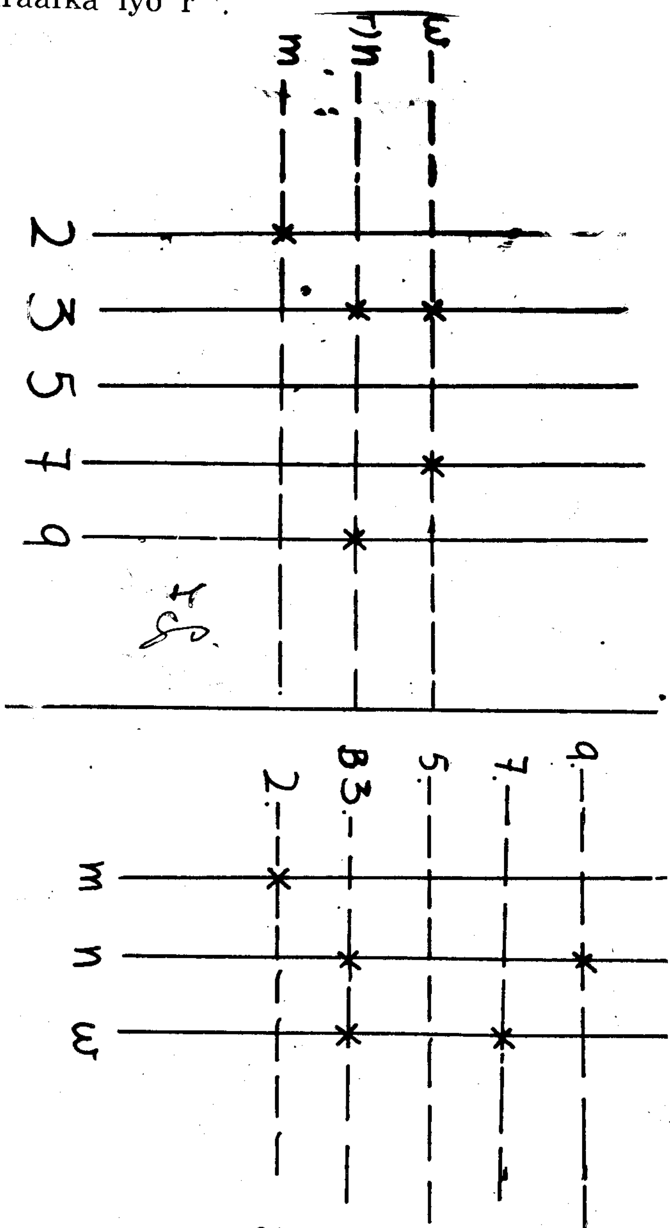
Haddii B iyo T ay yihiin ururro aan madhneyn; r tahay xiriir min B ilaa T, markaa isweydaarka r, oo loo qoro r^{-1} waa xiriir min T ilaa B ah, oo loo qeexo sidan: $r^{-1} = \{(y, x) \mid y \in T,\ x \in B\}$ isla markaa $(x, y) \in r\}$

Tusaale 1:

Haddii B = {2, 3, 5, 7, 9}, T = {m, n, w}
r_1 = {(2, m), (3, w), (9, n), (3, n), (7, w)}

markaa: $r_1^{-1} = \{(m, 2), (w, 3), (n, 9), (n, 3), (w, 7)\}$ ogow in r_1^{-1} ay tahay xiriir min T ilaa B ah, isla markaa $r_1^{-1} \subset T \times B$.

Shaxanada hoos ku yaal, waxay u kala taagan yihiin Garaafka iyo r^{-1}.



Tusaale 2:

markaa:

Haddii $S = \{(1,2), (3,2), (4,8), (5,9), (7,2)\}$

$S^{-1} = \{(2,1), (2,3), (8,4), (9,5), (2,7)\}$

Tusaalaha 1aad horaadka r_1, $H(r_1) = \{2, 3, 7, 9\}$, danbeedka r_1. $D(r_1) = \{m, n, w\}$. Waxan aragnaa in $H(r_1)$ uu hormo u yahay B isla markaa $D(r_1)$ uu hormo u yahay T.

Bal u fiirso $H(r_1^{-1}) = \{m, n, w\} = D(r_1)$. Sidaa oo kale $D(r_1^{-1}) = \{2, 3, 7, 9\} = H(r_1)$.

Tusaalaha 2aad, horaadka S, $H(S) = \{1, 3, 4, 5, 7\}$ danbeedka S, $D(S) = \{2, 8, 9\}$, laakiin horaadka S^{-1}, $H(S^{-1}) = \{2, 8, 9\}$ danbeedka S^{-1}, $D(S^{-1}) = \{1, 3, 4, 5, 7\}$ markaa, $H(S) = D(S^{-1})$, $D(S) = H(S^{-1})$.

Tusaalaha 3:

Haddii r tahay xiriir min B ilaa B ah:

$B = \{1, 2, 3, 4, 5, 6\}$ haddii r loo qeexo sidan:

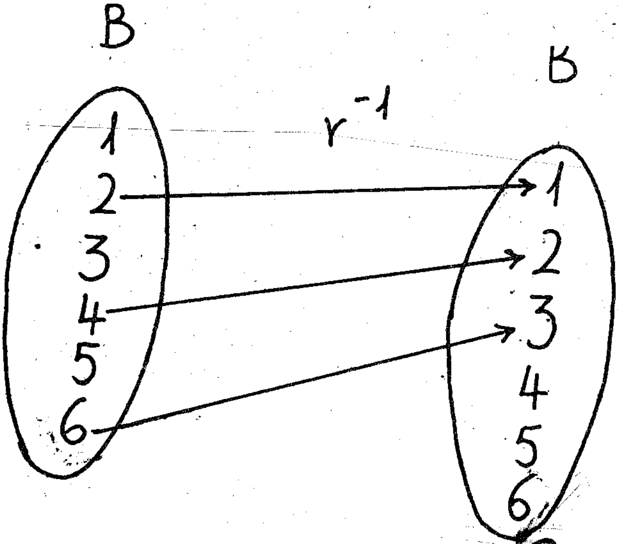
$r = \{(x, y) | x \in B, y \in B$, waliba $y = 2x\}$, raadi isweydaarka r.

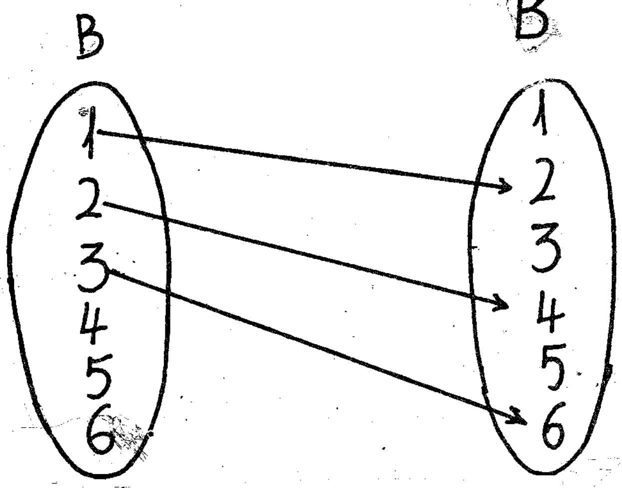
Furfuris:

Marka hore tax kutirsaneyaasha r:

$r = \{(1, 2), (2, 4), (3, 6)\}$ imika waxaan arkaynaa in $r^{-1} = \{(2, 1), (4, 2), (6, 3)\}$.

Shaxannada hoos ku yaal waa sawiro u taagan sida r iyo r^{-1} ay kutirsaneyaasha B ugu aaddiyaan kuwa B





Tusaalaha 4 :

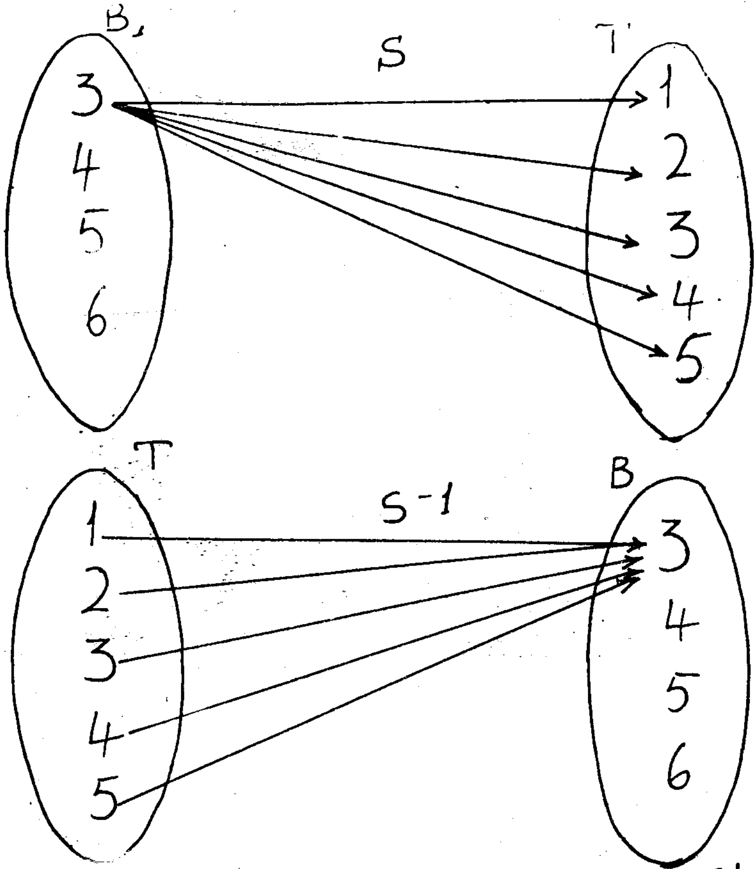
Haddii $B = \{3, 4, 5, 6\}$ $T = \{1, 2, 3, 4, 5\}$

$S = \{(x, y) \mid x \in B,\ y \in T,\ x = 3\}$.

markaa $S = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5)\}$

hadda $S^{-1} = \{(1, 3), (2, 3), (3, 3), (4, 3), (5, 3)\}$

shaxannada hoos ku yaal waxay muujinayaan sida S iyo S^{-1} ay isugu aaddiyaan kutirsaneyaasha B iyo kuwa T.



Sh.14

Layli:

1. Sheeg in xiriiryada S_1, S_2, S_3, S_4, S_5, ay yihiin xiriiryo B iyo in kale, B = {2, 4, 6, 8, 10, 12}.
   S_1 = {2, 4), (2, 2), (4, 2), (10, 10)}
   S_2 = {(2, 2), (1, 1), (3, 3), (4, 4), (5, 5)}
   S_3 = {(6, 8), (8, 6)}
   S_4 = {(3, 4), (2, 2), (4, 3)}
   S_5 = {(1, 10), (2, 10), (10, 10)}

2. Calaamadee dhibcaha garaafka $B \times B$ ee u taagan xiriiryada masalada 1aad.

3. Adigoo isku aaddinaya kutirsaneyaasha B iyo kuwa T, samee xiriiryada suuragalka ah ee min B ilaa T ah, $B = \{b, t, j\}$ $T = \{1, 2\}$.

4. Samee garaafka xiriiryada masalada 3aad.

5. Sheeg danbeedka iyo horaadka xiriir kasta oo soo socda.

b) $r = \{(1, 3), (2, 5), (3, 7)\}$

t) $s = \{(-3, 2), (-2, 4)\}$

j) $f = \{(-1, 1), (-1, 0), (-1, 1)\}$

x) $g = \{(1, 3), (2, 3), (3, 3), (4, 4), (5, 4)$
$(6, 4)\}$

kh) $h = \{(1, 2), (1, 3), (1, 4), (1, 5)\}$

6. Samee garaafka xiriiryada masalada 5aad.

7. Haddii $B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ samee garaafka r_1 iyo r_2
b) $r_1 = \{(x, y) \mid x, y \in B, y = x\}$
t) $r_2 = \{(x, y) \quad x, y \in B, y < x\}$.

8. Maxaa u dhexeeya $\{1, 2\}$, $\{(1, 2)\}$ $(1, 2)$ (9).

9. Tax kutirsaneysha xiriir kasta oo B, dabadeedna sheeg horaadkiisa iyo danbeedkiisa.

b) $r_1 = \{(x, y) \mid x = 3\}$;

$B = \{1, 2, 3\}$

t) $r_2 = \{(x, y) \mid y = 3\}$;

$B = \{1, 2, 3\}$

j) $r_3 = \{(x, y) \mid x + y = 1\}$;

$B = \{1, 2, 3, 4, 5, 6\}$

x) $r_4 = \{(x, y) \mid x + y = 1\}$;

$B = \{-1, 0, 1\}$

kh) $r_5 = \{(x, y) \mid x - y = 1\}$;
$B = \{-1, 0, 1\}$

d) $r_6 = \{(x, y) \mid y - 2x = 0\}$;
$B = \{1, 2, 3, \cdots, 12\}$

r) $r_7 = \{(x, y)\ 2y - x = 0\}$;

r) $r_7 = \{(x, y) \mid 2y - x = 0\}$;
$B = \{1, 2, 3, \cdots, 12\}$

s) $r_8 = \{(x, y) \mid y = x\}$;
$B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

sh) $r_9 = \{(x, y) \mid y = x^2\}$;
$B = \{1, 2, 3, \cdots, 20\}$

dh) $r_{10} = \{(x, y) \mid 2y = 3x\}$;
$B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

10. Raadi weydaarka xiriir kasta oo masalada qa- ad dabadeedna raadi horaadka iyo danbeedka xiriir kasta.

11. Raadi weydaarka xiriir kasta oo hoos ku yaal.

b) $s_1 = \{(1, 1), (2, 2), (3, 2), (3, 1)\}$

t) $s_2 = \{(1, 2), (1, 3), (2, 4)\}$

j) $s_3 = \{(1, 2), (2, 1), (3, 2), (2, 3)\}$

x) $s_4 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

kh) $s_5 = \{(1, 2), (1, 1), (1, 3), (1, 4)\}$

12. Haddii $B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3,$ samee garaafka $B \times B$, dabadeedna cal' dee baraha ku beegan xiriiryadan.

b) $f_1 = \{(x, y) \mid x, y \in B,\ y = x\}$

t) $f_2 = \{(x, y) \mid x, y \in B,\ y = 2x\}$

j) $f_3 = \{(x, y) \mid x, y \in B,\ y = x^2\}$

x) $f_4 = \{(x, y) \mid x, y \in B,\ y = x^4\}$

kh) $f_5 = \{(x, y) \mid x, y \in B,\ y = -1\}$

d) $f_6 = \{(x, y) \mid x, y \in B,\ y < x\}$

r) $f_7 = \{(x, y) \mid x, y \in B,\ y \geqslant x\}$

s) $f_8 = \{(x, y) \mid x, y \in B,\ -1 < y < 3\}$

sh) $f_9 = \{(x, y) \mid x, y \in B,$
$x > -1,\ y < 3,\ y > x\}$

dh) $f_{10} = \{(x, y) \mid x, y \in B,\ y = 3\}$

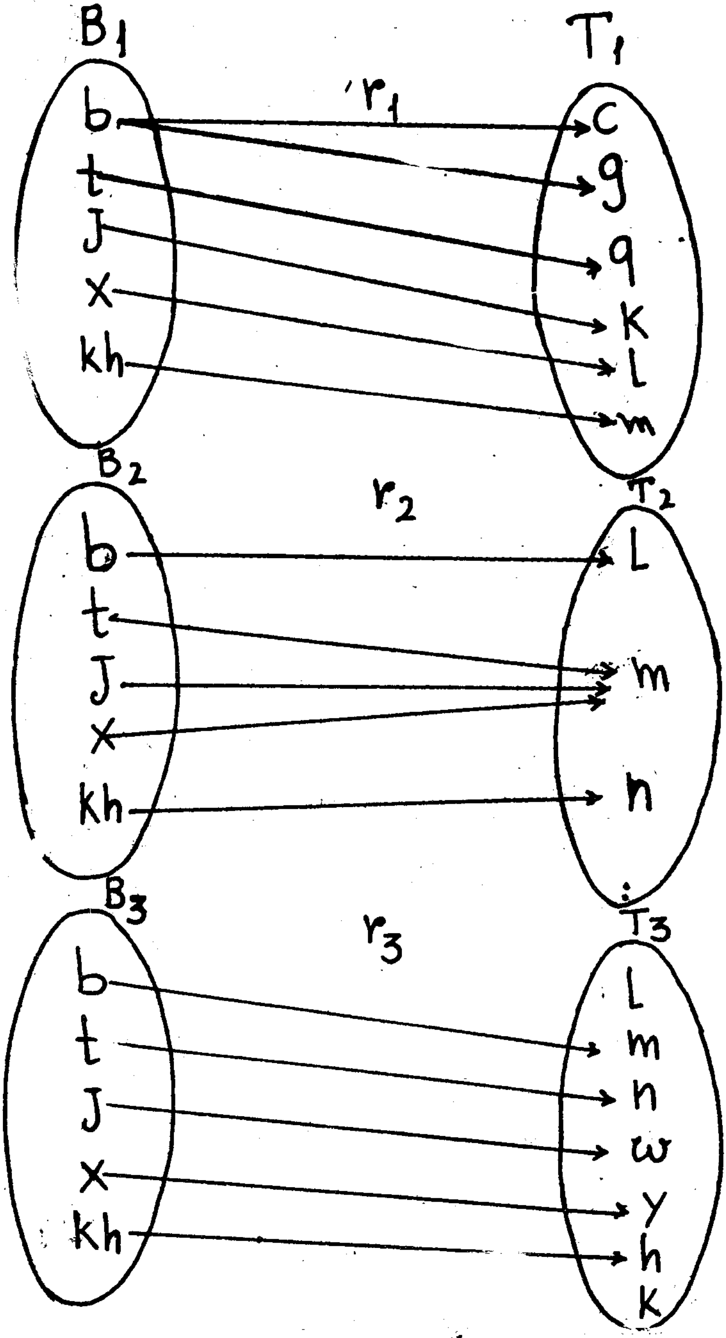
13. Tus sida $f_1, f_2, \cdots, f_{10}$, ay iskugu aaddiyaan ku-tirsaneyaasha B ee masalada 12.

14. Masalada 12, raadi horaadka iyo danbeedka xiriir kasta.

15. Waa maxay xiriir min Y ilaa H ahi?

FANSAARRO

U fiirso xiriiryada soo socda ee min B_i ilaa T_i marka $(i = 1, 2, 3, \cdots, 7)$.


B1
T1
r1
b
t
J
x
kh
c
g
q
k
l
m
B2
T2
r2
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t
J
x
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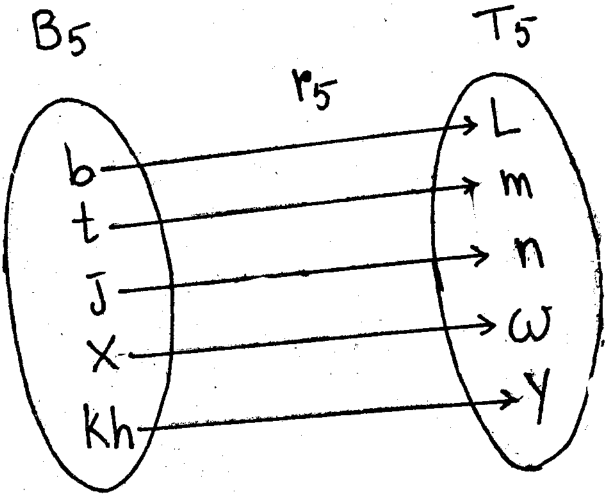
B5
T5
r5
b
t
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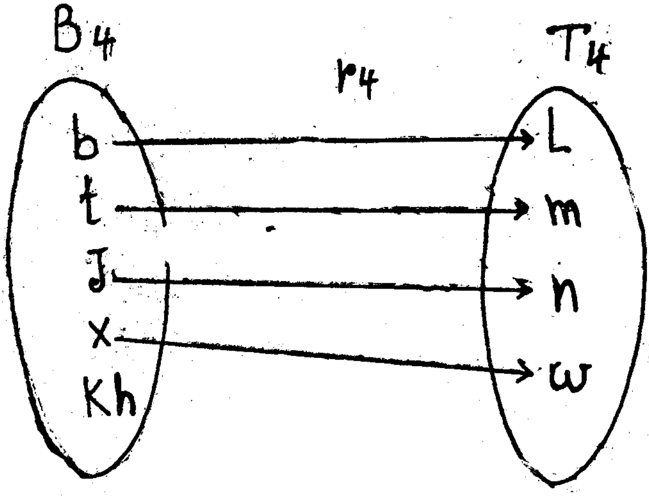
y


B4
T4
r4
b
t
J
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Kh
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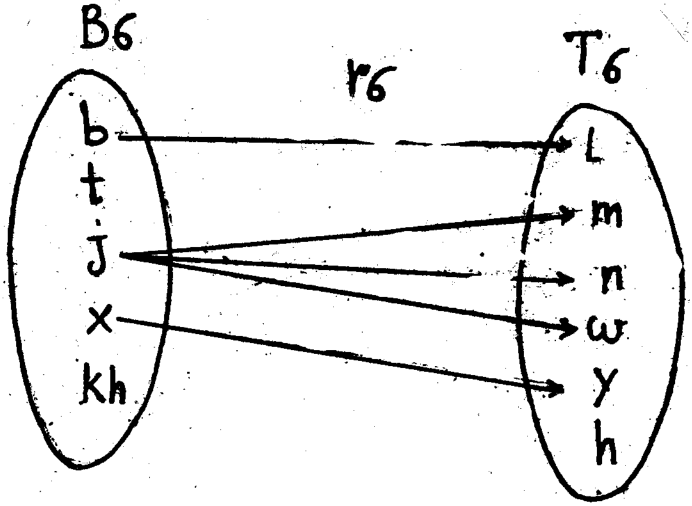
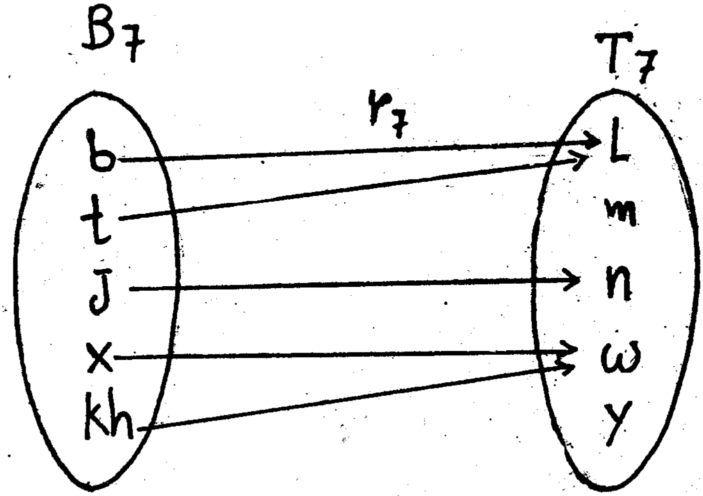




Xiriiryada r_2, r_3, r_5, iyo r_6 waa xiriiryo gaar ah oo xisaabta qaayo weyn ku leh. Bal u fiirso xiriiryadaa. Horaadka xiriir kasta iyo ururka B way isle'eg yihiin, isla markaa kutirsane kasta oo horaadku wuxuu ku aaddan yahay kutirsane keliya oo danbeedka.

Xiriiryada caynkaas ah waxa loo yaqaan fansaarro.

Qeexid:

Fansaarka f, oo min B ilaa T ahi, oo loo qoro

$f : B \longrightarrow T$, waa xiriir min B ilaa T ah, oo labadan sifo leh.

i) Horaadka f, $H(f) = B$
ii) Ma jiro kutirsane horaadka f oo ku aaddan wax ka badan, hal kutirsane oo danbeedka f ka mid ah.

Haddaba, haddii aan dib ugu noqono xiriiryadii hore ee $r_1, r_1, \cdots r_7$, waxan arkaynaa in r_1 uuna fansaar ahayn waayo waxa jira kutirsane horaadka r_1 oo ku aaddan labo kutirsane oo danbeedka r_1, taasi waa b, waxayna ku aaddan tahay e, iyo f oo danbeedka r_1. Kutirsane r_4 maaha fansaar waayo waxa jira kutirsane B_4, ee aan kutirsaneyn horaadka r_4, markaa, $H(r_4) = \{b, t, j, x\}$ mana le'eka B_4. B_6 maaha fansaar waayo waxa jira kutirsaneyaal B_6, sida t iyo kh oo aan kutirsaneyn horaadka r_6, $H(r_6)$, markaa $H(r_6) \neq B_6$. Weliba, waxa jira kutirsane horaadka r_6 oo ku aaddan in ka badan hal kutirsane oo danbeedka r_6.

Haddii aan taxno lammaaneyaasha horsan ee xiriiryada r_2, r_3, r_5, iyo r_7 waxaan arkaynaa in ayna jirin laba lammaane horsan oo xubnahooda horena isku mid yihiin, kuwooda danbena kala geddisan yihiin. Ugu danbeyn, bal aan is garab dhigno r_1 iyo r_2.

$$r_1 = \{(b, c), (b, f), (t, q), (j, k), (x, l), (kh, m)\}$$

$$r_2 = \{(b, l), (t, m), (j, m), (x, m), (kh, n)\}$$

Haddii aad eegtid lammaaneyaasha horsan ee r_1, waxaad arkaysaa in (b, c) iyo (b, f) ay xubnahooda hore isku mid yihiin kuwooda danbena kala geddisan yihiin. Laakiin ma jiraan lammaaneyaal horsan oo r_2 oo xubnahooda hore isku mid yihiin kuwooda danbena kala gaddisan yihiin, sidaa daraadeed, r_1 maaha fansaar; laakiin r_2 waa fansaar.

Haddii f tahay fansaar min B ilaa T ah, oo u qeexan sidan:

$f = \{(x,y) \mid x \in B,\ y \in T,\ y = x^2\}$ oo ay $B = \{1, 2, 3, 4, 5\}$ $T = \{1, 4, 9, 16, 25\}$, markaa waa la taxi karaa kutirsaneyaasha f, f waxay isku lammaaneysaa 1 iyo 1, 2 iyo 4, 3 iyo 9 iwm. Markaa, waxan qori karnaa $f(1) = 1$ ama f waxay 1 oo kutirsan horaadka ku lammaaneysaa 1 oo kutirsan danbeedka. Sidaas oo kale waxan qori karnaa $f(2) = 4$ ama f waxay 2 oo kutirsan horaadka ku lammaanaysaa 4 oo kutirsan danbeedka. Guud ahaan, waxan oran karnaa $f(x) = x^2$ ama f waxay x kasta oo kutirsan horaadka ku lammaaneysaa x^2 oo kutirsan danbeedka. $f(1)$, $f(2)$, $f(3)$, $f(4)$, $f(x)$ waxa loo akhriyaa f-da 1, f-da 2, f-da 3, f-da 4 iyo f-da x. $f(2)$ waa kutirsane horaadka f. Sidaas oo kale, $f(3)$ waa kutirsane danbeedka f oo ku lammaan x oo ah kutirsane horaadka f.

Imika, fansaarka f ee kor ku qeexan waxan u qori karnaa sidan:

$f = \{(x, f(x)) \mid f(x) = x^2,\ x \in B,\ f(x) \in T\}$. Waan soo gaabin karnaa oo waxaan u qori karnaa: $f(x) = x^2,\ x \in B$.

Badanaaba, summadda fansaarradu waa xaraf keliya, sida f, g, h ama f. Marmarka qaarkood, waxa la isku xiraa summadda fansaarka iyo doorsame u taagan kutirsaneyaasha horaadka si ay kuu siiyaan kutirsane danbeedka. Matalan: $f(x)$ waa kutirsanaha danbeedka f ee ku lammaan kutirsanaha horaadka x.

Waxa dhici kara in aad ogaan karto in xiriir u fansaar yahay iyo in kale adigoon tixin kutirsaneyaashiisa. Matalan: waxa laguu sheegay in f tahay xiriir min B ilaa T ah, isla markaa waxa laguu sheegay ururrada B, T iyo qeexda f, dabadeed waxa lagu weydiiyay in f fansaar tahay iyo in kale. Markaa labadan su'aalood ee soo socda jawaababahooda uunbaa kuu sheegi kara in ay f fansaar tahay iyo in kale.

1. Haddii halka x aan ku beddelno kutirsane kasta oo B, y ma noqonaysaa kutirsane T?

2. Haddii halka x aan ku beddelno kutirsane kasta oo B, y hal qiime oo keliya ma yeelanaysaa?.

Haddii jawaabta labadani su'aalood ay «haa» noqoto, markaa f waa fansaar min B ilaa T ah, haddii jawaabta mid ka mid ah, ama labadooduba ay maya noqoto markaa f ma aha fansaar min B ilaa T ah.

Tusaale ahaan, haddii $B = \{-3, -2, -1, 0, 1, 2, 3\}$ $T = \{0, 1, 4, 9\}$ $f_1 = \{(x, y) \mid x \in B, y \in T, y = x\}$
$f_2 = \{(x, y) \mid x \in B, y \in T, y = x^2\}$

Markaa f_1 maaha fansaar min B ilaa T ah waayo (i) marka aan x ku beddelno -2, $y = -2$, laakiin $-2 \notin T$. f_2 waa fansaar min B ilaa T ah waayo (i) markaa aan x ku beddelno kutirsane kasta oo B, y waxay noqonaysaa kutirsane T. Matalan, haddii $x = -2$, $y = (-2)^2 = 4$, $4 \in T$. (ii) marka aan x ku beddelno kutirsane kasta oo B, waxan helaynaa tiro keliya oo y u taagan.

Tusaale kale, haddii: $f_3 = \{(x, y) \mid x \in B, y \in T, y^2 = x\}$ isla markaa $B = \{0, 1, 4, 9, 16, 25, 36\}$ $T = \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$, f_3, ma fansaar baa? Bal aan eegno jawaabta labadii su'aalood ee ahaa. (i). Haddii halka x aan ku beddelno kutirsane kasta oo B. y ma noqonaysaa kutirsane T? Jawaabtu waa haa, matalan, haddii x ay tahay $4, y^2 = 4$ markaa y waa 2 ama -2. U fiirso in $2 \in T$ isla markaa in $-2 \in T$. (ii) haddii halka x aan ku beddelno kutirsane kasta oo B, y hal qiime oo keliya ma yeelanaysaa? Jawaabtu waa «maya» waayo marka $x = 16$, y waxay noqonaysaa 4 ama -4 markaa, f_3. maaha fansaar.

Layli:

1. Xiriiryadan B. kuwee baa fansaarro ah, haddii $B = \{1, 2, 3, 4, 5\}$

b) $= \{(1,2), (2,3), (3,4), (4,1), (5,2)\}$
t) $= \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$
j) $= \{(2,2), (4,3)\}$
x) $= \{(1,2), (3,4), (5,5), (1,5)\}$
kh) $= \{(1,2), (2,1), (3,1), (4,2), (5,2)\}$
d) $= \{(4,3), (5,2), (1,2), (2,2), (3,1)\}$
r) $= \{(1,2), (2,1), (1,3), (3,1), (1,4), (4,1)\}$
s) $= \{(1,1), (2,1), (3,1), (4,1), (5,1)\}$
sh) $= \{(2,1), (2,3), (2,2), (2,5), (2,4)\}$
dh) $= \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$

2. Haddii $B = \{1, 2, 3, \cdots, 8\}$, s_1, s_2, s_3, s_4 ay yihiin xiriiryo B, tax kutirsaneyaasha xiriir kasta, dabadeedna sheeg in uu fansaar yahay iyo in kale.

b) $s_1 = \{(x,y) \mid 2x - y = -1\}$

t) $s_2 = \{(x,y) \mid x = y\}$

j) $s_3 = \{(x,y) \mid y = 2x\}$

x) $s_4 = \{(x,y) \mid y = \frac{x}{2}\}$

3. Haddii r_1, r_2, r_3, r_4, r_5 ay yihiin xiriiryo N, oo N ay tahay tirsiimo, t. a. $N = \{1, 2, 3 \cdots\}$ sheeg in ay fansaarro min N ilaa N yihiin.

$r_1 = \{(x,y) \mid x = y\}$
$r_2 = \{(x,y) \mid y = 2x\}$

$r_3 = \{(x,y) \mid y = \frac{x}{2}\}$

$r_4 = \{(x,y) \mid x - 1 = 1\}$
$r_5 = \{(x,y) \mid x - y = 2\}$
$r_6 = \{(x,y) \mid x = 2\}$
$r_7 = \{(x,y) \mid y = 3\}$
$r_8 = \{(x,y) \mid y = x^2\}$
$r_9 = \{(x,y) \mid y > x\}$
$r_{10} = \{(x,y) \mid y = 3x\}$

4 Raadi f(3), f(1) iyo f(2).

$F = N \longrightarrow N$ weliba $N = \{1, 2, 3, \cdots\}$

b) $f = \{(x, y) \mid x, y \in N$, weliba $y = 2x\}$
t) $f = \{(x, y) \mid x, y \in N$, weliba $y = x^2\}$
j) $f = \{(x, y) \mid x, y \in N$, weliba $y = x^3\}$
x) $f = \{(x, y) \mid x, y \in N$, weliba $y = x\}$
kh) $f = \{(x, y) \mid x, y \in N$, weliba $2y = 3x\}$
d) $f = \{(x, y) \mid x, y \in N$, weliba $y = 3\}$
r) $f = \{(x, y) \mid x, y \in N$, weliba $-x + y = 2\}$
s) $f = \{(x, y) \mid x, y \in N$, weliba $x + y = 0\}$
sh) $f = \{(x, y) \mid x, y \in N$, weliba

$$y = \frac{1}{2}x + 3\}$$

dh) $f = \{(x, y) \mid x, y \in N$, weliba $y = x + 2\}$

5 Xiriiryada soo socda ee N, kuwee baa fansaarro ah:

b) $r_1 = \{(x, y) \mid x, y \in N, y = 2x\}$
t) $r_2 = \{(x, y) \mid x, y \in N, x + y = 0\}$
j) $r_3 = \{(x, y) \mid x, y \in N, y = x^2\}$
x) $r_4 = \{(x, y) \mid x, y \in N, x = y^2\}$
kh) $r_5 = \{(x, y) \mid x, y \in N, y = 2x + 1\}$

8. JAADADKA FANSAARADA

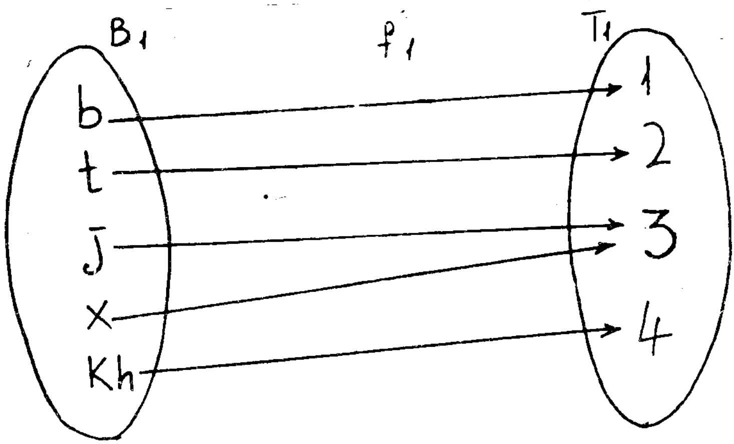
B — Fansaar mid - mid ah:

Qeex:

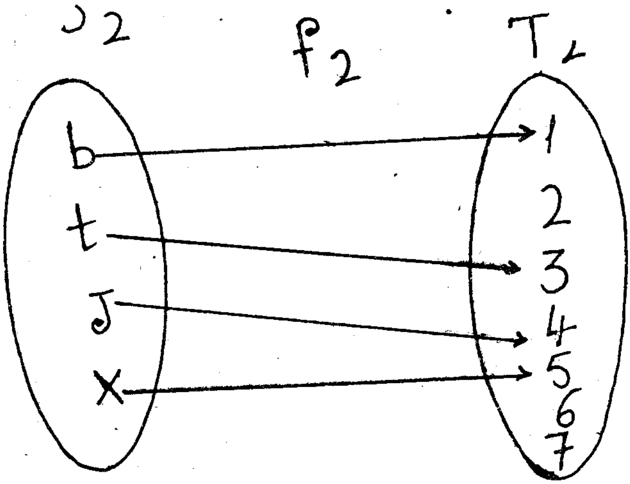
Haddii B iyo T ay yihiin ururro, f-na tahay fansaar min B ilaa T ah markaa (i) f waa fansaar mid-mid ah (oo loo soo gaabiyo 1 — 1) haddii ayna jirin labo lam maane horsan oo xubnahooda danbe isku mid yihiin kuwooda horena kala geddisan yihiin. (ii) f waa fansaar badi-mid ah, haddii ayna ahayn fansaar mid-mid ah.

Tusaalooyin:

Fansaarradan kuwee baa mid-mid ah (1 — 1), kuweebaana badi-mid ah.

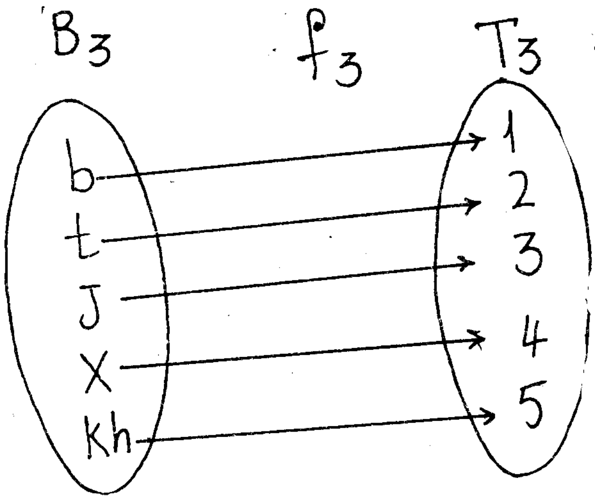


$f_1 = \{(b, 1), (t, 2), (j, 3), (x, 3), (kh, 4)\}$
f_1 maaha fansaar 1 — 1 ah, waayo waxa jira labo lammaane oo horsan, sida (j, 3) iyo (x, 3) oo xubnahooda danbe isku mid yihiin, kuwooda horena kala geddisan yihiin. f_1 waa fansaar badi-mid ah.



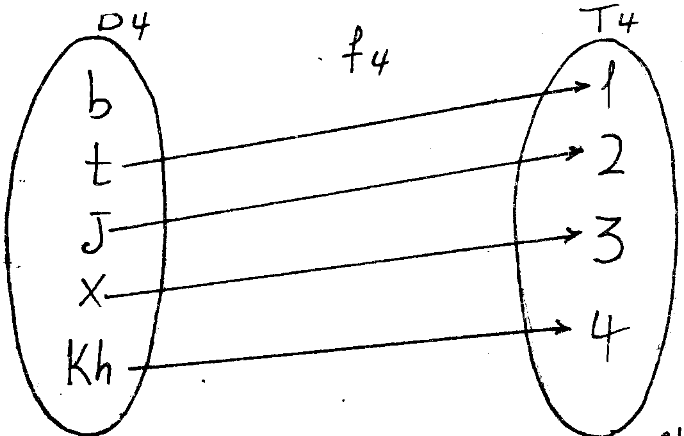
$f_2 = \{(b, 1), (t, 3), (j, 4), (x, 5)\}$

f_2 waa fansaar 1 — 1 ah, waayo ma jiraan laba lammaane horsan oo xubnahooda danbe isku mid yihiin, kuwooda horena kala geddisan yihiin.



$f_3 = \{ (b, 1), (t, 2), (j, 3), (x, 4), (kh, 5) \}$

f_3 waa fansaar 1 — 1 ah, waayo ma jiraan lammaaneyaal horsan oo xubnahooda danbe isku mid yihiin, kuwoodan horena kala geddisan yihiin.



Sh.10

$f_4 = \{ (t, 1), (j, 2), (x, 3), (kh, 4) \}$

f_4 maaha fansaar 1 – 1 ah, waayo f_4 maaha fansaar. Bal u fiirso horaadka f_4. $H(f_4) = \{t, j, x, kh\}$ markaa $H(f_4) \neq B_4$.

5. **Haddii $B = \{1, 2, 3, \cdots, 10\}$ $f = \{(x, y) \mid x, y \in B, y = x\}$ markaa la taxo kutirsaneyaasha f_5, waxan helaynaa in $f_5 = \{(1, 1), (2, 2), (3, 3), \cdots, (10, 10)\}$ f_5 waa fansaar waayo $H(f_5) = B$, mana jiraan labo lammaane horsan oo f_5 oo xubnahooda hore isku mid yihiin. kuwooda danbeña kala geddisan yihiin.**

Waliba f_5 waa 1 – 1, waayo ma jiraan labo lammaane horsan oo f_5 oo xubnahooda danbe isku mid yihiin. kuwooda horena kala geddisan yihiin.

6. Haddii $B = \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$ $T = \{0, 1, 4, 9, 16, 25, 36\}$

$$f_6: \{(x, y) \mid x \in B, y \in T, y = x^2\}$$

f_6 waa fansaar min B ilaa T ah waayo:

i) Haddii x ay noqoto kutirsane kasta oo B, markaa y waa kutirsane T.

ii) Haddii x ay noqoto kutirsane kasta oo B, markaa y waxay yeelanaysaa hal qiime oo keliya.

Haddaba. f_6 ma tahay 1 – 1? Bal aan taxno kutirsaneyaasha $f_6 = \{(-6, 36), (-5, 25), (-4, 16), (-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36)\}$ f_6 maaha 1 – 1, waayo lammaaneyaasha horsan ee $(-4, 16)$ iyo $(4, 16)$ xubnahooda danbe waa isku mid kuwooda horena way kala geddisan yihiin.

Innaga oo aan taxin kutirsaneyaasha fansaar, waan ogaan karnaa in ay 1 – 1 tahay iyo in kale, su'aashan jawaabteedaana ina siinaysa. Su'aashu waa: «Haddii y ay qaadato kutirsane kasta oo T, x hal qiime oo kaliya ma leedahay?» Jawaabtu haddii ay noqoto "haa" fansaarku waa mid-mid, haddii kalena maaha mid-mid.

Tusaale ahaan, f_5 waa 1 — 1 waayo aan halka y ku beddelno kutirsane kasta oo T, x hal qiima oo kaliya bay leedahay. Matalan, marka ay $y = 3$, x waxay le'eg tahay 3 marka y ay tahay 4, x-ina waa 4, iwm. F_6 maaha 1 — 1 waayo waxa dhici kara in la helo kutirsane T oo marka halka y lagu beddelo, siiya x laba qiime, matalan, marka 9 lagu beddelo halka y, x waxay yeelanaysaa labo qiime, kuwaas oo ah 3 ama — 3, markaa. f_6 maaha fansaar 1 — 1.

Layli:

1. Haddii $B = \{1, 2, 3, 4, 5\}$, oo $f_1, f_2, \cdots, f_5$ ay yihiin fansaarro B, sheeg kuwa mid-midka ah.

   b) $f_1 = \{(1, 1), (2, 1), (3, 2), (4, 2), (5, 2)\}$

   t) $f_2 = \{(x, y)\ x, y \in B, y = x\}$

   j) $f_3 = \{(x, y)\ y = 4\}$

   x) $f_4 = \{(1, 1), (2, 2), (3, 4), (4, 5), (5. 3)\}$

   kh) $f_5 = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$

2. Ka soo qaad in f_1, f_2, f_3, f_4 iyo f_5 ay yihiin xiriiryo min N ilaa N ah, $N = \{1, 2, 3, \cdots\}$. tus inay fansaarro yihiin iyo in ay 1 — 1 yihiin.

   x) $f_4 = \{(x, y)\ \ y = 8\}$

   t) $f_2 = \{(x, y) \mid y = x\}$

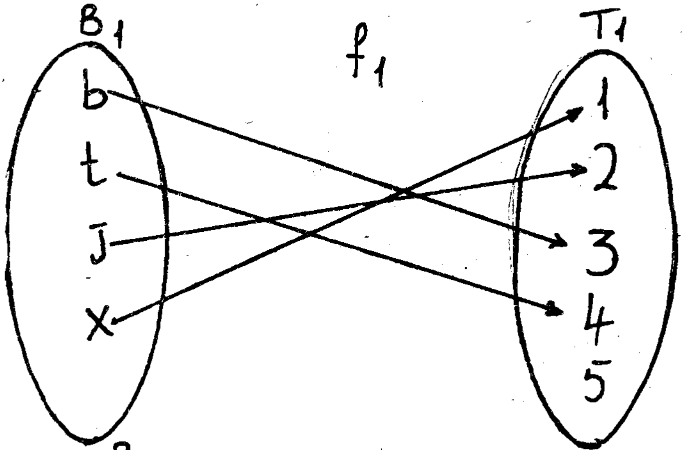
   kh) $f_5 = \{(x, y)\ \ y = 5x\}$

   j) $f_3 = \{(x, y)\ \ y = x^2\}$

   b) $f_1 = \{(x, y) \mid y = 2x\}$

3. Haddii f_1, f_2, f_3, f_4 iyo f_5 ee masalada 2aad ay yihiin xiriiryo Q, Q-na ay tahay ururka abyooneyaasha. Tus in ay fansaarro yihiin iyo in ay mid-mid yihiin.

b.



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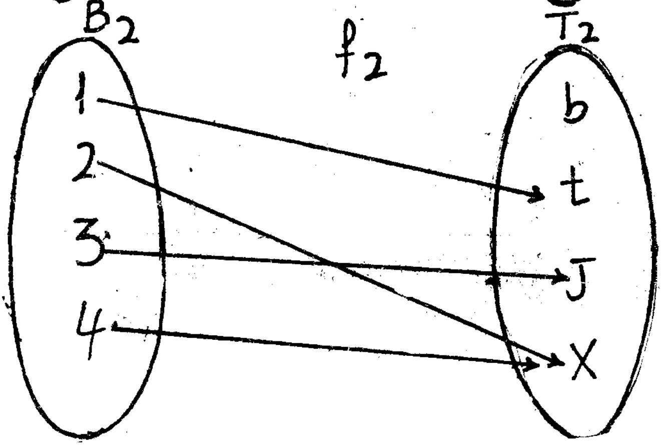

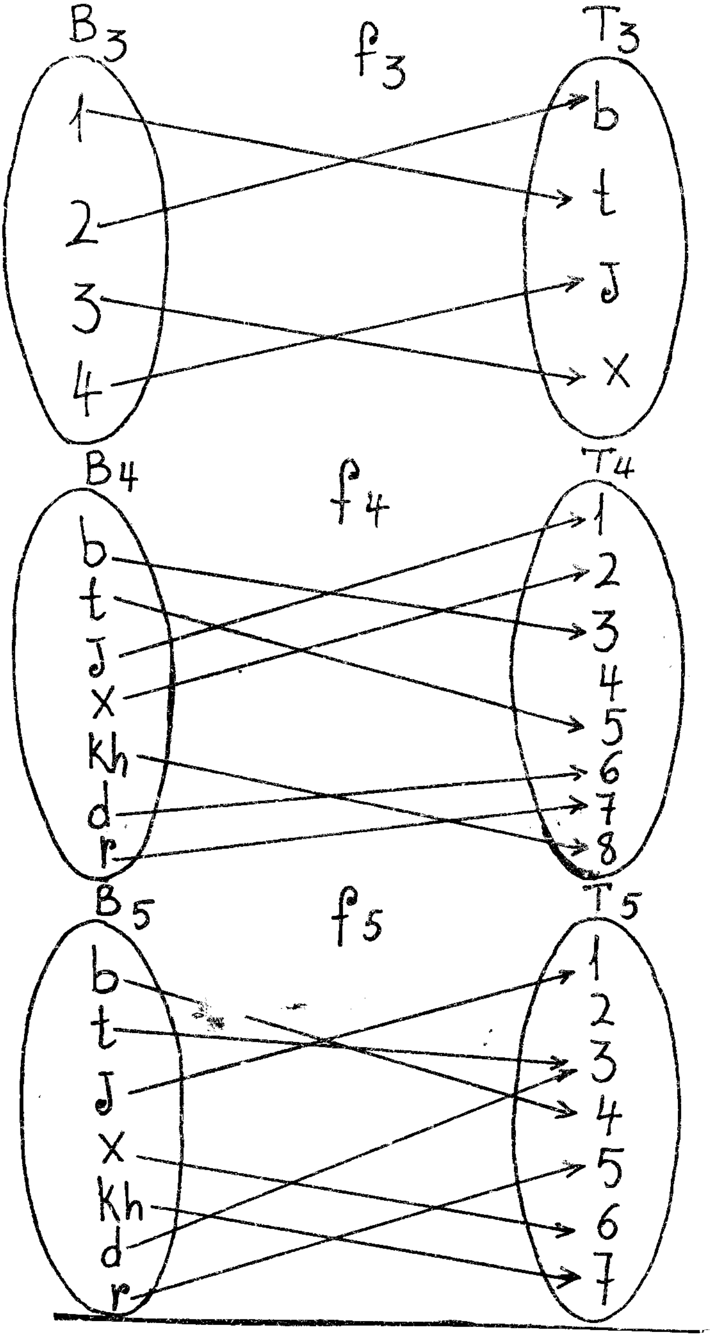
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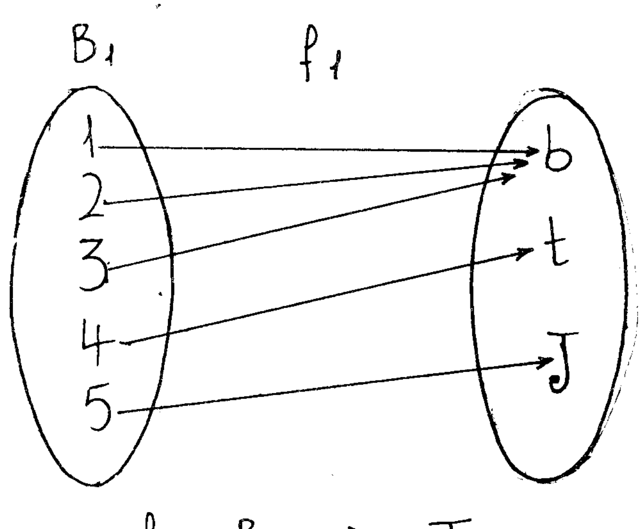
T. FANSAAR DHAMMAYS AH

Qeex:

Haddii B iyo T ay yihiin ururro, f-na tahay fansaar min B ilaa T ah, f waa fansaar dhammays ah oo min B ilaa T ah, haddii danbeedka f, D(f) = T. Waxa loo qoraa $f: B \xrightarrow{dm} T$.

Tusaalooyin:

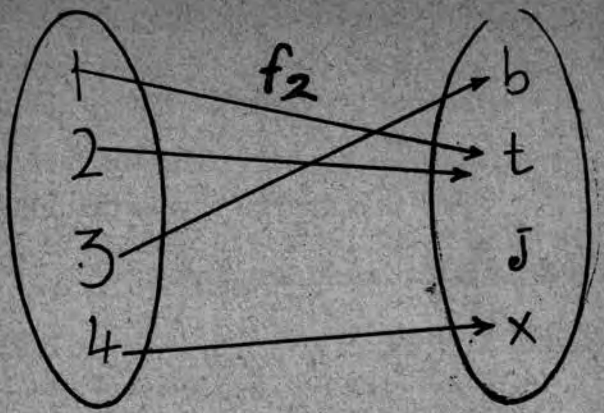
Sheeg in fansaarradan dhammays yihiin iyo in kale



$$f = \{(1,b), (2,b), (3,b), (4,t), (5,j)\}$$

$$D(f_1) = \{b, t, j\} = T_1$$

f_1 waa fansaar dhammays ah oo min B ilaa T ah.

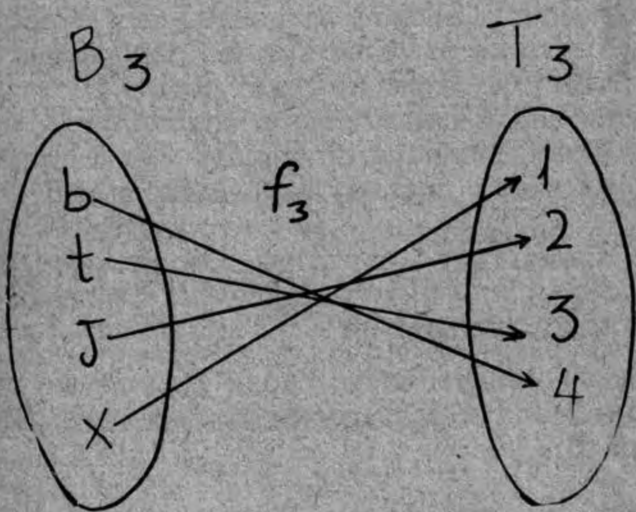


$f_2: B_2 \longrightarrow T_2$

$f_2 = \{(1, t), (2, t), (3, b), (4, x)\}$

$D(f_2) = \{b, t, x\} \neq T_2$

markaa f_2 maaha dhammays waayo danbeedkaa ma le-ekaa T_2



$D(f_3) = \{1, 2, 3, 4\} = T_3$

$f_3 = \{(b, 4), (t, 3), (j, 2), (x, 1)\}$

f_3 waa fansaar dhammays ah oo min B_3 ilaa T_3.

4. Haddii $B = \{1, 2, 3, 4, 5\}$; f_4 ay tahay fansaar min B ilaa B oo u qeexan sidan:

$f_4 = \{(x, y) \mid x, y \in B,\ y = x\}$, f_4 ma tahay dhammays? haddii aan taxno kutirsaneyaasha f_4 waxan helaynaa in

$$f_4 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$$

$D(f_4) = \{1, 2, 3, 4, 5\} = B$, marka f_4 waa dhammays.
$f_4 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$ $D(f_4) = \{1, 2, 3, 4, 5\} = B$, marka f_4 waa dhammays.

5. Haddii $B = \{1, 2, 3, 4, 5, 6\}$; $T = \{2, 4, 6, 8, 10, 12, 14, 16\}$ $f_5 = \{(x, y) \mid x \in B,\ y \in T,\ y = 2x\}$. F_5 fansaar dhammays ah ma tahay?

$f_5 = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12)\}$
$D(f_5) = \{2, 4, 6, 8, 10, 12\}$. Danbeedka f_5 iyo T isma leka, $D(f_5) \neq T$; markaa f_5 maaha fansaar dhammays ah oo min B ilaa T ah.

Adiga oo aan tixin kutirsaneyaasha fansaar, waad ogaan kartaa in ay dhammays tahay iyo in kale. Su'aashan soo socota jawaabteeda ayaa kuu sheegi karta dhammaysnimada fansaar, su'aashu waa: «Haddaii y ay qaadato kutirsane kasta oo T, x ma noqonaysaa kutirsane B? Haddii jawaabtu ay "haa" noqoto, fansaarku waa dhammays, haddii kalese maaha dhammays.
Tusaale ahaan, f_5 maaha dhammays waayo marka ay y tahay 14, x waxay noqonaysaa 7, laakiin $7 \notin B$.

6. Haddii f_6 ay tahay fansaar B_6, $B_6 = \{0, 1, 2, 3, 4, 5\}$ $f_6 = \{(x, y) \mid x, y \in B_6,\ y = 3\}$ markaa, f_6 ma tahay dhammays?

f_6 maaha dhammays waayo haddii y ay noqoto kutirsane B_6 oo aan 3 ahayn, x ma qeexna mana oran karno waa kutirsane B_6.

Haddii aan taxno kutirsaneyaasha f waxay noqonayaan sidan:

$f_6 = \{(0, 3), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3)\}$ markaa $D(f_6) = 3$. U fiirso $D(f_6) \neq B_6$.

Layli:

1. Haddii f_1, f_2, f_3 iyo f_5 ay yihiin fansaarro B, B-na ay tahay $\{1, 2, 3, 4, 5, 6\}$ sheeg in fansaarrada-ni yihiin dhammays iyo in kale.

   b) $f_1 = \{(x, y) \mid y = x\}$
   t) $f_2 = \{(x, y) \mid y = 1\}$
   j) $f_3 = \{(1, 1), (2, 3), (3, 2), (4, 5), (5, 4), (6, 6)\}$
   x) $f_4 = \{(1, 6), (2, 5), (3, 4), (4, 3) (5, 2), (6, 1)\}$
   kh) $f_5 = \{(x, y) \mid y - x = 0\}$

2. Haddii f_1, f_2, f_3, f_4 iyo f_5 ay yihiin fansaarro N, $N = 1, 2, 3, \cdots$, ma yihiin fansaarro dhammays ah.

   b) $f_1 = \{(x, y) \mid y = 2x\}$
   t) $f_2 = \{(x, y) \mid y = 7x\}$
   j) $f_3 = \{(x, y) \mid y - x = 1\}$
   x) $f_4 = \{(x y) \mid y = x^2\}$
   kh) $f_5 = \{(x, y) \mid y = x\}$

3. Haddii $f_1, f_2, \cdots, f_5$ ta ee masalada kowaad ay yihiin fansaarro I marka I ay tahay ururka abyoo-neyaasha, ma yihiin fansaarro dhammays ah?.

4. Fansaarradan hoos ku qoran ee ururka tirsii-mada N, kuweebaa dhammays ah.

   b) $f = \{(x, y) \mid x, y \in N, x = y - 1\}$
   t) $g = \{(x, y) \mid x, y \in N, x = y + 1\}$
   j) $h = \{(x, y) \mid x, y \in N, y = x^2 + 3\}$

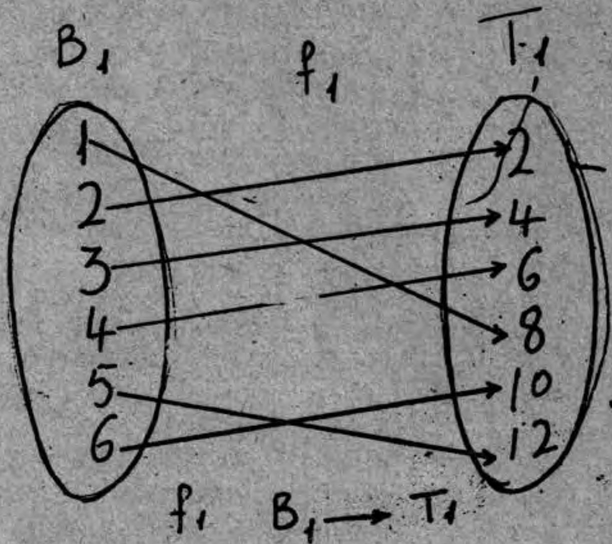
J. ISKUBEEGNAAN MID-MID AH

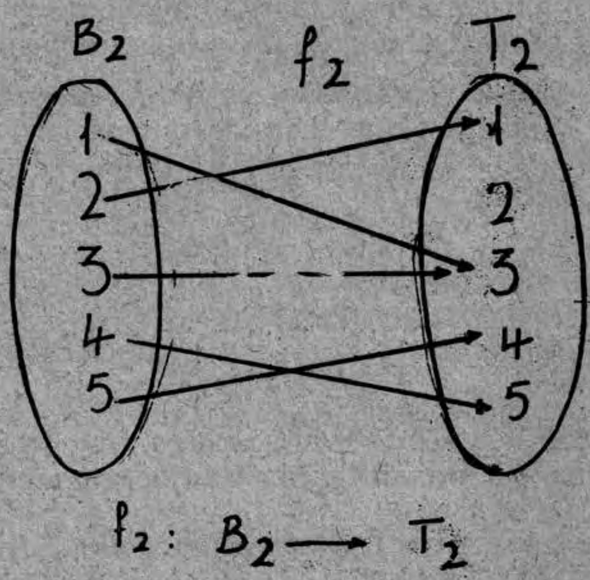
Qeex:

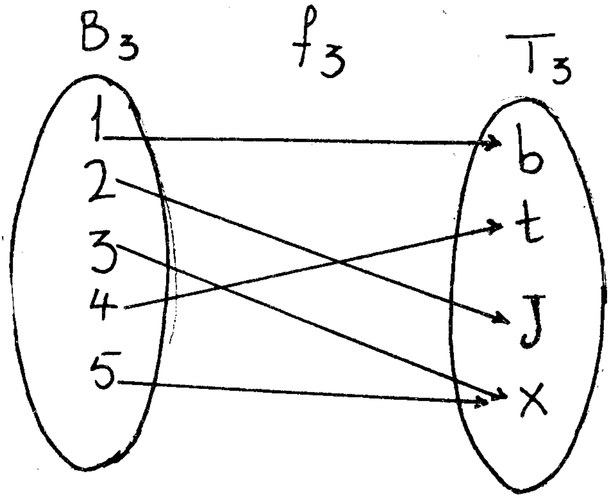
Haddii B iyo T ay yihiin ururro, f-na tahay fansaar min B ilaa T, f waxa la yiraa **Isku beegnaan mid-mid ah** oo ka dhexaysa B iyo T, waxaana loo qoraa $f : B \xrightarrow{dm} T$ haddii f tahay fansaar 1 — 1 ah, isla markaana tahay fansaar dhammays ah.

Layli:

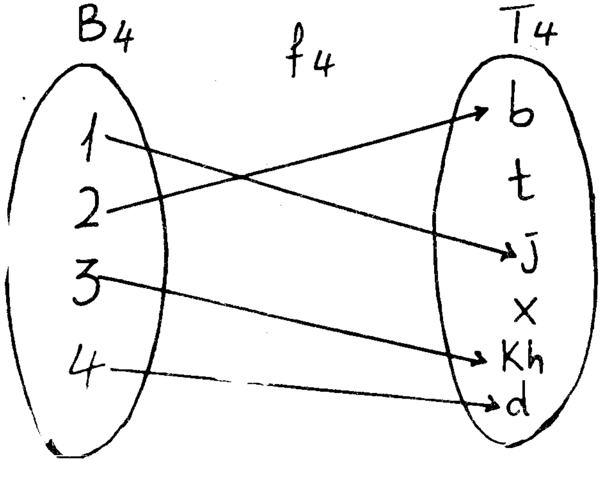
1. Fansaaradan soo socda, kuwee baa ah isku beegid mid-mid ah oo ka dhexaysa B iyo T.

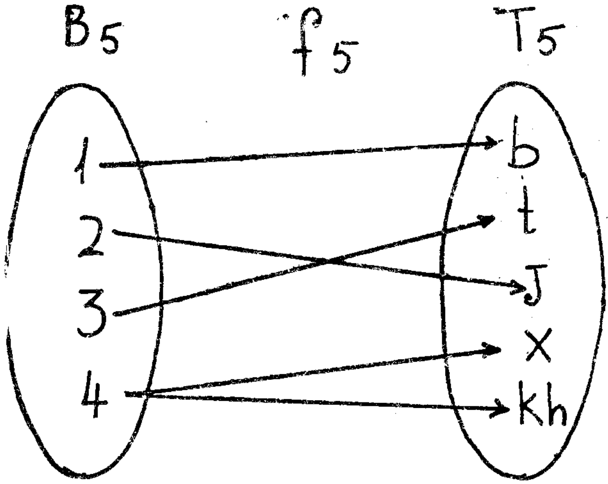






$f_3 : B_3 \longrightarrow T_3$

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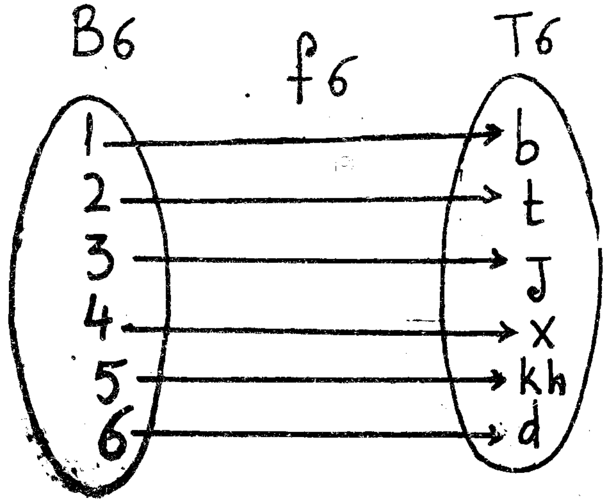
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Shaxannada 24aad:

2. Haddii B ay tahay $\{1, 2, 3, 4, 5, 6, 7, 8\}$, fansaarrada soo socda ma yihiin isku beegid mid-mid ah oo ka dhexaysa B_i iyo T_i.

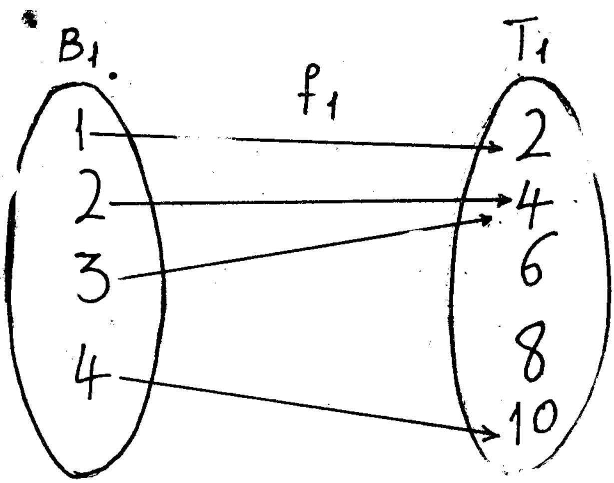
   b) $f = \{(x, y) \mid x, y \in B,\ y = x\}$

   t) $f = \{(x, y) \mid x, y \in B,\ y = 3\}$

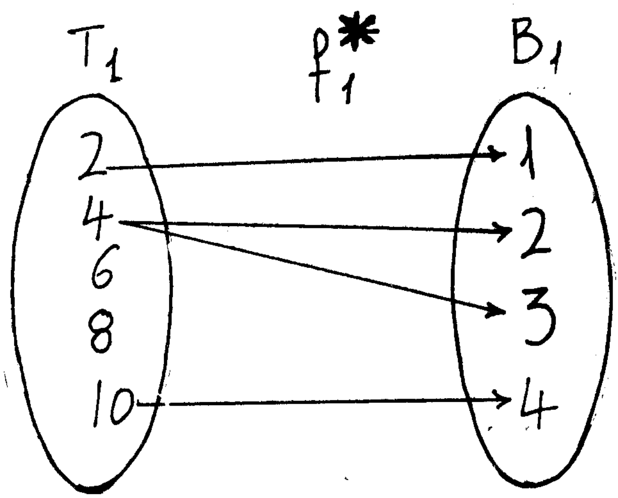
5. FANSAARRO ISWEYDAAR AH:

Haddii f ay tahay fansaar min B $\longrightarrow$ T ah, oo u qeexan sida soo socota:
$f = \{(x, y) \mid x \in B,\ y \in T\}$, weydaarka f waa xiriirka $f^{-1} = \{(y, x) \mid x \in B,\ y \in T,\ (x, y) \in f\}$ weydaarka fansaar wuxu noqon karaa fansaar laakiin taasi waajib maaha. Bal u fiirso tusaalooyinka soo socda:

Tusaale 1:



$f_1 = \{(1, 2), (2, 4), (3, 4), (4, 10)\}$



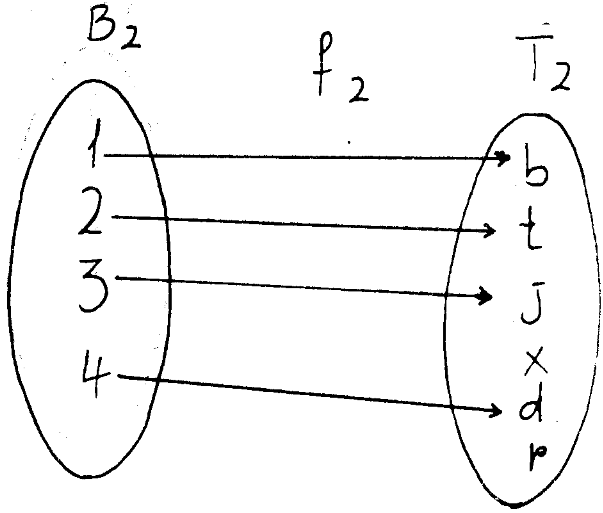
$f_1 = \{(2, 1), (4, 2), (4, 3), (10, 4)$

f_1 waa fansaar, la'akiin mid-mid maaha waayo waxa jira laba lammaane oo horsan sida: $(2, 4)$ iyo $(3, 4)$ oo xubnahooda danbe isku mid yihiin, kuwooda horena kala geddisan yihiin, weliba f_1 maaha dhammays waayo $D(f_1) = \{2, 4, 10\}$ mana le'eka T_1.

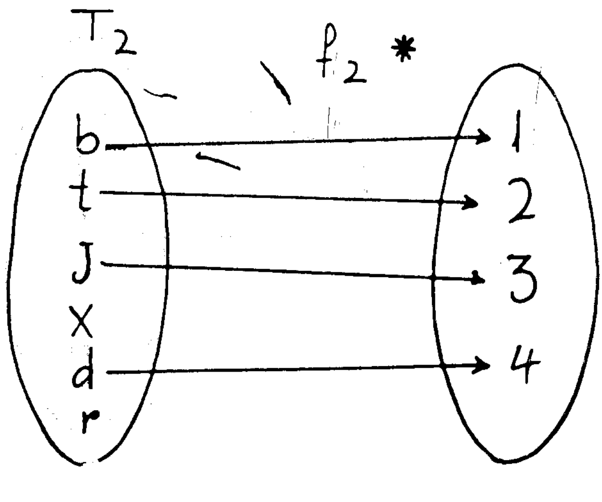
Bal u fiirso weydaarka f_1. Weydaarka f_1 t. a. f_1^{-1} ma aha fansaar waayo $H(f_1^{-1}) = \{2, 4, 10\}$ mana le'eka T_1, ama $(H(f_1^{-1}) \neq T_1$.

Tusaale 2:

1.



2.

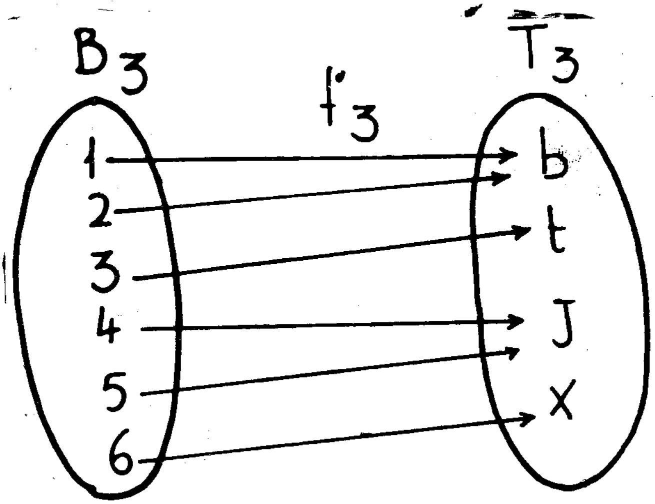


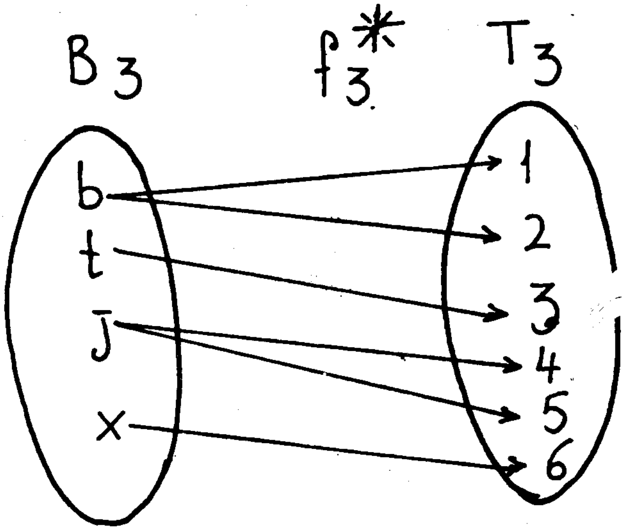
$f_2 = \{(1, b), (2, t), (3, j), (4, d)\}$

$f_2^{-1} = \{(b, 1), (t, 2), (j, 3), (d, 4)\}$

f_2 waa fansaar mid-mid ah laakiin f'_2 maaha dhammays F_2 waafansaar mid-mid ah; laakiin f_2 maaha dhammays waayo $D(f_2^{-1}) = \{b, t, d\} = T_2$. Bal u fiirso weydaarka f_2, t. a. f_2^{-1}. Weydaarka f_2 maaha fansaar waayo $H(f_2) =$ f_2, t. a. f'_2. Weydaarka f_2 maaha fansaar waayo $H(F'_2) =$ $\{b, t, j, x\}$. Marka, $H(f'_2) \neq T_2$.

Tusaale 3:



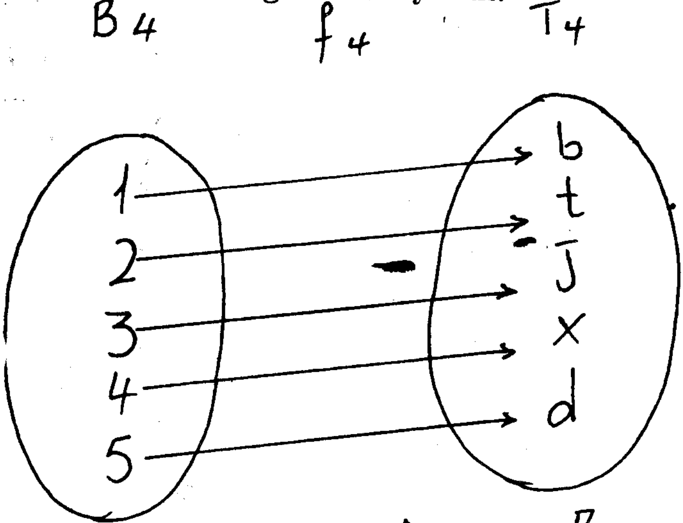


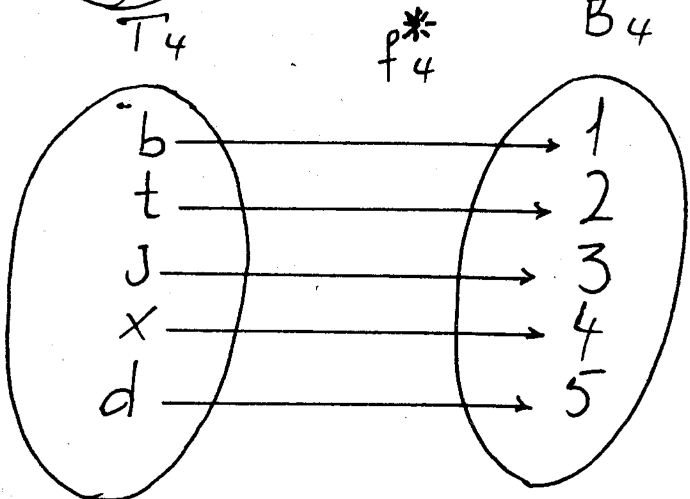
$F_3 =$ (1, b), (2, b), (3, t), (4, j), (5, j), (6, x)

$F_3^{-1} =$ (b, 1), (b, 2), (t, 3), (j, 4), (j, 5), (x, 6)

F_3 waa fansaar dhammays ah oo min B ilaa T ah, laakiin F_3^{-1} maaha 1 — 1, waayo waxa jira labo lammaane horsan sida: (4, j) iyo (5, j) oo xubnahooda danbe isku mid yihiin kuwooda horena kala geddisan yihiin.

Weydaarka f_3 oo ah f'_3 maaha fansaar waayo waxa jra labo lammaane horsan f_3' sida: (b, 1) iyo (b, 2) ama (j, 4) iyo (j, 5) oo xubnahooda hore isku mid yihiin kuwooda danbena kala geddisan yihiin.





$f_4 = \{(1, b), (2, t), (3, j), (4, x), (5, d)\}$

$f_4^{-1} = \{(b, 1), (t, 2), (j, 3), (x, 4), (d, 5)\}$

f_4 waa isku beegan mid-mid ah oo ka dhexeysa B_4 iyo T_4 waayo f_4 waa fansaar 1 - 1 ah, isla markaa dhammays.

Bal ka waran weydaarka f_4 oo ah f'_4 waa fansaar waayo $H(f'_4) = T_4$ isla markaa ma jiraan labo lammaane horsan oo f'_4 ee xubnahooda hore isku mid yihiin kuwooda danbena kala geddisan yihiin.

Haddii weydaarka fansaar uu fansaar yahay, sida f'_4 oo kale labada fansaar waxa la yiraa fansaarro isweydaar ah. Markaa f'_4 waxa loo qori f_4^{-1}, waxana loo akhri fansaar isweydaarka f_4.

Guud ahaan, haddii g ay tahay weydaarka fansaarka S, isla markaa ay tahay fansaar, g waxa loo qoraa S^{-1} waxaana loo akhriyaa fansaar isweydaarka S.

Afarta tusaale ee kor ku yaal waxay inoo sheegayaan astaantan fansaarrada iyo isweydaarkooda.

Haddii f ay tahay fansaar min B ilaa T ah, isla markaana ay tahay isku beegnaan mid-mid ah oo ka dhexaysa B iyo T, weydaarka f waa fansaar min T ilaa B ah, waxaana loo qoraa f^{-1}. Haddii f ayna ahayn isku beegnaan mid-mid ah, weydaaka f maaha fansaar.

1.3. XIRIIRYADA IYO FANSAARADA TIRADA MAANGALKA AH:

Ilaa hadda, waxan badnaaba ka hadlayney xiriiryo iyo fansaarro kooban. Fansaarro koobah waxan uga jeednaa kuwa lammaaneyaashooda horsan la tirin karo ama la tixi karo. Waxa jira fansaarro tiro beel ah. Haddaba, sidee baa loo ogaan karaa in xiriir tiro beel ihi uu fansaar yahay iyo in kale? Weliba, sidee baan u sameyn karnaa garaafka fansaar tiro beel ah? Inta aynaan u gelin jawaabta su'aashan, bal tusaalooyinkan soo socda u fiirso.

Tusaale 1:

$$f = \{(x, y) \mid x, y \in R,\ y = 2x\}$$

f ma tahay fansaar R, haddii R ay tahay ururka tirooyinka maangal ah?. Bal aan is weydiino labadii su'aalood ee fansaarka aan ku garan jirnay.

1) Haddii x ay noqoto tiro kasta oo maangal, y tiro maangal ah ma noqonaysaa? Jawaabtu waa "haa" waayo haddii x ay tahay tiro maangal ah tirada maangalka ahi waxay ku oodan tahay isku dhufashada.

2) Haddii x ay noqoto tiro kasta oo maangal ah, y hal qiime oo kaliya ma leedahay? Jawaabtu waa "haa" waayo waxan ognahay in tirada maangalka ahi ay ku oodan tahay isku dhufashada.

Noocyada Fansaarro:

Badanaaba, haddii lagu siiyo xiriir ama fansaar xubnaha lammaaneyaashiisa horsani ay yihiin tirooyin maangal ah, sida $f = \{(x, y) \mid x, y \in R,\ y = 2x\}$ waxa la qoraa isle'egta xiriirka ama fansaarka sifeynaysa oo keliya. Matalan: f waxan u qoraynaa $y = 2x$ ama $f(x) = 2x$, halkii aan ka qori lahayn $f = \{(x, y) \mid x, y \in R,\ y = 2x\}$ ama $f = \{(x, f(x)) \mid x \in R\ \ f(x) = 2x\}$

Haddii aan haysanno:

i) $f_1 = \{(x, y) \mid x, y \in R.\ y = x + 1\}$

ii) $f_2 = \{(x, y) \mid x, y \in R,\ y = x^2\}$ waxan u qoraynaa sidan: $f_1(x) = x + 1$ iyo (ii) $f_2(x) = x^2$

Tusaale 2:

$f(x) = x^2$ ma tahay fansaar R? $f(x) = x^2$ waxay la mid tahay $f = \{(x, y) \mid x, y \in R,\ y = x^2\}$ markaa, haddii ay x noqoto tiro kasta oo maangal ah, x^2 oo la mid ah $x \cdot x$, waa tiro maangal ah oo weliba madiya, markaa haddii x ay tahay tiro maangal ah, y waxay leedahay hal qiime oo keliya, isla markaa waa tiro maangal ah.

Tusaale 3:

$f = \{(x, y) \mid x, y \in R,\ y^2 = x\}$ ma tahay fansaar? Bal labadii su'aalood aan isweydiinno, haddii x ay noqoto tiro kasta oo maangal ah, y tiro maangal ah ma tahay? Jawaabtu waa "maya" waayo haddii x noqoto tiro taban, y maaha tiro maangal ah. Matalan: haddii $x = -3$, markaa $y^2 = -3$ $y = \sqrt{-3}$ laakiin $\sqrt{-3}$ maaha tiro maangal ah. Weliba haddii $x = 4$, $y^2 = 4$ markaa

$$y = 2 \text{ ama } y = -2$$

Ogow:

1. Fansaarrada tusaalaha 1aad iyo tusaalaha 2aad iyo xiriirka tusaalaha 3aad, mid wal kutirsaneyaashiisu waa tiro beel (∞)

2. Haddii aan lagu oran fansaarka f waa min ururkaas ilaa ururkaas, waxan u qaadanaynaa in f tahay min horaadka ilaa danbeedka, oo waliba kutirsaneyaashiisu ay yihiin lammaaneyaal horsan oo tirooyinka maangalka ah.

Si aan u ogaanno in f tahay fansaar iyo in kale, waxaan isweydiinaynaa hal su'aal oo keliya, taas oo ah, haddii x ay tahay tiro kasta oo horaadka kutirsan, y hal qiime oo keliya ma leedahay?

Haddii jawaabtu haa noqoto f waa fansaar. haddii kalena maaha fansaar.

Tusaale ahaan: $y^2 = x^2$ MAAHA fansaar, waayo marka ay x noqoto 2 $x^2 = 4$, marka $y^2 = x^2 = 4$. Markaa y waxay noqonaysaa 2 ama -2.

11. FANSAARRO CAADIYA:

Waxa jira fansaarro xisaabta kugu soo maray ama kugu soo mari doona oo loo yaqaan magacyo gaar ah. Bal qaar ka mid ah, aan sheegno.

B. FANSAARRO TIBXAALE:

Fansaar tibxaale f(x), waa fansaar sidan u qoran: $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_0$. a_i waa tirooyin lakab ah, x waa doorsoome, n waa abyoone togan.

Tusaalooyin ku saabsan fansaarro Tibxaale:

$$f(x) = 5x^3 - 7x^2 + 5$$

$$g(x) = 3x^{10} - 8x^5 + \frac{2}{3}x^4 - 6$$

$$h(x) = 5$$

Tus in f(x), g(x) iyo h(x) ay yihiin fansaarro. Bal aan mid-mid u qaadno. $f(x) = 5x^3 - 7x^2 + 5$ waxay la mid tahay $f = \{(x, f(x) \mid x, f(x) \in R,\ f(x) = 5x^3 - 7x^2 + 5\}$. Markaa haddii ay x tahay tiro maangal ah, f(x) waa tiro madiya oo maangal ah, waayo ururka tirooyinka maangalka ahi wuxu ku oodan yahay 4ta xisaab fale. Matalan haddii:

$x = 2, f(x) = f(2) = 5(2)^2 - 7(2)^2 + 5 = 40 - 28 + 5 = 17$. Ogow in f(2) ay tahay 17 oo keliya oo aan noqon karin tiro kale.

Sidaas oo kale, $g(x) = 3x^{10} - 8x^5 + \frac{2x^4}{3} - 6$ waxay la mid cahay $g = \{(x, y) \mid x, y \in R, \; y = 3x^{10} - 8x^5 + \frac{2x^4}{3} - 6\}$ g waa fansaar waayo, marka ay x noqoto tiro kasta oo maangal ah, y waa tiro madiya oo maangal ah, ee laba qiime ma yeelan karto.

$h(x) = 5$ waxa loo qori karaa $h = \{(x, y) \; x, y \in R, \; y = 5\}$ haddii x ay noqoto tiro kasta oo maangal ah, y qiima keliya bay leedahay, kaasoo ah 5. Markaa haa waa fansaar.

Sidii aan hore u sheegnay, $f(x)$, $j(x)$, $h(x)$ waa fansaarro tibxaale. Fansaarrada tibxaale qaarkood baa magacyo leh, sida $f(x) = a_1x + a_0$ oo la yiraa fansaar toosan, ama $f(x) = a_2x^2 + a_1x + a_0$ oo la yiraa fansaar saabley ah.

Tusaalooyin ku saabsan fansaar toosan:

i. $f(x) = \frac{2x}{3} + 4$

ii. $f(x) = 4x + 5$

iii. $f(x) = -\frac{5x}{3} + \frac{1}{2}$

iv. $f(x) = 6$

v. $f(x) = 8x.$

Tusaalooyin ku saabsan fansaarro saabley ah:

i. $f(x) = \frac{1x^2}{2} + 3x - \frac{3}{5}$

ii. $f(x) = x^2$

iii. $f(x) = 3x^2 + 8$

iv. $f(x) = -4x^2 - 8x + 7$

Tus fansaarradaa kor ku qoran in ay yihiin fansaarro R (R waa ururka tirooyinka maangalka ah).

FANSAAR JIBBAAR:

Haddii $f(x) = b^x$, oo b ay tahay tiro togan, x-na tahay tiro maangal ah, markaa f waxa la yiraa fansaar jibbaar.

Tusaalooyin fansaar jibbaar:

i. $f(x) = 2^x$

ii. $g(x) = (\frac{1}{2})^x$

iii. $m(x) = 4^x$

Bal aan eegno fansaarka $f(x) = 2^x$. Waxa loo qori karaa $f = \{(x, y)\ y = 2^x\}$

Haddii x ay noqoto tiro kasta oo maangal ah, 2^x waa tiro maangal ah. Waliba 2^x waa tiro togan. Matalan, haddii $x = 3$, $y = 2^3 = 8$. Haddii x ay tahay

$$-4,\ y = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}.$$

Ogow in $\sqrt{2}$ uu yahay xidid doorka laba-jibbaarka ee 2. Marka waxan araghaa in f tahay fansaar. Sidaas oo kale waxan ogaan karnaa in g iyo m ay yihiin fansaarro.

$$\text{Marka ay } x = \frac{1}{2},\ y = \frac{1}{2} = \sqrt{2}.$$

J. FANSAAR LOGARDAM:

Fansaarka logardamka, $L = \{(x, y) \mid x = b^y\}$ oo $b > 1$, x iyo y ay yihiin tirooyin maangal ah, L waa fansaar min $+R$ ilaa R ah.

Tusaalooyin:

i. $L_1 = \{(x, y) \mid 10^y = x\}$

ii. $L_2 = \{(x, y) \mid 2^y = x\}$

iii. $L_3 = \{(x, y) \mid \left(\frac{1}{2}\right)^y = x\}$

Bal L_1 aan soo qaadanno, haddii x ay noqoto tiro kasta oo maangal ah, y waa tiro maangal ah oo madi ah. Hawraartan halkan laguma caddayn karo ee bal aan tusaalooyin qaadanno. Matalan, haddii x tahay 100, x waxaan u qori karnaa 10^2, markaa y waa 2. Ma jirtaa tiro kale oo maangal ah oo marka 10 lagu jibbaaro ku siinaysa 100? Jawaabtu waa "maya" sidaas oo kale, haddii x ay tahay 100000, waxan u qori karnaa 10^5. Markaa y waa 5. Haddii x ay noqoto 0.0001 waxan u qoraynaa 10^{-4}, y-ina waa -4.

L_2 iyo L_3 naftooduna waa fansaarro logardam oo horaadkoodu yahay ururka tirooyinka maangalka ah ee togan, danbeedkooduna yahay, ururka tirooyinka maangalka ah.

X. FANSAARKA QIIMAHA SUGAN:

Fansaarka qiima sugan, f(x) waa fansaarka u qeexan sidan: $f = \{(x, y) \mid x, y \in R, \ y = b\,|x|\}$, b waa tiro maangal ah.

f ma tahay fansaar min R ilaa R ah?

1. Haddii x ay noqoto tiro kasta oo maangal ah, $|x|$ waa tiro maangal ah.

2. Haddii x ay tahay tiro kasta oo maangal ah, y oo ah $|x|$ waa tiro madiya oo maangal ah. Taa waxa inna siiya qeexda qiime sugan. Ogow in ayna jirin hal tiro oo labo qiime oo sugan leh. Markaa, mar haddii f ay oofinayso labadii shardi ee fansaarka, f waa fansaar min R ilaa R ah.

Layli:

1. Xiriiryadani hoos ku qoran ma yihiin fansaarro R?

   b) $f_1(x) = x^2 + 2x^2 + 5$

   t) $f_2(x) = -x^2$

   j) $f_3(x) = |x| + 3$

   x) $f_4(x) = 2^x$

   kh) $f_5(x) = 4$

   d) $f_6(x) = \frac{1}{2}x^3 + 3$

   r) $f_7(x) = 10^{-x}$

   s) $f_8 = \{(x, y) \mid x, y \in R,\ x = 8^y\}$

   sh) $f_9 = \{(x, y) \mid 10^y = x\}$

   dh) $f_{10} = \{(x, y) \mid y = 2\ |x|\}$

2. Haddii $B = \{1, 2, 3, 4, 5, 6\}$ $T = \{3, 6, 9, 12, 15, 18\}$

   $F = \{(x, y) \mid x \in B,\ y \in T,\ y = 3x\}$ raadi f^{-1}.

3. Raadi weydaarka fansaarradan, dabadeedna sheeg in ay fansaarro yihiin iyo in kale.

   b) $f_1 = \{(x, y) \mid y = x\}$

t) $f_2 = \{(x, y) \mid y = \frac{1}{2}x\}$

j) $f_3 = \{(x, y) \mid y = x^3\}$

x) $f_4 = \{(x, y) \mid y = x^2\}$

kh) $f_5 = \{(x, y) \mid y = |x|\}$

4. Fansaarrada masalada 1aad, kuwee baa:

i. mid-mid ah;

ii. dhammays ah;

iii. isku beegnaan mid-mid ah oo ka dhexaysa R ilaa R.

12. FANSAARRO LAKAB AH:

Fansaarka f oo loo qeexo $f = \{(x, y) \mid y = \frac{S(x)}{H(x)}\}$ ee S(x) iyo H(x) ay yihiin tibxaaleyaal x, H(x) ayna ahayn tibxaale eber, waxa la yiraa fansaar lakab ah. Horaadka f waa urur ka, dhammaan tirooyinka maangalka ah ee x marka $H(x) \neq 0$.

Fiiro:

Xannibaadda horaadka f la xannibay waa lagamaarmaan waayo summadda $\frac{S(x)}{H(x)}$ ma laha micno marka x ay ka dhigto H(x) eber. Waxa la yiraa fansaarku kama qeexna meelaha qiimaha x uu H(x) ka dhigo eber. Garaafka fansaarku iskama haysto barahaa, mana jirto bar garaafka ka mid ah oo ku beegan qiimaha x ee eber ka dhigga H(x).

Waajib maaha in had iyo jeer aan doorsamaheenna x ka dhiganno. Waxan qaadan karnaa y, r, d, iwm.

Markaa, $\frac{x-1}{2x-1}$ iyo $\frac{y-1}{2y-1}$ iyo $\frac{r-1}{2r-1}$ waxay wada qeexaan isla fansaar qura. Mar kasta horaadka fansaarku waa ururka tirooyinka maangalka ah oo dhan marka laga reebo $\frac{1}{2}$.

Tusaale:

Haddii $f(x) = \frac{3x-1}{x^2-9}$, markaa f waa fansaar lakab ah.

Haddii aan u qorno qormo urur, fansaarku wuxu noqonayaa sidan:

$$f = \{(x, y) \mid x, y \in R,\ y = \frac{3x-1}{x^2-9}\}\ \text{ama}$$

$$f = \{x, f(x) \mid f(x) = \frac{3x-1}{x^2-9}\}.$$ F waa fansaar lakab ah.

Horaadkeedu waa ururka tirooyinka maangalka ah oo laga reebay 3 iyo -3, waayo $x^2 - 9$ waa eber haddii $x = +3$ ama $x = -3$. Summadda $\frac{3x-1}{x^2-9}$ waxa la yiraa tibaax lakab.

Haddii x_1 ay tahay tiro horaadka ka mid ah, tibaaxdu waxay inna siinaysaa tirada danbeedka kutirsan, ee ku ıammaan tirada horaadka ama qiimaha x.

Tusaale ahaan. Haddii x = 5, waxan heleynaa in:

$$f(x) = f(5) = \frac{3 \times 5 - 1}{5^2 - 9} = \frac{14}{16} = \frac{7}{8} \text{ markaa } \frac{7}{8}$$

axa la yiraa qiimaha fansaarka marka ay x tahay 5.
Ogow in lammaanaha horsan ee $\left(5, \frac{7}{8}\right)$ uu yahay kutir-

Layli:

1. Tibaaxahan, kuwee baa fansaarro lakab ah qeexaya.

b) $\frac{6x - 8}{5x - 10}$ t) $\frac{a^2}{3a - 1}$

j) $y^2 + 6y + 1$ x) $\frac{2b^3 - 5b + 8}{3b}$

kh) $Log_2\ x$ d) 2^x

s) $\frac{1}{y + 3}$ sh) $\frac{3}{a} + \frac{7}{a}$

dh) $\frac{x^2 + 8x + 3}{5}$ g) $\sqrt{\frac{x - 2}{x + 2}}$

c) $3x$ f) $\frac{(a - 1)\ (a + 1)}{2a - 3}$

q) $\frac{3}{2}$

2. b) **Fansaar kasta oo tibxaale ma yahay fansaar lakab ah? Waayo?**

t) **Fansaar kasta oo lakab ah ma yahay fansaar tibxaale? Waayo?**

3. **Haddii fansaarka f. ee lakabka ah loo qeexo $f(x) = \frac{x^2 + 8}{x - 4}$ $(x \neq 4)$ raadi qiimaha fansaarka**

Markaa:

b) x ay tahay 6 j) x ay tahay 4

t) x ay tahay 1 x) x ay tahay 0

4. Sheeg lammaaneyaal horsan oo kutirsan fansaarka masalada 3aad.

5. Haddii lagu siiyo fansaarka $y = \frac{x}{x - 1}$, raadi qiimaha x ee, qiimaha fansaarka 6 ka dhiga.

6. Haddii $f(x) = \frac{x}{bx + 2}$, raadi b haddii $f(x) = 7$.

7. $f(x) = \frac{bx + 2}{x^2 + b}$. Raadi b haddii $(-2,1)$ ay tahay lammaane horsan oo fansaarka kutirsan.

8. $f(x) = \frac{bx + t}{x - 1}$. Raadi b iyo t haddii $f(-1) = 2.5$, isla markaa $f(2) = 1$.

9. b) Haddii bedka saddexagal u yahay 40 m^2. salkiisuna uu yahay x m. Qor tibaaxda sheegaysa joogga saddexagalka.

t) Wareegga layli waa 20 m, dhererkiisu waa x m. Qor tibaaxda sheegaysa bedka.

j) Nin baa daaq ku qodi kara x saacadood, yarkiisu wuxu u baahan yahay 2 saacadood oo dheeraad ah si u isla daaqqa u qodo. Sheeg inta daagga ka mid ah ee

i) ninku saacad ku qodi karo?

ii) yarku saacad ku qodi karo?

kh) Tareyn baa xawaarihiisu yahay x mayl saacaddiiba. Qor tibaaxda sheegaysa inta saacadood ee tareynku ku goyn karo 340 mayl?

d) Bedka labajibbaarane waa x mitir oo labajibbaarane waa x mitir oo labajibbaaran. Qor tibaaxda sheegaysa dhinaca labajibbaaranaha.

r) Tibaaxda b, t, j, kh iyo j, kuwee baa fansaarro qeexaya? Kuwee baa fansaarro lakab ah qeexaya?

13. HORAADKA IYO DANBEEDKA FANSAARKA LAKABKA AH:

Haddii $f = \left\{ (x,y) \mid y = \frac{S(x)}{H(x)} \right\}$, horaadka f waa dhammaan tirooyinka maangalka ah x, ee aan H(x) ka dhigin eber. Matalan, haddii $f(x) = \frac{1}{x}$, f(x) waa fansaar lakab ah oo $S(x) = 1$, $H(x) = x$. Marka $H(x) = 0$, waxan leenahay $x = 0$, markaa horaadka f, H(f) waa ururka, dhammaan tirooyinka maangalka ah ee aan ahayn eber, t. a. $H(f) = \{x \mid x \in R,\ x \neq 0\}$.

Tusaale 1:

Haddii $F = \left\{ (x, y) = \dfrac{3}{x - 3} \right\}$, raadi:

b) qiimaha x ee f ayna ku qeexnayn?
t) horaadka f?

Furfuris:

b) $S(x) = 3$, $H(x) = x - 3$ marka $H(x) = 0$, $x - 3 = 0$ ama $x = 3$.
$\therefore$ f ma qeexna marka ay x tahay 3.

t) $H(f) = \{x \mid x \in R, x \neq 3\}$. Haddii x ay tahay tiro kasta oo maangal ah oo aan 3 ahayn. y waa tiro maangal ah ama micnay leedahay, laakiin haddii x tahay 3,

$$y = \frac{3}{3 - 3} = \frac{3}{0}$$

Ogow in $\dfrac{3}{0}$ ayna ahayn tiro.

Tusaale 2:

Haddii $G = \left\{ (x, y) \mid y = \dfrac{1}{(x + 2)\ (x - 3)} \right\}$, raadi:

b) qiimaha x ee g ayna ka qeexnayn.

t) horaadka g.

Furfuris:

b) $S(x) = 1$, $H(x) = (x + 2)\ (x - 3)$. Marka $H(x) = 0$, $(x + 2)\ (x - 3) = 0$. Haddii aan furfurno isle'egtan $(x + 2)\ (x - 3) = 0$. waxan helaynaa in $x = -2$ ama 3. Marka, x ay tahay -2 ama 3, g ma qeexna.

t) horaadka g waa ururka, dhammaan tirooyinka maangalka ah x ee aan ahayn -2 iyo 3 $\therefore H(g) = \{x \mid x \in R, x, \neq -2\}$ isla markaa $x \neq 3$.

Tusaale 3:

Haddii $F = \left\{ (x, y) \mid y = \dfrac{3}{x-2} \right\}$, raadi danbeedka f.

Furfuris:

x ka dhig yeelaha jidka, waxan naqaan in $y = \dfrac{3}{x-2}$ markaa, sidan u shaqee.

$(x-2) \cdot y = \dfrac{3}{(x-2)} \cdot (x-2)$ labada dhinac ee isle'egta ku dhufo $(x-2)$.

$$\therefore xy - 2y = 3 \quad \therefore xy = 3 + 2y,$$

$$\therefore xy - 2y = 3 \quad \therefore xy = 3 + 2y,$$

$$x = \frac{3 + 2y}{y} = \frac{3}{y} + 2$$

x micna ma le, marka $y = 0$, markaa danbeedka f.
$D(f) = \{y \mid y \in R, y \neq 0\}$.

Tusaale 4:

Haddii $g = \left\{ (x, y) \mid x, y \in R, y = \dfrac{1}{x-9} \right\}$, raadi horaadka iyo danbeedka g.

$\dfrac{1}{x^2-9}$ y micno ma le marka $x^2 - 9 = 0$.

Haddii aan furfurno $x^2 - 9 = 0$, waxan helaynaa in x tahay 3 ama -3, waayo $x^2 = 9$, $x = \pm\sqrt{9}$. $\therefore$ horaadka g, $H(g) = \{x \mid x \in R, x \neq \pm 3\}$.

Si aan u helno danbeedka, waa in y aan ka dhignaa yeelaha jidka isle'egta $y = \dfrac{1}{x^2 - 9}$

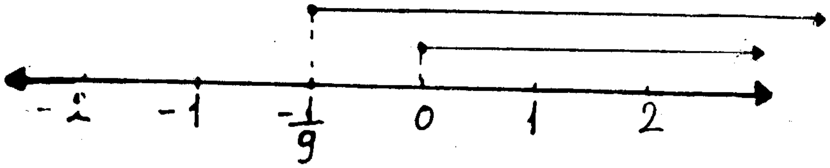
$$\therefore\ y = \frac{1}{x^2 - 9} \longrightarrow x^2 - 9 = \frac{1}{y}$$

$$\longrightarrow x^2 = \frac{1}{y} + 9 \longrightarrow x = \pm\sqrt{\frac{1}{y} + 9}$$

x waa tiro maangal ah haddii $\dfrac{1}{y} + 9 \geqslant 0$. Bal aan furfurno dheelliga $\dfrac{1}{y} + 9 \geqslant 0$ waxay malagelisaa in $\dfrac{1}{y} \geqslant -9$.

Xaaladda 1aad:

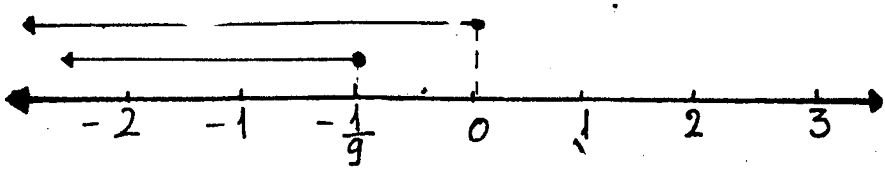
Haddii y ay tahay tiro togan, t·a $y > 0$, markaa $\dfrac{1}{y} \geq -9$ laakiin $1 \geq -9y \longrightarrow -\dfrac{1}{9} \leq y$.



Dhexyaalka $y > 0$ iyo $y \geq -\dfrac{1}{9}$ waa $y > 0$.

Xaaladda 2aad:

Haddii y ay tahay tiro taban, t·a, $y < 0$, markaa $\frac{1}{y} \ge -9$ waxay noqonaysaa $1 \ge -9y$. Laakiin, $1 \ge -9y$ ——, $-\frac{1}{9} \le -y$.



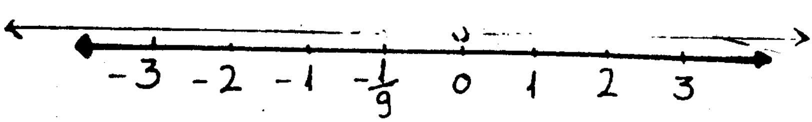
Dhexyaalka $y < 0$ iyo $y \le -$ — waa $y \le -\frac{1}{9}$

Xaaladda 3aad:

Haddii y tahay eber, t·a, $y = 0$. $\frac{1}{y}$ micna ma le, marka y eber ma noqon karto.

jadeeyada saddexda xaalo waa, $y > 0$ ama $y \le -\frac{1}{9}$

danbeedku wuxuu noqonayaa, $\left\{ y \mid y \in R,\ y > 0 \text{ ama } y \le -\frac{1}{9} \right\}$. Haddii aan ku muujinno danbeedka g xarriiqda tiro waxan helaynaa jawaabta hoos ku taal.



U fiirso. Haddii $y > 0$ ama $y \le -\frac{1}{9}$, x waa tiro maangal ah, haddii kale, x maaha tiro maangal ah. Tusaale ahaan, haddii:

$$y = -\frac{1}{45},\ x = \pm\sqrt{-45+9} = \pm\sqrt{-36}$$

Tusaale 5:

Haddii $M = \{(x, y) \mid xy - 4y = 1,\ x, y \in R\}$ raadi horaadka iyo danbeedka M.

Furfuris:

$$x^2y - 4y = 1 \longrightarrow y(x-4) = 1 \longrightarrow y = \frac{1}{x^2-4}$$

y micna ma le marka $x^2 - 4 = 0$ ama $x = \pm 2$. Markaa, $H(M) = \{x \mid x \in R,\ x \neq \pm 2\}$.

Si aan danbeedka u helno waa in x aan ka dhignaa yeelaha jidka.

$$\therefore\ x^2y - 4y = 1 \quad x^2y = 1 + 4y \quad x^2 = \frac{1+4y}{y}$$

$$\therefore\ x^2 = \frac{1}{y} + 4 \quad \therefore\ x = \pm\sqrt{\frac{1}{y}+4}$$

x waa tiro maangal ah haddii:

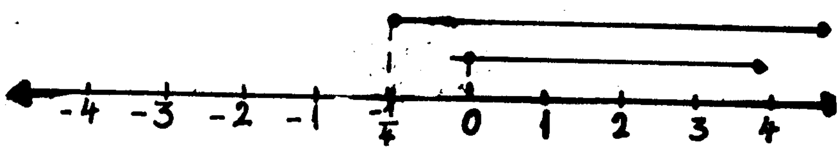
$$\frac{1}{y} + 4 \geq 0,\ \text{t·a}\ \frac{1}{y} \geq -4$$

Xaaladda 1aad:

Haddii $y > 0$, markaa $\frac{1}{y} \geq -4$,

$$\therefore\ 1 \geq -4y \longrightarrow -\frac{1}{4} \leq y$$

$\therefore\ y > 0$ isla markaa $y \geq -\frac{1}{4}$.

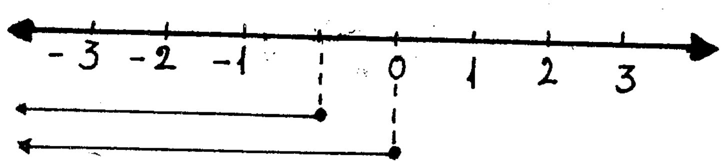


Dhexyaalka $y > 0$ iyo $y \geq -\frac{1}{4}$ waa $y > 0$

Xaaladda 2aad:

Haddii $y < 0$, marka $\frac{1}{y} \geq -4 \longrightarrow 1 \leq -4y$

$\longrightarrow -\frac{1}{4} \geq y \quad \therefore\ y < 0$ isla markaa $y \leq -\frac{1}{4}$.

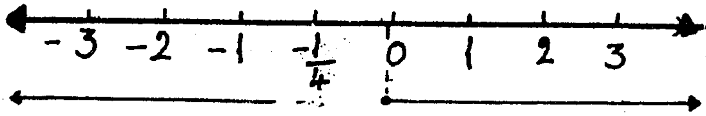


Dhextaalka $y < 0$ iyo $y \leq -\frac{1}{4}$ waa $y \leq -\frac{1}{4}$

Xaaladda 3aad:

Haddii y tahay eber. t·a, $y = 0$, markaa $\frac{1}{y}$ macna ma le, x na micna ma le. $\therefore$ jadeeyada 3 xaalo waxa weeyi $y > 0$ ama $y \leq -\frac{1}{4}$. $\therefore$ markaa, danbeedka M,

$$D(M) = \left\{ y \mid y \in R,\ y > 0 \text{ ama } y \leq -\frac{1}{4} \right\}.$$



Jaantuska kor ku taal waa garaafka D(M)

Layli:

1. Raadi qiimaha x ee fansaarku uuna ku qeex-
nayn.

b) $\left\{ (x, y) \mid y = \dfrac{1}{2x + 3} \right\}$

t) $\left\{ (x, y \mid y = \dfrac{1}{x^2 - 49} \right\}$

t) $\left\{ (x, y \quad y = \dfrac{1}{x^2 - 49} \right\}$

j) $\{ (x, y) \mid xy + y = 4 \}$

x) $\{ (x, y) \mid x^2y - 9y = 1 \}$

kh) $\left\{ (x, y) \mid y = \dfrac{x - 1}{x^2 + 5x + 4} \right\}$

d) $\left\{ (x, y) \mid y = \dfrac{1}{(x + 4)^2} \right\}$

r) $\left\{(x, y) \mid y = \dfrac{1}{(x-6)\ (x-7)}\right\}$

s) $\left\{(x, y) \mid y = \dfrac{x-5}{10^2 - 13x + 5}\right\}$

dh) $\left\{(x, y) \mid y = \dfrac{1}{x}\right\}$

sh) $\left\{(x, y) \mid y = \dfrac{5}{x(x-2)}\right\}$

2. Raadi horaadka iyo danbeedka fansaar kasta oo hoos ku yaal.

b) $\left\{(x, y) \mid y = \dfrac{1}{x-2}\right\}$

t) $\{(x, y)\ yx + y = -3\}$

$\{(x, y) \mid yx + y = -3\}$

j) $\left\{(x, y) \mid y = \dfrac{1}{2x-3}\right\}$

ẋ) $\left\{(x, y) \mid y = \dfrac{3}{2x}\right\}$

kh) $\{(x, y) \mid 4y + xy = 20\}$

d) $\{(x, y) \mid 2y + 3x = 5\}$

r) $\left\{(x, y) \mid \dfrac{5y + 3x^2 + 5x}{8} = \dfrac{1}{2}\right\}$

s) $\left\{(x, y) \mid y = \dfrac{4}{x^2 - 81}\right\}$

sh) $\left\{(x, y) \mid y = \frac{x}{2} + 3\right\}$

dh) $\left\{(x, y) \mid y = 2 + \frac{1}{x}\right\}$

14. ISLE'EG KU SAABSAN TIBAAXO LAKAB AH:

Waxan niri tibaaxda u qoran sansaanka $\frac{S(x)}{H(x)}$ oo

S(x) iyo H(x) ay yihiin tibxaaleyaal x, waxa la yiraa tibaax lakab ah, hadda, bal aan eegno sida looga shaqeey isle'eg ku saabsan tibaaxo lakab ah.

Tusaale 1:

Furfur isle'egtan tibaaxaha lakabka ah le:

$$\frac{3}{x} + \frac{5}{x+1} = \frac{7}{4}$$

Dh. Y. W. hooseyaasha oo dhan waa 4x (x + 1). Markaa, jajabyada oo dhan isla hooseeye ka dhig.

$$\frac{3}{x} \cdot \frac{4x(x+1)}{4x(x+1)} + \frac{5}{x+1} \cdot \frac{4x(x+1)}{4x(x+1)} = \frac{7}{4} \frac{4x(x+1)}{4x(x+1)}$$

$$\longrightarrow \frac{12(x+1)}{4x(x+1)} + \frac{20x}{4x(x+1)} = \frac{7x(x+1)}{4x(x+1)}$$

Labada dhinac ee isle'egta waxad ku dhufataa 4x(x + 1). (Ogow in $4x(x+1) \neq 0$, t.a, $x \neq 0$, isla markaa $x \neq -1$) waxan helaynaa isle'egtan $12(x+1) + 20x = 7x(x+1)$

$$12x + 12 + 20x = 7x^2 + 7x.$$

Haddaba waa in aan furfurnaa isle'egtan saableyga ah (xusuuso xannibaadda horaadka:

$$x \neq 0, \text{ isla markaa } x \neq -1).$$

$$\therefore\ 12x + 12 + 20 = x \quad 7x^2 + 7x$$

$$0 = 7x^2 + 7x - 12x - 12 - 20x.$$

$$7x^2 - 25x - 12 = 0$$

$$(7x + 3)\ (x - 4) = 0$$

$$\therefore\ (7x + 3) = 0 \text{ ama } (x - 4) = 0$$

$$\therefore\ x = - \text{—} \text{ ama } x = 4.$$

$$\therefore \text{ Urur furfurista isle'egta waa: } \left\{-\frac{3}{7}, 4\right\}$$

Tusaale:

$$\text{Furfur isle'egtan, } \frac{x^2 + x + 2}{2x - 2} = \frac{2x}{x - 1}$$

U fiirso:

Haddii $2x - 2 = 0$ ama $x - 1 = 0$, isle'egtu micna ma le, markaa $x \neq +1$.
Labada dhinac ee isle'egta waxaad ku dhufataa $2(x - 1)$:

$$\therefore\ \frac{x^2 + x + 2}{2x - 2} \cdot 2(x - 1) = \frac{2x}{x - 1} \cdot 2(x - 1)$$

$$\frac{x^2 + x + 2}{2(x - 1)} \cdot 2(x - 1) = \frac{2x}{x - 1} \cdot 2(x - 1)$$

$$x^2 + x + 2 = 4x$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)\ (x - 1) = 0$$

$$\therefore\ x = 2 \text{ ama } x = 1 \ (x \neq 1)$$

Mar haddii 1 uuna kutirsanayn horaadka, urur rumeedka isle'egta waa {2}.

Layli:

1. Raadi urur rumeedka $\frac{x}{x-4} = 6$ (x waa abyoone ka weyn)

2. Raadi urur rumeedka $\frac{x}{2x+28} = \frac{1}{9}$ (x waa abyoone togan)

3. Raadi urur rumeedka $\frac{60}{x} + \frac{72}{2x} = 8$ (x waa tiro togan)

4. Raadi urur rumeedka $\frac{25}{x} - \frac{25}{2x} = \frac{1}{4}$ $(x > 0)$

5. Raadi urur rumeedka $2(x + \frac{12}{x}) = 4$ $(x > 0)$

6. Raadi urur rumeedka $\frac{1}{b-4} + b^2 - 16 = \frac{10}{b+4}$

7. Raadi urur rumeedka $\frac{6}{x^2+3x-4} = \frac{3}{5x-5} + \frac{2}{5}$

8. Raadi urur rumeedka $\frac{2t-9}{2t-14} = \frac{3t}{t^2-7t} - \frac{1}{2t-14}$

9. Raadi urur rumeedka $\frac{x-3}{x} = 0$

Furfur isle'egyadan soo socda:

10. $$\frac{b}{5-b} - = \frac{4}{5}$$

11. $$\frac{3}{x} + \frac{14}{x^2} = \frac{2}{3} + \frac{1}{3x}$$

12. $$\frac{9}{x^2-4} = \frac{5}{x} - \frac{4}{x+2}$$

13. $$2 + \frac{6}{x^2-11x+10} = \frac{-13}{2x-2}$$

14. $$\frac{1}{3} = \frac{7}{3x+9} - \frac{x}{x^2+6x+9}$$

15. $$\frac{\frac{10}{x}+3}{\frac{5}{4}+4} = \frac{1}{6-x}$$

16. $$\frac{2}{x} + \frac{5}{3} = \frac{7}{3x}$$

17. $$\frac{3}{x} = \frac{5}{x-2}$$

18. $$\frac{5}{x-5} = \frac{12}{x^2} + \frac{3}{x^2-5x}$$

19. $$\frac{4}{a^2 - a - 6} = \frac{2}{3a - 9} - \frac{1}{3a - 6}$$

20. $$\frac{1}{x^2 - 6x} = \frac{1}{7}$$

21. Halkan waxa ku qoran laba isle'eg:

b) $\frac{x^3 - 1}{x - 1} = 7$ t) $x^3 - 1 = 7(x + 1)$

tirooyinka -3, -1, -2 kuwee baa raalli geliya (b);

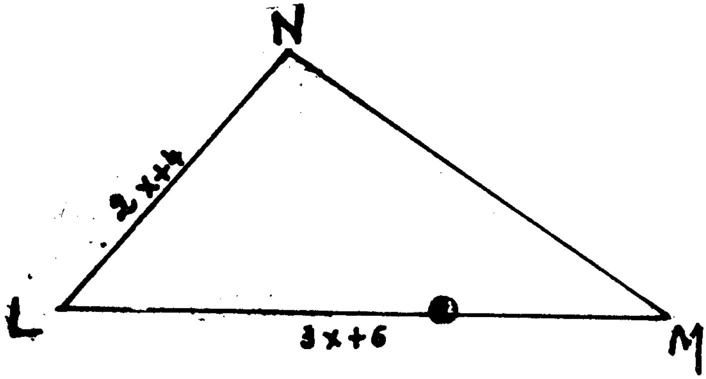
tirooyinka -3, -1, 1,2 kuwee baa raalli geliya (t).

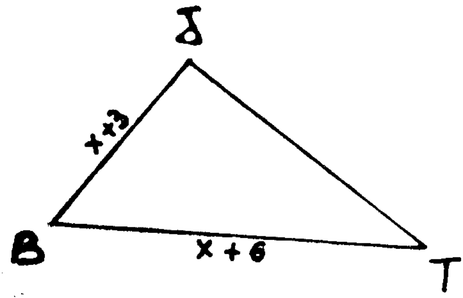
22. Tiro togan oo ah $\frac{1}{x}$ baa loo geeyey tiro kale oo ah $\frac{1}{x+2}$ wadarkoodu waa 1, raadi tirada.

23. Ninbaa qandaraas ku qaatay in uu 72 tan oo sonkor ah ka qaado beer oo uu geeyo Wershad. Haddii uu gaarigiisa ku shaqaysto dhowr tirib bay ku qaadanaysaa, laakiin haddii uu gaari weyn oo mirkiiba qaadi kara 2 tan oo dheeraad u isticmaalo, 3 tirib baa ka dhinmaaya. Imisa tan buu gaarigiisu qaadi karaa markiiba?

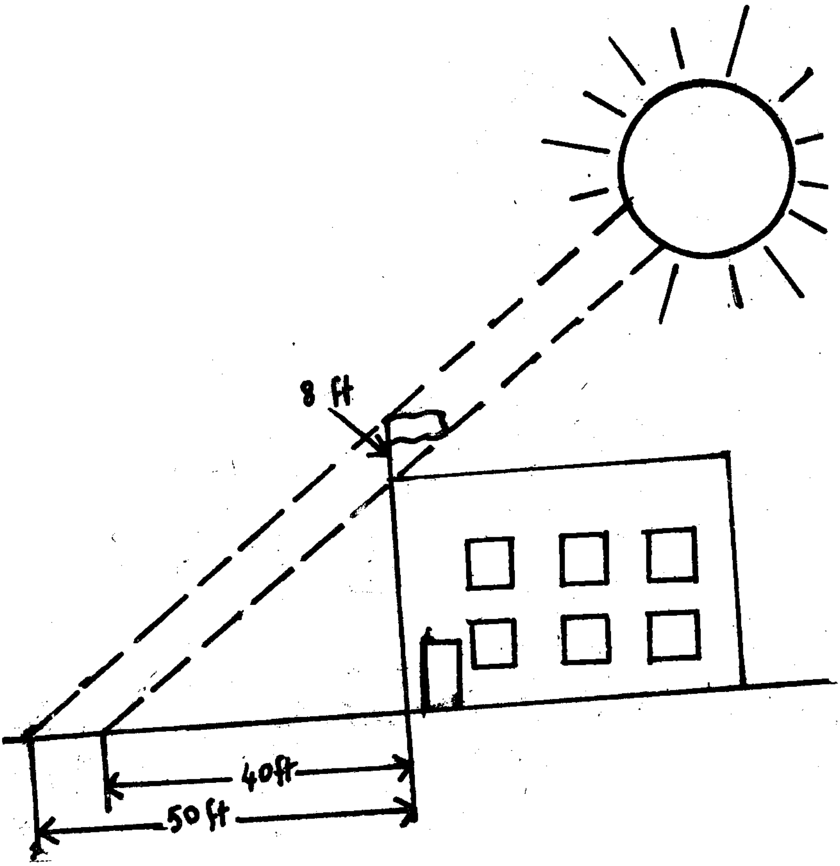
24. Saddexgalka BTJ waxu u egyahay saddexgal-

ka LMN. BJ = x + 3, BT = x + 6, LN = 2x + 4, LM = 3x + 6. Raadi cabbirka BJ, BT, LN, iyo LM?





25. Daar baa hooskeedu yahay 40 ft. Bir-calan 8 ft. ah baa ku dul taagan, isla ammintaa cirifka hooska calanku wuxu daarta u jiraa 50 m. Raadi joogga daarta?



26. Wiil baa hawl ku dhammeeya x saacadood, haddii uu keli shaqeeyo mid kale oo ka gaabiya wuxuu u baahan yahay 4 saacadood oo dheeraad ah, si uu isla hawshii u dhammeeyo. Haddii ay wada shaqeeyaan waxay u baahan yihiin 6 saacadood si ay hawsha u dhammeeyaan. Imisa saacadood bay wiilka hore hawshu ku qaadataa marka uu keli shaqeeyo?

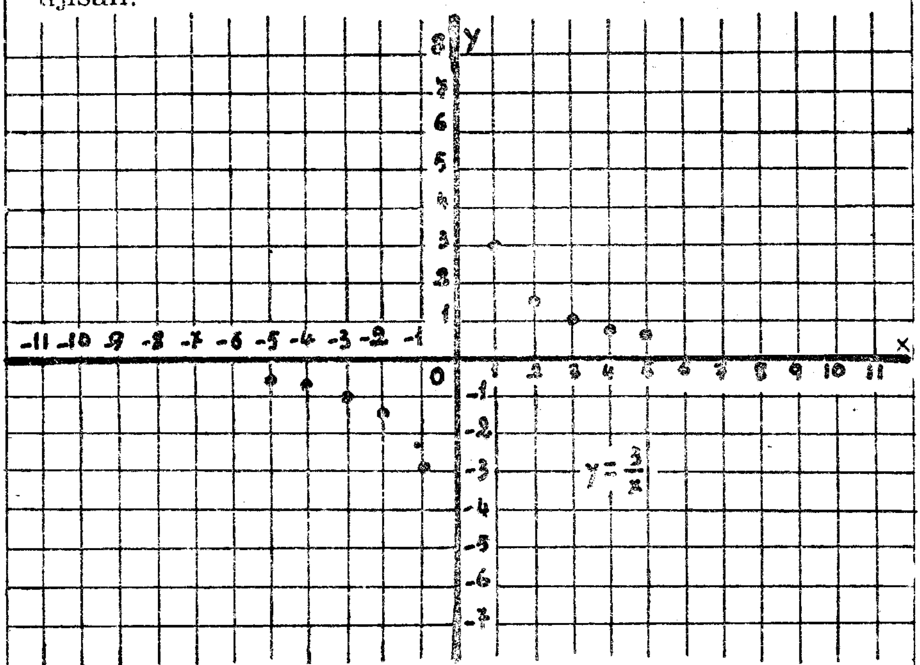
15. GARAAFKA FANSAAR LAKAB AH:

Xasuuso in garaafka fansaar u yahay ururka dhibcaha ku beegan ururka lammaaneyaasha horsan ee fansaarka. Hore waxaad u aragtay garaafka fansaar toosan iyo mid saabley ah. Hadda, bal aan eegno garaafka fansaar lakab ah. Marka aan rabno in aan samayno garaafka $y = \frac{3}{x}$ waa in aan helnaa lammaaneyaasha horsan ee fanaasrka qayb ka mid ah, sida tusahan ku muujisan.

$y = \frac{3}{x}$

x	− 5	− 4	− 3	− 2	− 1	0	1	2	3	4	5
y	$-\frac{3}{5}$	$-\frac{3}{4}$	− 1	$-\frac{3}{2}$	− 3		3	$\frac{3}{2}$	1	$\frac{3}{4}$	$\frac{3}{5}$

Baraha ku beegan lammaaneyaasha horsan ee tusaha ku qoran, waxay noqonayaan kuwa shaxankan ku muujisan.

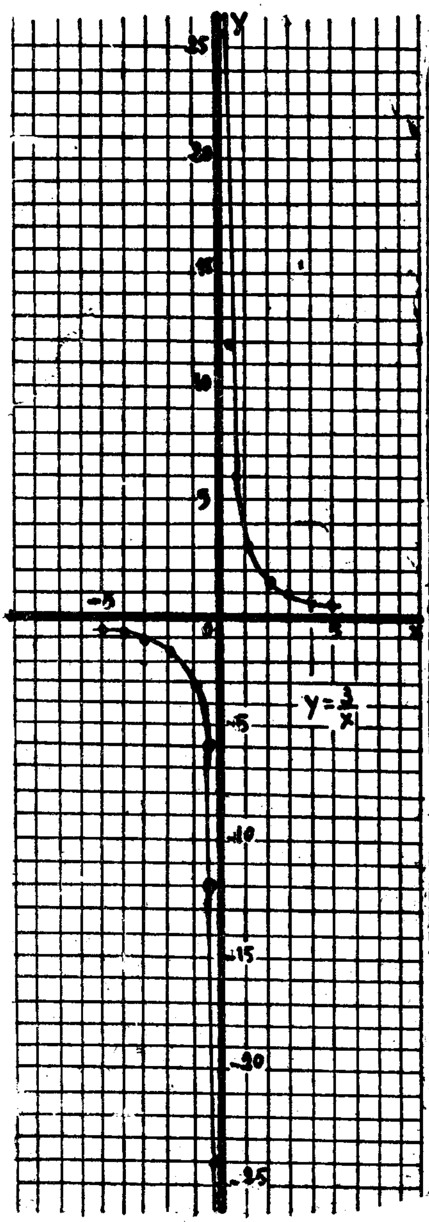


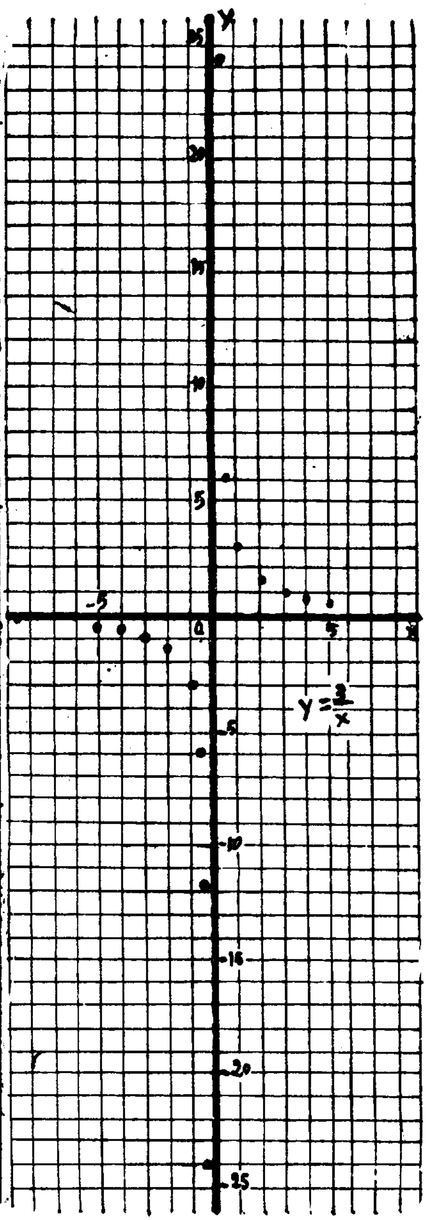
Bal u fiirso marka x = 0; y qiima ma le, waayo

$\frac{3}{0}$ micna ma le. Markaa, ma jirto baragaraafka ku taal markaa ay x = 0. Waxa la yiraa garaafku iskama haysto meesha ay x = 0. Haddaba maxaa ku dhaca garaafka marka ay x u dhawaato 0? Bal aan qaadanno qiimayaal x oo eber u dhow, sida tusahan hoose ku muujisan.

$y = \frac{3}{x}$	x	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{8}$	$+\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
	y	-4	-6	-12	-24	$+24$	$+12$	16	4

Imika, garaafku wuxu u ekaanayaa ka ku muujisan Sh. 32. Mar haddii eber u yahay qiimaha keliya ee x ee fansaar uuna ka qeexnayn, waxan filaynaa in garaafku meelaha kale iska haysto. Markaa, barahii aan dhignay oo dhan waa in aan iskugu xirnaa sidan shaxanka ku muujisan.


Y
25
20
15
10
5
-5
0
5
X
y = 3/x
-5
-10
-15
-20
-25


Y
25
20
15
10
5
-5
0
5
X
y = 3/x
-5
-10
-15
-20
-25

Layli:

1. Barahan soo socda kuwee baa ku yaal garaafka $y = \frac{1}{x-2}$?

$\left(4, \frac{1}{2}\right)$, $\left(-4, \frac{-1}{2}\right)$, $\left(0, \frac{-1}{2}\right)$, $(8, 1)$,

$\left(-1, \frac{-1}{3}\right)$, $(2, 0)$ $\left(\frac{3}{2}, -2\right)$, $\left(\frac{1}{2}, \frac{2}{3}\right)$,

$\left(7, \frac{1}{9}\right)$, $(1, -1)$, $\left(\sqrt{5}, \sqrt{5}+2\right)$

2. Barahan soo socda kuwee baa ku yaal garaafka $y = \frac{2x-3}{x+4}$?

$\left(0, \frac{-3}{4}\right)$, $(1,5)$, $(-3, -9)$, $(-5, -13)$,

$\left(2, \frac{1}{6}\right)$, $\left(\frac{1}{6}, 2\right)$, $(-4,11)$, $\left(\frac{1}{2}, \frac{-4}{9}\right)$, $\left(\frac{3}{2}, 0\right)$

3. Samee garaafka $y = \frac{1}{x-2}$ adoo raacaya dariiqaddii loo sameeyey garaafka $y = \frac{3}{x}$.

Aad ugu fiirso meesha fansaarku uuna ka qeex-nayn, dabadeedna raadi lammaanayaal horsan oo kugu filan oo u dhow barahaa si aad u arag-tid waxa garaafkaa ku dhacaya.

4. Ka soo qaad, in alla intii la doono la fidin kara-ayo xaashida u ku taswiiran yahay garaafka

$y = \frac{1}{x-2}$. Markaa, dhammee tusahan, wax-na ka sheeg meelaha ay barahaasi ku dhacaya-an.

$y = \frac{1}{x-2}$	$\frac{x}{y}$	12	+50	−88	102	1002	10,002	1,000,002
$y = \frac{1}{x-2}$	$\frac{x}{y}$	−12	−50	−88	−99	−998	−999,998	

5. Tusahan waxad uga shaqaysaa sida ka xidhiidh-ka masalada 4aad.

$y = \frac{1}{x-2}$	$\frac{x}{y}$	1.5	1.9	1.999	1.99999
$y = \frac{1}{x-2}$	$\frac{x}{y}$	2.5	2.1	2.01	2.001	2.00001

Masalooyinka 6, 7, 8 iyo 9, raac dariiqada hoos ku sharaxan si aad u falanqaysid una heshid garaafka fansaarrada lagu weydiiyay.

b) Samee tuse muujinaaya qiimayaasha abyoon ee x qaarkood.

t) U fiirso qiimaha x ee fansaarku uuna ka qeexnayn.

j) Samee tuse kale oo muujinaaya qiimayaasha x ee u dhow qiimaha x ee fansaarku uuna ka qeexnayn.

d) U fiirso inta ay y noqoto marka x ay noqoto 1,000,000 ama — 1,000,000.

r) Weliba, u fiirso inta ay y noqoto marka x ay qaadato qiimayaal aad iyo aad ugu dhow qiimaha x ee fansaarku uuna ku qeexnayn,

5. Dabadeedna, samee washirka garaafka adoo isticmaalaya warka aad ka ogaatay dariiqooyinka b, t, j, d, iyo r.

6. $y = \frac{1}{x}$ 7. $y = \frac{8}{x-4}$ 8. $y = \frac{2}{x+3}$

9. $y = \frac{36}{x^2}$

16. W A N Q A R :

Haddii aad u fiirsatay masalada 9aad waxaad aragtay in garaafka $y = \frac{36}{x^2}$ u ku wanqaran yahay dhidibka y Taa micnaheedu waxa weeye, bar kasta oo garaafka ku taal, sida (2,9) waxay leedahay bar kale oo isk garaafka ku taal, sida (— 2,9) dhidibka y-na wr qotomaha badhe u yahay xariijinta labadaa barood, isku xirta.

Guud ahaan, haddii garaafka fansaar ku wanqaran yahay dhidibka y, bar kasta, (b, t) oo garaafka ku taal waxay leedahay bar kale $(-b, t)$ oo isla garaafkaa ku taal. Taas oo kale waxay dhacdaa marka jibbaarka x ee tibaaxdu uu dhaban yahay. Markaa, garaafyada $\frac{4x^2}{x^2-4}$, $6+x^2$, $\frac{x^2-5}{3+x^6}$, waxay ku wanqaran yihiin dhidibka $-$ y.

Haddii aad u fiirsatay masalada 6aad, hubaal waxaad aragtay in garaafka $y = \frac{1}{x}$ u ku wanqaran yahay unugga. Tusaale ahaan, baraha $\left(5, \frac{1}{5}\right)$ iyo $\left(-5, -\frac{1}{5}\right)$ labaduba waxay ku jiraan garaafka, unuggana wuxu kala badhaa xarriijinta isku xiraysa. Guud ahaan, haddii garaafka fansaar ku wanqaran yahay unugga, bar kasta (b, t) oo garaafka ku taal waxay leedahay bar kale $(-b, -t,)$ oo isla garaafka ku taal. Taas oo kale waxay dhacdaa marka.

1. Tibix kasta oo sarreeyaha tibaaxda lakabka ah ba uu heerkeedu kisi yahay, isla markaana tibix kasta oo hooseeyaha tibaaxda lakabka ah ka mid ah ba uu heerkeedu dhaban yahay ama,

2. Marka tibix kasta oo ka mid ah sarreeyaha tibaaxda lakabka ah, uu heerkeedu yahay dhaban, isla markaa tibix kasta oo ka mid ah hooseeyaha tibaaxda lakabka ah uu heerkeedu kisi yahay markaa garaafyada $\frac{2x^3}{x4} + 8$, $\frac{3x^2-5}{x^3+6x}$, $x^5 - x$ waxay ku wanqaran yihiin unugga.

Layli:

1. Isle'egyadan soo socda; kuwee baa garaafkoodu ku wanqaran yahay dhidibka — y? Kuwee baana garaafkoodu ku wanqaran yahay unugga?

b) $y = \dfrac{x^2}{x^3 + x}$ x) $y = \dfrac{x}{x^4 + 1}$

t) $y = \dfrac{6}{x^2 - 9}$ kh) $y = \dfrac{x^2 + 5x + 2}{x^3 + 1}$

j) $y = x^2$ d) $y = \dfrac{x^4 + 2}{x^6 - 2}$

2. b) Ma jirtaa fansaar ku wanqaran dhidibka x?

t) Ma jiraa xiriir ku wanqaran dhidibka x?

17. TIKRAAR:

Haddii aad u fiirsatid masalada 2 ee layliga 10aad waxad arkaysaa in baraha kulammadoodu yihiin $\left(0, -\frac{3}{4}\right)$ iyo $\left(\frac{3}{2}, 0\right)$ ay ku jiraan garaafka labada barood. Ta hore waxay ku taal dhidibka — y, ta danbana waxay ku taal dhidibka — x.

Badanaaba, baraha ku yaal dhidibyada dhib yaraan baa loo helaa, garaafkana aad bay inooga caawiyaan. Tikraarka — y, waa qiimaha y ee ku beegan marka x ay eber tahay (waayo?) garaafka fansaar wuxu leeyahay hal tikraar — y. Si aan u helno tikraarka — y, x baan ka dhignaa eber, dabadeedna waxan raadinaa qiimaha y.

Sidaas oo kale, si aan u helno tikraarka — x, y baa ka dhignaa eber, dabadeedna waxan raadinaa qiimaha x. Tusaale ahaan, haddii aan haysanno fansaarka u qeexan sidan:

$$y = \frac{3x + 4}{x - 2},$$ waxan heli karnaa tikraarka — y iyo ka x.

Marka $x = 0$,

$$y = \frac{3(0) + 4}{0 - 2} = -2.$$ Ogow in barta $(0, -2)$ ay ku jirto garaafka

$$y = \frac{3x + 4}{x - 2},$$ weliba in tikraar — y u yahay — 2. Marka ay $y = 0$,

$$0 = \frac{3x + 4}{x - 2}.$$ Hawraartaasi waxay run tahay marka ay

$$x = \frac{-4}{3}$$ (waayo?)

Marka, barta $\left(\frac{-4}{3}, 0\right)$ waxay ku jirtaa garaafka, tikraarka — x, na waa $\frac{-4}{3}$.

Layli 12:

Fansaar kasta oo hoos ku yaal, raadi Tikraar — x iyo Tikraar — y.

1. $y = \dfrac{x - 8}{x + 2}$

2. $y = \dfrac{3x}{x^2 + 4}$

3. $y = \dfrac{2x + 5}{3x^2 + x + 7}$

4. $y = \dfrac{6x - 8}{2x^2 - 3x}$

5. $y = \dfrac{x^2 - 7x - 18}{x - 18}$

6. $y = \dfrac{3x^2 + 12x}{x + 1}$

7. $y = \dfrac{8}{x + 2}$

8. $y = \dfrac{x^2 + 8}{2x - 1}$

9. $y = \dfrac{x^2 + 8}{x + 1}$

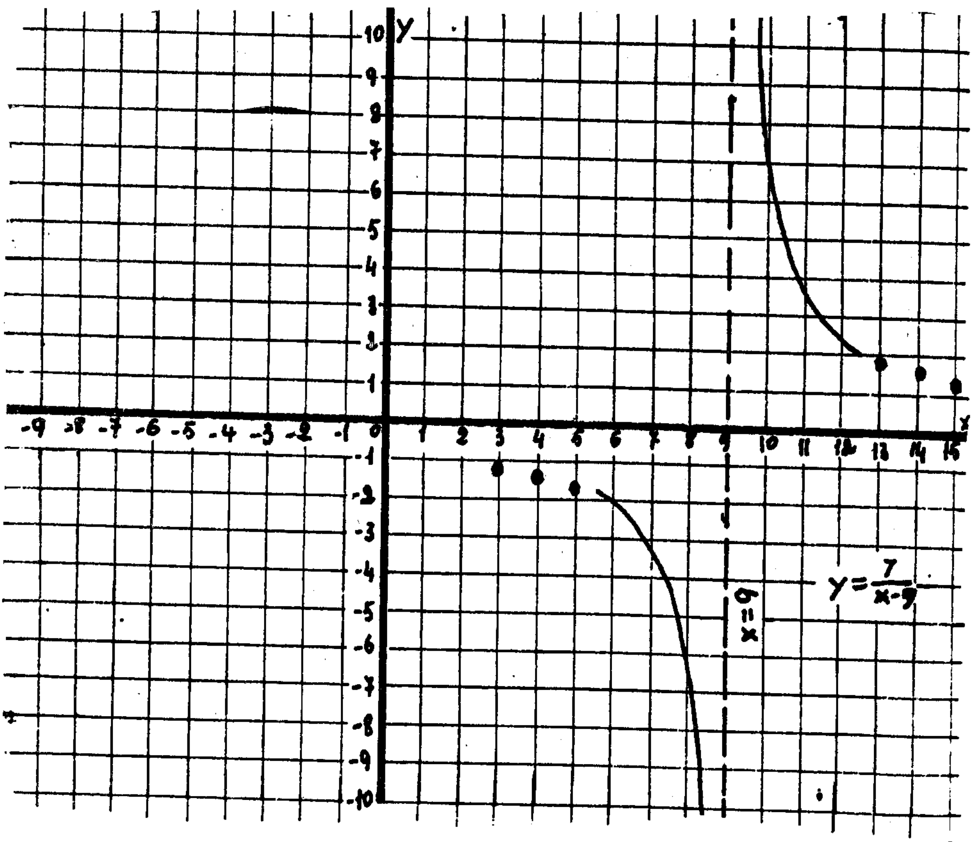
10. $y = \dfrac{2}{x - 2}$

11. b) Sheeg waxa masalooyinka 7 iyo 8 ayna tikraar — x u lahayn?
    t) Sheeg waxa masalooyinka 4 uuna tikraar — y u ahayn?

18. MADE:

Layliska 10aad waxad aragtay in fansaar kastaa leeyahay qiime x uuna ka qeexnayn. Qiimeyaasha x ee u dhow qiimaha, garaafku dibadduu uga baxaa xaashida. Tusaale ahaan, haddii $y = \dfrac{7}{x - 9}$ waxa inoo muuqata in fansaarku uuna qeexnayn marka $x = 9$, haddii aan taswiirno xariiqada taagan ee $x = 9$, waxa caddaan ah in marna garaafku uusan taabanayn xarriiqdaa. Marka x ay woxoogay ka weyn tahay 9, garaafku saray buu u baxaa, marka ay woxoogay ka yar tahay 9 na, garaafku hoos ayuu u baxaa. Marka ay x, 9 u sii dhawaataba, garaafku saray ama hoos ayuu u sii baxaa. Shaxanka hoose ayaa muujinaya garaafka $y = \frac{7}{x - 9}$.

Xarriiqda garaafku una taaban laakiin u aad iyo aad ugu dhawaado, sida $x = 9$ waxa la yiraa made. Ogow, in xarriiqdu ayna kutirsanayn garaafka, laakiin garaafkaa ku siqa madaha marka uu saray ama hoos u sii baxo. Marka x aad ugu dhowaato 9, qiimaha sugan ee y, |y| aad iyo aad buu u weynaadaa. Marka $x = 9$, fansaarku ma qeexna, mana jirto bar garaafka ka tirsan, oo xubinta hore ee lammaaneheedu horsani yahay 9. Markaa, garaafku iskama haysto meesha x tahay 9.



Layli 13:

1. b) U fiirso garaafka $y = \dfrac{5}{x-7}$. Sheeg qiimaha x ee uu made jiro?

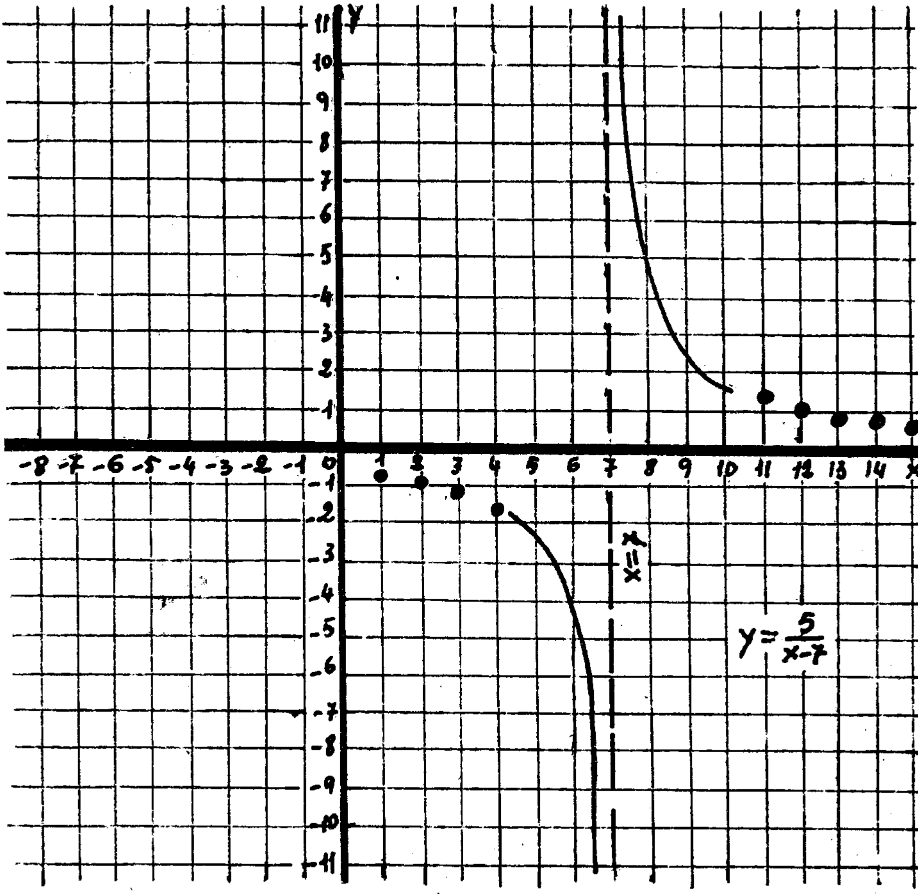
   t) Marka ay x qaadato qiime u dhow 7, qiimaha $(x - 7)$ eber buu u dhowaadaa, haddaba, maxaa ku dhaca qiimaha $\left|\dfrac{5}{x-7}\right|$?

   j) Qiimayaasha x ee woxoogay ka yar 7 (sida 6.8, 6.99), $(x - 7)$ ma tiro togan baa mise waa tiro taban? $\dfrac{5}{x-7}$ matiro togan baa mase waa tiro taban?

   x) Qiimayaasha x ee woxoogay ka weyn 7

   kh) Qiimayaasha x ee woxoogay ka weyn 7 (sida 7.2, 7.01), $(x - 7)$ ma tiro taban baa mase tiro togan? $\dfrac{5}{x-7}$ ma tiro togan baa mase waa tiro taban?

Adiga oo aan dhigin baraha waxad ka arki kartaa jawaabta, su'aasha kor ku taal, in garaafku made leeyahay marka $x = 7$, iyo in garaafku u aad hoos ugu baxo (y waa tiro taban, y aad bay u weyn tahay) marka xagga bidix uu madaha uga soo dhawaado, isla markaa, in u aad sarray ugu baxo, (y waa tiro togan, y aad bay u weyn tahay) marka uu madaha xagga midig uga soo dhawaado. Shaxanka hoos ku yaal wuxuu muujinayaa in garaafka ka mid an.



2. Adoo raacaya dariiqa masalada 1aad lagu sharaxay, raadi madaha taagan, sheeg sida uu garaafku noqdo marka uu u soo dhowaado madaha. Weliba, raadi tikraarka — y, dabadeedna washir garaafka.

b) $y = \dfrac{6}{x + 5}$ x) $y = \dfrac{-1}{x + 17}$

t) $y = \dfrac{-2}{x - y}$ kh) $y = \dfrac{2}{4 - 3x}$

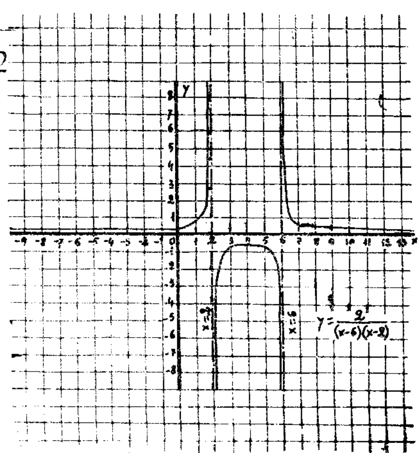
j) $y = \dfrac{11}{2x - 3}$ d) $y = \dfrac{-7}{4 - 3x}$

3. Garaafyada qaarkood waxay leeyihiin laba made ama ka badan.

Tusaale:

$$y = \frac{2}{x^2 - 8x + 12} \qquad y = \frac{2}{(x-6)\ (x-2)}$$

Fansaarkani ma qeexna marka $x = 6$ iyo marka $x = 2$. Bal aan eegno qiimaha y marka x ay qaadato qiime 6 ama 2 u dhow. Haddii $x = 1.9$, $(x - 6)$ waa tiro togan oo eber u dhow. Haddii $x = 1.9$, markaa $|y|$ wuxu weyn yahay, isla markaa y way togan tahay (Waayo?) Haddii $x = 2.1$ markaa $(x - 6)$ waa tiro togan laakiin $(x - 2)$ waa tiro taban oo eber u dhow. Haddaba, haddii $x = 2.1$ markaa $|y|$ waa tiro weyn, isla markaa y waa tiro taban; (Waayo?) Imika, ma sheegi kartaa waxa ku dhaca y iyo $|y|$ marka x ay tahay 5.9 iyo 6.1? Haddii aad su'aalaha sare oo dhan si sax ah uga jawaabtay, waxad aragtay in madeyaal jiraan marka $x = 6$ iyo marka $x = 2$, shaxanka hoos ku yaal waa washirka garaafka $y = \dfrac{2}{x^2 - 8x + 12}$



1. Sheeg madeyaalka isle'eg kasta oo hoos ku qoran, dabadeedna washir garaafkeeda.

b) $y = \dfrac{1}{(x-3)(x-5)}$

t) $y = \dfrac{-3}{(x-3)(x-6)}$

j) $y = \dfrac{5}{x^2 - 6x + 5}$

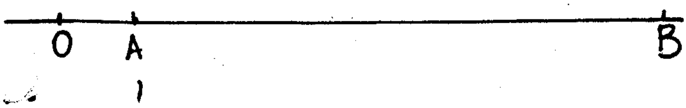
x) $y = \dfrac{x-2}{(x+3)(x-5)}$

CUTUB 2

JOOMETERIGA SOOFAN

1. FOGAAN JIHAN:

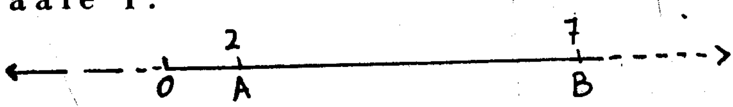
Haddii aan haysanno xarriiqda AB fogaanta u dhaxaysa A iyo B waxay noqon kartaa fogaanta A ilaa B ama B ilaa A. Waxa lagama maarmaana jihada kolba aad u socotid.



Qeex:

Fogaanta jihan ee u dhexaysa A iyo B oo loo qoro AB waxay tahay fogaanta A ilaa B. Fogaanta jihan ee u dhexaysa B iyo A waa fogaanta B ilaa A oo loo qoro BA. Haddaba haddii fogaanta A ilaa B ay togan tahay, fogaanta B ilaa A way taban tahay ta BA = – AB

Tusaale 1:



Fogaanta jihan ee min A ilaa B waa + 5.

Fogaanta jihan ee min B ilaa A waa – 5.

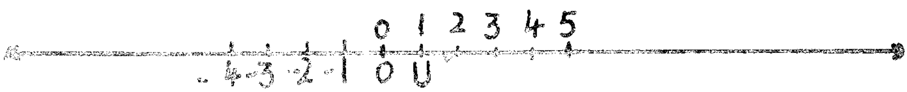
Tusaale 2:

Haddii AB ay togan tahay, BA ay taban tahay markaa BA = – BA.

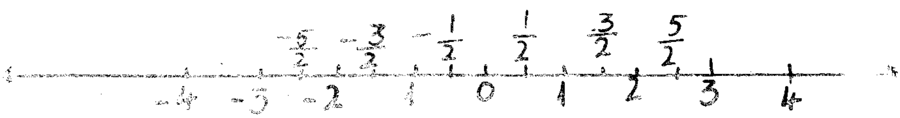
2. HABDHISKA KULAMMADA XARRIIQEED:

Ka soo qaad inaan haysanno xarriiqda K waxaad qaadataa barta 0 oo ku taal xarriiqda K kuna beegan eber.

Waxaad qaadataa barta U oo xagga midigta ka ah eber, kuna beeg 1. Fogaanta u dhexaysa 0 iyo U waa halbeeg cabbiraadeed. Qaado bar kale oo midigta ka xigta 1 kuna beeg tirada ah 2. Sidaas oo kale qaado baro kale oo bidixda ka xiga eber oo mid kastaaba midka kale u jirto halbeeg cabbiraadeed. Ku beeg barahaasi tirooyinka — 1, — 2, — — — — —.



Waxaad aragtaa in tirooyinka ku beegan barahaas ku yaal xarriiqda ay ka kooban yihiin ururka abyoonayaasha. Tirooyinka midigta ka xiga eber way togan yihiin kuwa bidixduna way taban yihiin. Haddaba haddii aan sii qaybino inta u dhexaysa laba barood oo is xigaba waxa ku samaymaaya xarriida ururka tirooyin lakab.

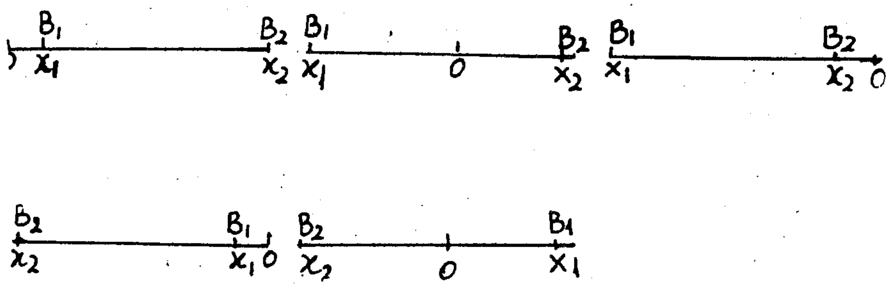


Qaado baro kale oo ku beegan tirooyinka lakab la'.

Waxa muuqata in ay jirto isku beegnaan mid-mid ah oo ka dhexaysa baraha xarriiqda ku yaal iyo tirooyinka maangalka ah.

Haddaba haddii aan haysanno tirada x_1 oo ku beegan barta x_1 iyo tirada x_2 oo ku beegan barta x_2, kolkaa tirada $|x_2 - x_1|$ waxay cabbirtaa fogaanta u dhexaysa labada barood ee x_1 iyo x_2.

Sidaas oo kale haddii aan haysanno laba barood B_1 iyo B_2 oo ku yaal xarriiqda oo kulammadeedu yihiin x_1 iyo x_2 kolkaa fogaanta jihan ee u dhexaysa labada barood waxay mar kasta tahay $B_1 B_2 = x_2 - x_1$.

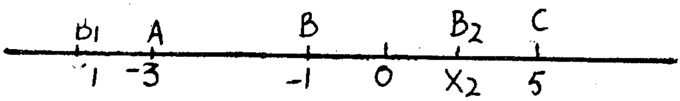


Ogow in fogaanta jihan ah $B_1 B_2$ ay togan tahay haddii ay $x_2 > x_1$. Sidaas waxay noqotaa haddii B_2 ay xagga midigta ka xigto B_1. Haddii $x_2 < x_1$ fogaanta jihani way taban tahay. Sidaas waxay noqotaa haddii B_2 ay xagga bidixda ka xigto B_1.

Haddaba fogaantaan jiha lahayn haddaan sheegayno waxaan u qornaa sidan:

$$B_1B_2 = |B_1B_2| = |B_2B_1| = |x_2 - x_1| = |x_1 - x_2|$$

Tusa[illegible]



Fogaanta jihan ee:

1. $AB = (-1) - (-3) = 2$
2. $BC = 5 - (-1) = 6$
3. $B_1B_2 = x_2 - x_1$
4. $CA = -3 - (+5) = -8$
5. $B_2B_1 = x_1 - x_2$

Haddii aan rabno inaan soo saarro fogaanta ah K H ee u dhexaysa labada barood H iyo K, waxaan ka goynaa kulanka baraha la hor taxo kulanka ta mar labaadka la taxo. Sida tusaalaha kor ku qoran qaybtiisa 3aad iyo ta aad.

Tusaale 2:

Baraha $B_1 B_2$ iyo fogaanta jihan $B_1 B_2$. Fogaanta jihan $B_2 B_1$ waa intee?

Furfuris:

$$\text{Fogaanta } B_1 B_2 = |x_2 - x_1| = \left| -\frac{5}{2} - 3 \right| = \left| -\frac{11}{2} \right| = \frac{11}{2}$$

$$\text{Fogaanta jihan } B_1 B_2 = x_2 - x_1 = -\frac{5}{\;} + 3 = -\frac{11}{\;}$$

$$\text{ama } B_2 B_1 = x_1 - x_2 = 3 - \left(-\frac{5}{2} \right) = 3 + \frac{5}{2} = \frac{11}{2}$$

Layli:

1. Soo saar fogaanta iyo fogaanta jihan ee u dhexaysa baraha B_1 iyo B_2.

j)	b , a	x)	6 , 10
kh)	8 , 2	d)	-4 , 3
r)	5 , -7	c)	$\sqrt{63}$, $-\sqrt{28}$

2. Kulammada baraha A iyo B siday u kala horreeyaan.

b) tus inay AB = OB – AB.

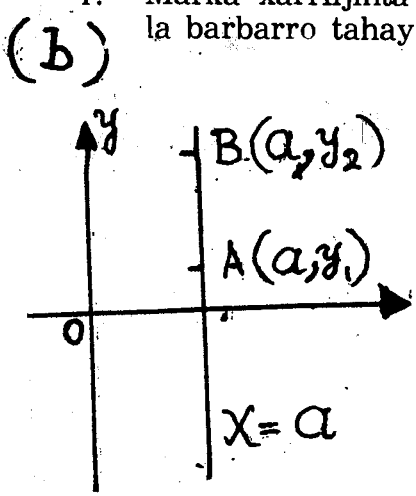
t) haddii D tahay bar – badhtanka A B, tus in kulammada D ay yihiin $\left(d, = \frac{a+b}{2}\right)$.

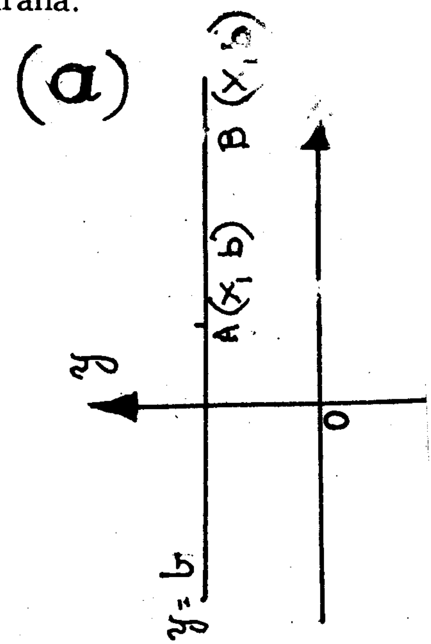
3. FOGAANTA KU TAAL SALLAXA:

Waxaan naqaannaa sida loo helo fogaanta iyo fogaanta jihan ee u dhexaysa laba barood oo ku yaal xarriiqda tiro. Marka halkan waxan ku baranayna sida loo soo saaro fogaanta u dhexaysa laba barood oo ku yaal sallax.

Fogaanta u dhexaysa laba barood oo ku yaal sallax waa dhererka xarriiqda labada barood isku xirta. Kolkaa si loo helo fogaantaasi waa inaan tixgelinaa labadan xaaladood ee soo socda:

1. Marka xarriijinta isku xirta labada barood ay la barbarro tahay baraha.



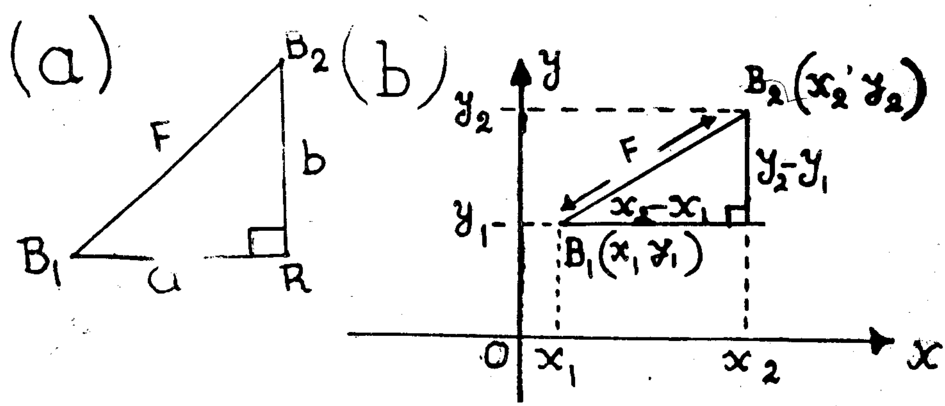


b) **Marka ay xirriijintu la barbarro tahay dhidibka $-x$, una jirto "b" halbeeg unugga, waxaan aragnaa in fogaanta u dhexaysa labada barood, A iyo B ay tahay $AB = |x_2 - x_1|$ ama $|x_1 - x_2|$.**

t) Marka ay xarriijintu la barbarro tahay dhidibka $-y$, una jirto "a" halbeeg unugga, fogaantu waa $AB = |y_2 - y_1|$ ama $|y_1 - y_2|$.

2. Marka xarriijintu ayna la barbarro ahayn labada dhidib midnaba. Ka soo qaad laba barood oo ku yaal sallax ka soo qaad in xarriijinta isku xirta labada barood ayna la barbarro ahayn labada dhidib midnaba.

Tixgeli shaxankan:



Si loo soo saaro fogaantaasi waxa la isticmaalaa aragtiinka (Pythogras).

SADDEXAGAL QUMMAN:

b) Haddii aan qaadanno saddexagalka $B_1 R B_2$,
$F^2 = a^2 + b^2$
ama $F = \sqrt{a^2 + b^2}$

t) Sidaas oo kale qaado shaxanka kale,
$F^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
ama $F = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Tusaale 1:

Soo saar fogaanta u dhexaysa labadan barood (3,7) iyo (− 3,2).

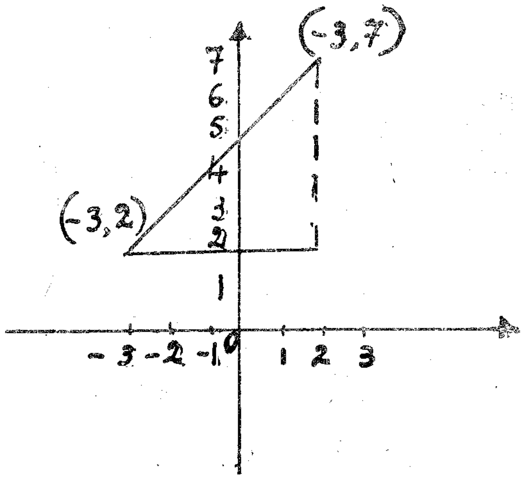
Furfuris:

$$F = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$F = \sqrt{[3 - (-3)]^2 + (7 - 2)^2}$$

$$= \sqrt{36 + 25}$$

$$= \sqrt{61}$$



Tusaale 2:

Tus in saddexagalka geesihiisu yihiin (0, 0) (− 3, 4) iyo (− 6 , 0) u yahay saddexagal labaale ah.

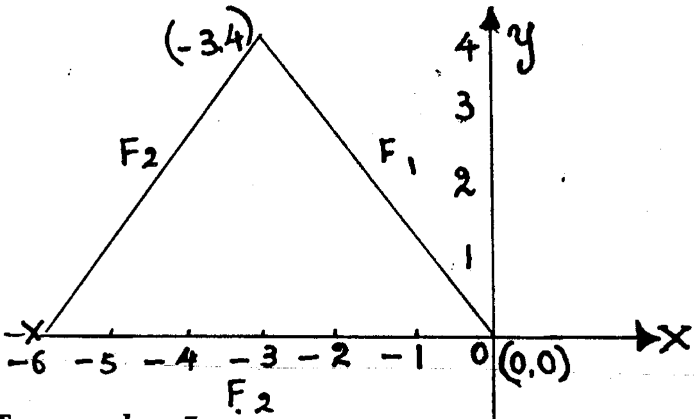
Furfuris:

$$F_1 = \sqrt{(-3^2) + (4)^2} = \sqrt{9 + 16} = 5$$

$$F_2 = \sqrt{(-6 + 3^2) + 4^2} = \sqrt{3^2 + 4^2} = 5$$

$$F_1 = F_2$$

Kolkaa saddexagalku waa labaale.

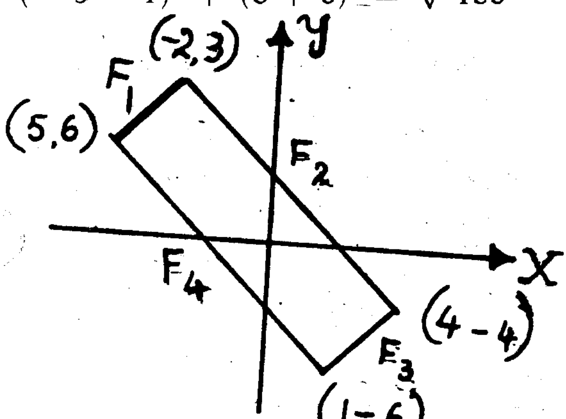


Tusaale 3:

Tus in afar geesalaha geesihiisu yihiin (− 5,6), (− 2,8), (4, − 4) iyo (1, − 6) u yahay barbarroole.

Furfuris:

$$F_1 = \sqrt{(-2+5)^2 + (8-6)^2} = \sqrt{13}$$
$$F_2 = \sqrt{(4+2)^2 + (-4-8)^2} = \sqrt{180}$$
$$F_3 = \sqrt{(1-4)^2 + (-6+4)^2} = \sqrt{13}$$
$$F_4 = \sqrt{(-5-1)^2 + (6+6)^2} = \sqrt{180}$$



Mar haddii $F_1 = F_3$, $F_2 = F_4$, afar geesooluhu waa barbarroole.

Layli:

1. Soo saar fogaanta u dhexaysa baraha:

b)	(− 3 , 1) iyo (9 , 6)	jawaab: 13
t)	(2 , 13) , (8 , 5)	10
j)	(− 5 , 3) , (0 , 8)	52
x)	(− 6 , 4) , (− 6 , 17)	13
kh)	(− 9 , − 2) , (− 3 , 6)	10
d)	(7 , 5) , (3 , 14)	$\sqrt{97}$
r)	(7 , 4) , (− 2 , 4)	9

2. Soo diir jidka fogaanta haddii aad haysatid laba barood.

3. Raadi dherarrada dhinacyada saddexagalka geesihiisu yihiin A (7 , 0) , B (1 , 6) iyo C (− 8 , 6).

4. Tus in saddexagalka geesihiisu yihiin A (4 , 7) . B (7 , 12) , C (9 , 10) u yahay saddexagal laba-ale ah.

5. Tus in saddexagalka geesihiisu yihiin A (6 , 1) , B (10 , 9) iyo C (− 6 , 7) u yahay saddexagal qumman. Raadi bedhka saddexagalka.

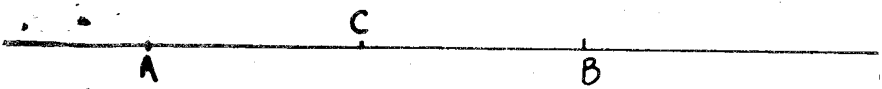
6. Raadi barta in u wada jirta baraha A (1 , 7) , B (8 , 6) , C (7, − 1). Jaw. x = 4, y = 3

7. Tus in barahani yaalaan xarriiq keliya: A (− 3 , − 2) , B (5 , 2) , C (9 , 4).

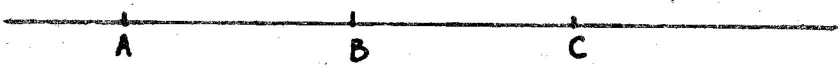
BARTA QAYBISKA;

Qeex:

Barta Qaybintu waa barta u qaybisa xarriijin sami la ogyahay oo ah $r_1 : r_2$. Barahaasi marna waxay ka qaybisaa xarriijinta gudaha marna dibedda.



qaybin guudeed



qaybin dibadeed

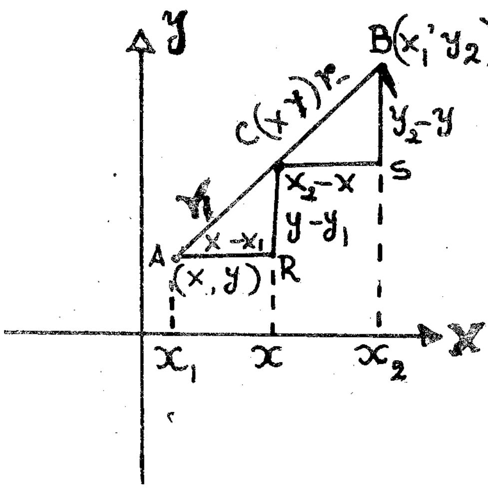
QAYBIN GUUDEED:

Ka soo qaad in barta C(x , y) ay u qaybiso xarriijinta AB saamiga $r_1 : r_2$.

Adoo adeegsanaya shaxankan, waxaad aragtaa in labada saddexagal CRA iyo CBS ay isku egyihiin.

Markaasi $\frac{AR}{CS} = \frac{AC}{CB}$; $\frac{AC}{SB} = \frac{AC}{CB}$

ama $\frac{x - x_1}{x - x_2} = \frac{r_1}{r_2}$, $\frac{y - y_1}{y_2 - y_1} = \frac{r_1}{r_2}$.



Kolkaa $r_2(x - x_1) = r_1(x_2 - x)$; $r_2(y - y_1) = r_1(y_2 - y)$

$$r_2 x - r_2 x = r_1 x_2 - r_1 x \; ; \; r_2 y - r_2 y_1 = r_1 y_2 - r_1 y$$

$$r_2 x + r_1 x = r_1 x_2 + r_2 x_1 \; ; \; r_2 y + r_1 y = r_1 y_2 + r_2 y_1$$

$$x(r_1 + r_2) = r_1 x_2 + r_2 x_1 \; ; \; y(r_2 + r_1) = r_1 y_2 + r_2 y_1$$

$$x = \frac{r_1 x_2 + r_2 x_1}{r_2 + r_1} \; ; \; y = \frac{r_1 y_2 + r_2 y_1}{r_2 + r_1}$$

Haddaba kulammada barta $C(x, y)$ waxay yihiin:

$$\left(\frac{r_1 x_2 + r_2 x_1}{r_2 + r_1} , \frac{r_1 y_2 + r_2 y_1}{r_2 + r_1} \right)$$

Ogow:

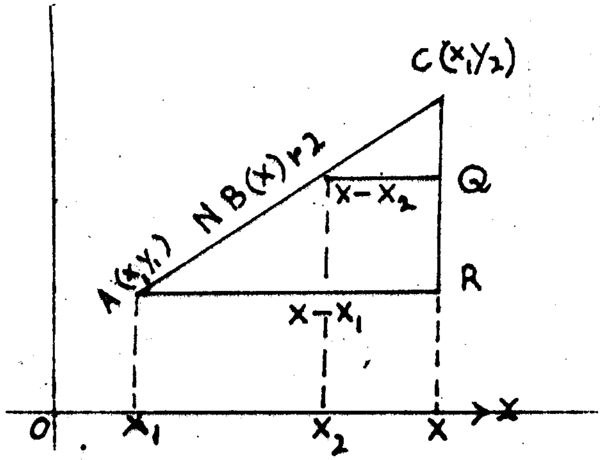
Haddii $r_1 = r_2 = 1$, kulammada bar badhtanka waa

$$x = \frac{x_1 + x_2}{2} , \quad y = \frac{y_1 + y_2}{2}$$

QAYBIN DIBADEED:

Ka soo qaad inay barta $C(x, y)$ ku taallo xarriijinta AB oo la fidiyay Ka soo qaad in bartaasi u qaybiso xarriijinta saamiga sida: $r_1 : r_2$. Adoo adeegsanaaya shaxanka waxad aragtaa in $\triangle$ ACR $\sim$ $\triangle$ BCQ.

Kolkaa $\frac{AC}{BC} = \frac{AR}{BQ}$



$$\frac{r_1}{r_2} = \frac{x - x_1}{x - x_2}$$

$$r_1 x - r_1 x_2 = r_2 x - r_2 x_1$$
$$r_1 x - r_2 x = r_1 x_2 - r_2 x_1$$
$$x(r_1 - r_2) = r_1 x_2 - r_2 x_1$$

$$x = \frac{r_1 x_2 - r_2 x_1}{r_1 - r_2}$$

Sidaas oo kale: $y = \dfrac{r_1 y_2 - r_2 y_1}{r_1 - r_2}$.

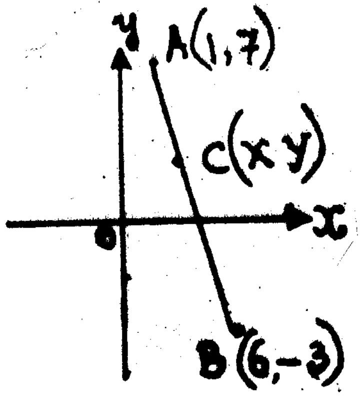
Tusaale 1:

Soo saar kulannada barta C(r , y) ee u qaybisa xarriijinta isku xirta labada barood A(1, 7) iyo B(6, − 3) marka saamiga $r = \dfrac{2}{3}$.

Furfuris:

Marba haddii uu saamigu togan yahay AC iyo AB waa isku jiho. Markaa waa in barta C(x, y) ay gudaha ka qaybisaa xarriijinta AB.

Waxaan ognahay in $r = \frac{AC}{CB} = \frac{2}{3}$.



Kolkaa: $x = \frac{r_1 x_2 + r_2 x_1}{r_2 + r_1} = \frac{2(6) + 3(1)}{3 + 2} = \frac{12 + 3}{5} = 3$

$y = \frac{r_1 y_2 + r_2 y_1}{r_1 + r_2} = \frac{2(-3) + 3(7)}{3 + 2} = \frac{-6 + 21}{5} = 3$

Kolkaa: $C(x, y) = C(3, 3)$

Tusaale 2:

Soo saar kulannada barta $C(x, y)$ ee qaybisa xarri-ijinta isku xirta labadan barood $A(-2, -1)$ iyo $B(3, -4)$.

Marka saamiga $r = -\frac{8}{3}$.

Furfuris:

Mar haddii saamigu taban yahay, AC iyo CB jihado-odu waa isku lid. Marka barta $C(x, y)$ waxay taal xarri-ijinta AB oo la fidiyay.

Waxan ognahay in $r = \frac{AC}{CB} = -\frac{8}{3}$.

Kolkaa:

$$x = \frac{r_1 x_2 - r_2 x_1}{r_1 - r_2} = \frac{-8(3) - -3(-2)}{-8 - (-3)} = -\frac{30}{-5} = 6$$

$$y = \frac{r_1 y_2 - r_2 y_1}{r_1 - r_2} = \frac{-8(-4) - [-3(1)]}{-8 - (-3)}$$

$$= \frac{32 + 3}{-5} = \frac{35}{-5} = -7$$

Kolkaa $C(x, y) = C(6, -7)$.

Tusaale 3:

Barta $B(-4, 1)$ waa $\frac{3}{5}$ ka fogaanta marka laga bilaabo barta $A(2, -2)$ ee xarriijinta ilaa bar dhammaadka $C(x, y)$.

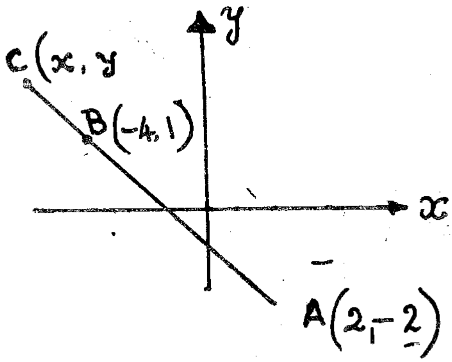
Furfuris:

$$\frac{AB}{BC} = \frac{3}{2}$$

Kolkaa $r = \frac{AC}{CB} = -\frac{5}{2}$

Mar haddii AC iyo CB jihooyinkoodu lid isku yihiin, saamigu wuu taban yahay

Kolkaa $x = \frac{r_1 x_2 - r_2 x_1}{r_1 - r_2} = 5$



$$x = \frac{-5(-4) - [-2(2)]}{-5 - (-2)} = \frac{20 + 4}{-3} = \frac{24}{-3} = -8$$

$$y = \frac{r_1 y_2 - y_2 y_1}{r_1 - r_2} = \frac{-5(1) - (-2)(-2)}{-5 - (-2)} = \frac{-9}{-3} = 3$$

Kolkaa $C(x, y) = C(-8, 3)$

Layli:

1. Soo saaṛ kulannada barta $C(x, y)$ ee u qaybisa xarriijinta AB saamiga $r = \frac{AC}{CB}$.

b) $A(4, -3)$, $B(1, 4)$, $r = \frac{3}{1}$

t) A(5, 3), B(−3 −3), r = $\frac{1}{3}$

j) A(−2, 3), B(3, −2), r = $\frac{2}{3}$

x) A(0, 3), B(7, 4), r = $\frac{-2}{7}$

2. Barta C(x, y) waa afar todobaadka fogaanta laga bilaabo barta A(3, 2), ee xarriijinta ilaa barta B(34, 74). Soo saar kulannada C(x, y).

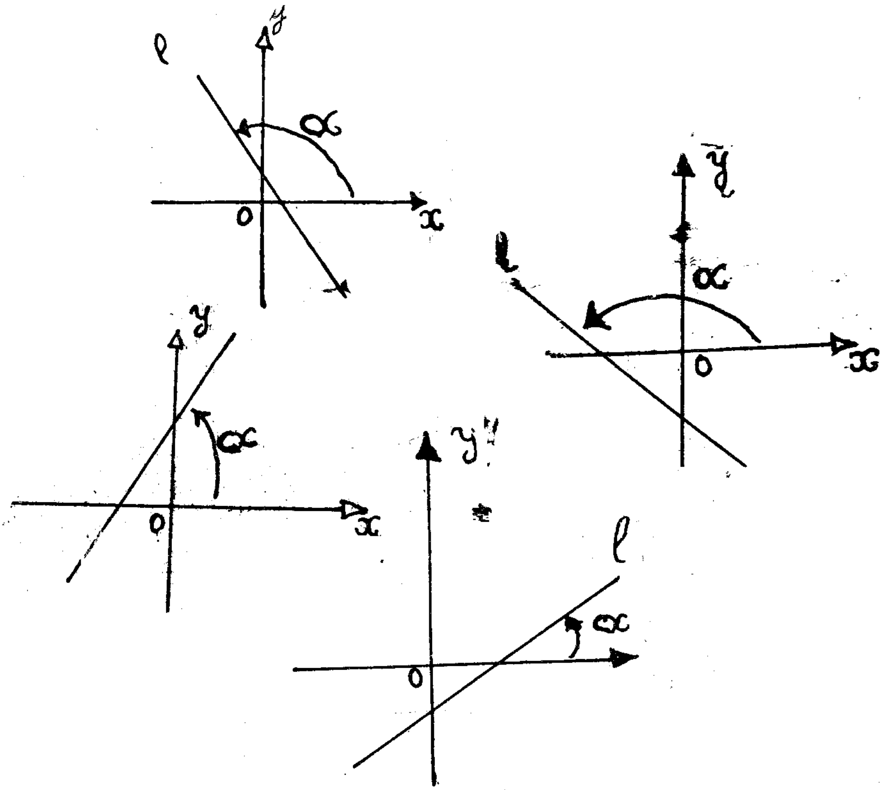
3. Soo saar saamiga ay barta (− 11,6) u qaybiso xarriijinta isku xirta labadan barood A(2, 7) iyo B(6, 8).

4. Soo saar kulannada barta u qaybiso xarriijinta isku xirta labada barood (22, 11) iyo (30, 44). Marka u yahay saamigu 3 : 2.

5. Xarriijinta isku xirta labadan barood A(−2, −1) iyo B(3, 3) waxa la fidiyay ilaa iyo barta C. Haddii C tahay barta (18,15), soo saar saamiga ay u qaybiso xarriijinta AB.

JANJEER IYO TIIRO:

Waxa lagama maarmaan ah inaan garanno jihada xarriiqda ku taal sallax waxan ku magacawnaa jihada xarriiqdaasi janjeerka xarriiqda. Janjeerka xarriiqda, α waa xagasha u dhexeysa dhidibka − X togan iyo xarrriiqda L. Waxa lagaga bilaabaa dhidibka − X togan waxana loo cabbiraa lid − saacad wareeg. Badanaa ba α ayaa loo taagaa xagal janjeerkaa.



Shaxannada sare waxay ku tusayaan xagal janjeer-ka xarriiqda L.

Haddii xarriiqdu la barbarro tahay dhidibka – X, xagal janjeerku waa eber. Had iyo jeer xagal janjeerku wuxuu u dhexeeya 0° iyo 180°. Sidaasi waxa loo qoraa $0 \leq \alpha \leq 180°$.

Haddaba haddii xagasha u dhexayso 0° iyo 90° xar-riiqdu waxay u janjeertaa medig kor. Haddii xagashu u dhexayso 90° iyo 180°, xarriiqdu waxay u janjeertaa mi-dig hoose.

Ilaa hadda waxaan naqaannaa xagal janjeerka xarriiqi wuxu yahay. Haddaba haddii aan haysanno xarriiqda xagal janjeerkeedu yahay α, markaa tiirada xarriiqda L waxay tahay:

$$M = \tan \alpha$$

Haddii xagashu tahay eber, kolkaa tiiradu waa

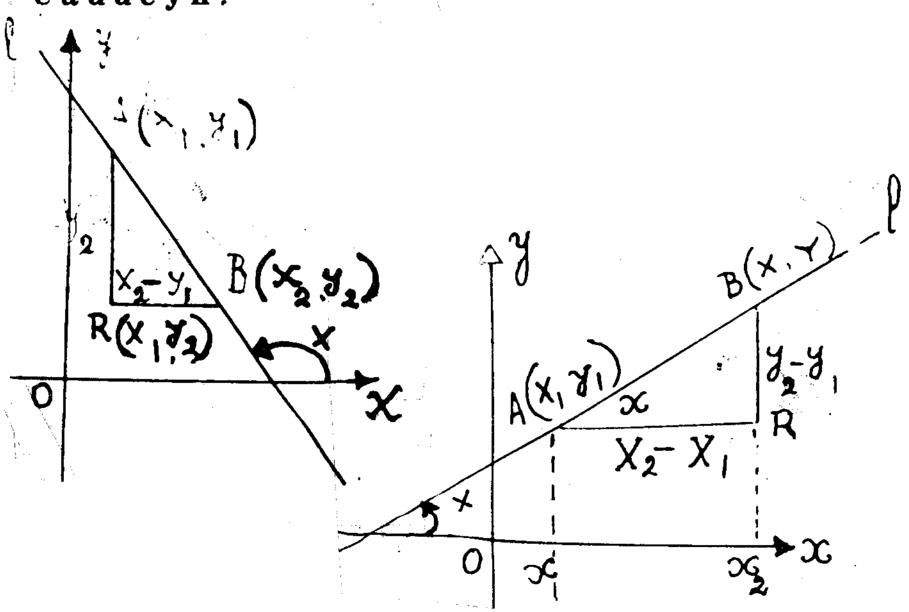
$$M = \tan 0° = 0$$

Haddii α tahay 0° ilaa 45°, tan α wuxuu u dhexeeyaa 0 ilaa 1. Markay α ku siqo 90°, tan α si aad ah ayuu u kordhaa ama tan α wuxuu ku siqaa tirobeel. Sidaasi waxay tahay in tan 90° una qeexnayn. Kolkaa tiirada xarriiq kasta oo qotonta ma qeexna. Haddaba haddii α tahay xagal furan $90° < \alpha < 180°$. tan α wuu taban yahay. Xarriiqduna waxay u janjeertaa midig hoose. Marka ay xagashu fiiqan tahay $0° < \alpha < 90°$ tan α wuu togan yahay. Xarriiqdu waxay u janjeertaa mid kor.

Xarriiqda marta barta (x_1, y_1) iyo (x_2, y_2) ee aan barbarro la ahayn labada dhidib midnaba, tiiradeedu waa

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

Caddeyn:



Shaxanka (a)

$$M = \tan \alpha$$

$$= \frac{RB}{AR} = \frac{y_2 - y_1}{x_2 - x_1}$$

Shaxanka (b)

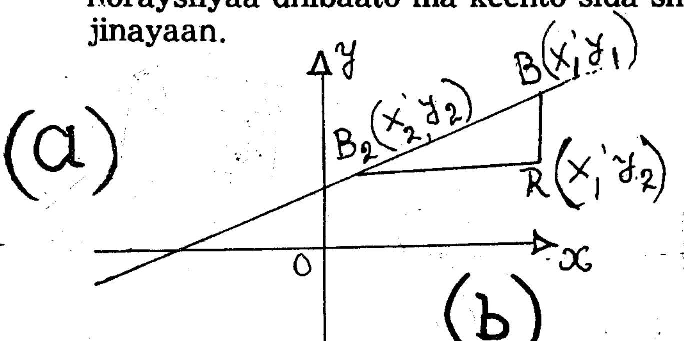
$$M = \tan \alpha = - \tan (180^\circ - \alpha).$$

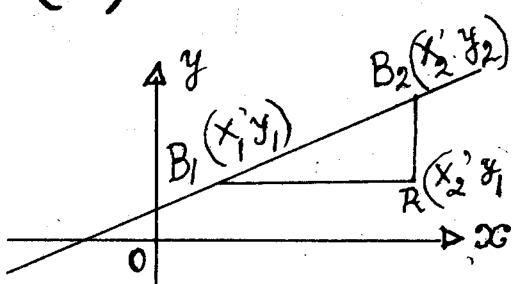
$$= - \frac{y_1 - y_2}{x_2 - x_1}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Markaa tiirada $M = \frac{y_2 - y_1}{x_2 - x_1}$ hadday xagasho x fiiqan tahay iyo hadday furan tahay ba.

Haddii xarriiqda L ay marto laba barood kolba tii la horaysiiyaa dhibaato ma keento sida shaxannadani muujinayaan.





Adoo adeegsanaya shaxanka (a):

$$M = \frac{RB_1}{B_2R} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{(-1)\ (y_1 - y_2)}{(-1)\ (x_1 - x_2)} = \frac{y_2 - y_1}{x_2 - x_1}$$

Adoo adeegsanaya shaxanka (b):

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

XARRIIQYADA BARBARRO AH IYO KUWA ISKU QOTOMA:

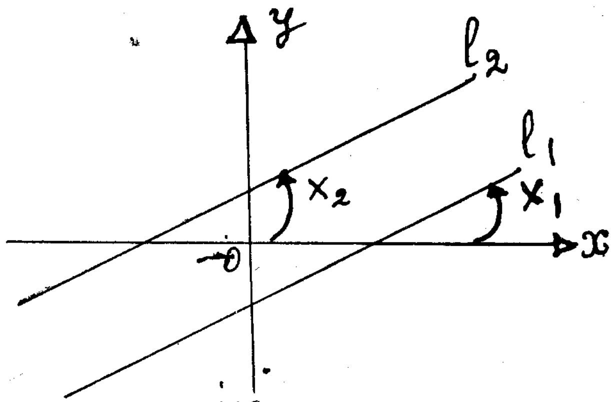
Aragtiin 1:

Laba xarriiqood oon taagnayn oo tiirooyinkoodu yihiin M_1 iyo M_2 waa barbarro haddii oo qura oo $M_1 = M_2$.

Caddayn:

a) Haddii L_1 iyo L_2 ay barbarro yihiin kolka $M_1 = M_2$ ·· U qaado in janjeeryada labada xarriiqood yihiin α_1 iyo α_2.
Haddii L_1 iyo L_2 ay barbarro yihiin, kolkaa $\alpha_1 = \alpha_2$ kolkaa tan α_1 = tan α_2.
Haddaba $M_1 = M_2$.

b) Haddii $M_1 = M_2$, kolkaa L_1 iyo L_2 waa barbarro.
Haddii $M_1 = M_2$ kolkaa tan α_1 = tan α_2.
Markaa $\alpha_1 = \alpha_2$.
Haddaba L_1 iyo L_2 waa barbarro.



Aragtiin 2:

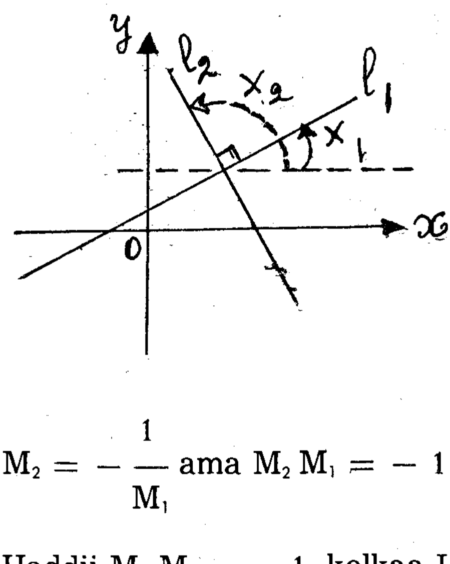
Laba xarriiqood oo la'aagnayn tiirooyinkooduna yihiin M_1 iyo M_2 way isku qotommaan haddii iyo haddii oo qura oo $M_1 \cdot M_2 = -1$.

Caddayn:

a) Haddii L_1 iyo L_2 ay isku qotommaan, kolkaa $M_1 \cdot M_2 = -1$.

Haddii $L_1 \perp L_2$, kolkaa $\alpha_2 = \alpha_1 + 90°$. Haddaba $\tan \alpha_2 = \tan (\alpha_1 + 90°) = \dfrac{-1}{\tan \alpha_1}$.

Kolkaa, haddaba inagoo adeegsanayna Midaal trignoometeri oo ah $\tan \alpha_2 = \tan (\alpha_1 + 90°) = \dfrac{1}{\tan \alpha_1}$ waxan helaynaa



$M_2 = -\dfrac{1}{M_1}$ ama $M_2 M_1 = -1$.

b) Haddii $M_2 M_1 = -1$, kolkaa $L_1 \perp L_2$. Haddii $M_2 = -\dfrac{1}{M_1}$, kolkaa $\tan \alpha_2 = -\dfrac{1}{\tan \alpha_1}$

Midaalku wuxuu **ina** siiyaa **in tan** $\alpha_2 = -\dfrac{1}{\tan \alpha_1} = \tan(\alpha_1 + 90^\circ)$. Haddii $\alpha_2 = (\alpha_1 + 90^\circ)$ ama $\alpha_2 - \alpha_1 = 90^\circ$ kolkaa $L_1 \perp L_2$. Maxaa dhacaaya haddii labada xarriiq oo barbarro ahi ay taagan yihiin? Waxaan aragnay in xarriiqda jiirteedu aanu qeexnayn. Sidaas darteed, haddii M_1 iyo M_2 qeexnayn L_1 iyo L_2 waxay la barbarro yihiin dhidibka $- y$, iyaguna waa barbarro. Haddii $M_1 = M_2$, kolkaa $\tan \alpha_1 = \tan \alpha_2$ isla markaa $\alpha_1 = \alpha_2$.

Maxaa dhacaaya haddii labada xarriiqood ee isku qotoma midkood u taagan yahay sida dhidibka sallax?

Haddii L_1 ay taagan tahay L_1 iyo L_2 ay isku qotommaan kolkaa waa inay L_2 jiiftaa. Tiirada L_1 ma jirto, ta L_1 waa eber. Sidaas darteed M_1, M_2 ma jirto.

Tusaale 1:

Soo saar tiirada xarriiqda marta labadan barood $(-3,4)$ iyo $(5,2)$. Xarriiqdaasi ma la barbarraa mise way ku qotontaa xarriiqda marta barta $(-1,7)$ iyo $(3,-4)$.

Furfuris:

$$M_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{5-(-3)} = \frac{-6}{8} = \frac{-3}{4}$$

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-7}{3-(-1)} = \frac{-11}{4}$$

Kolkaa $M_1 \neq M_2$. Markaa L_1 iyo L_2 maaha barbarro. $M_1 \cdot M_2 \neq -1$. Markaa L_1 iyo L_2 iskuma qotomaan.

Tusaale 2:

Tus inuu barbarroole yahay, afargeesoolaha geesihi isu yihiin barahan:

A(− 2, − 1), B(3,3), C(9, − 1), iyo D(4, − 5).

Furfuris:

$$M_1 = \frac{3 - (-1)}{3 - (-2)} = \frac{4}{5}$$

$$M_2 = \frac{-1 - 3}{9 - 3} = \frac{-4}{6} = -\frac{2}{3}.$$

$$M_3 = \frac{-5 - (-1)}{4 - 9} = \frac{-4}{-5} = \frac{4}{5}$$

$$M_4 = \frac{-5\ (-1)}{4 - (-2)} = \frac{-4}{6} = \frac{-2}{3}$$

Mar haddii $M_1 = M_3$ isla markaa $M_2 = M_4$, kolkaa afargeesluhu waa barbarroole.

Layli:

1. Soo saar tiirada xarriiqda isku xirta barahan:

   b) (− 3 , − 2) , (− 4 , − 8)
   t) (5 , 0) , (− 5 , 9)
   j) (3 , − 5) , (− 7 , − 5)
   x) (− 1 , 9) , (0 , − 2)

2. Xarriiqda marta labada barood ee B iyo Q ma la barbarraa mise way ku qotontaa xarriiqda marta labada barood ee R iyo S.

   b) B(5 , − 2) Q(6 , 4);

t) B(5 , − 3) Q(9 , − 9);
j) B(− 2 , − 7) Q(− 2 , 3);
x) B(1 , − 1) Q(9 , 3);
kh) B(− 4 , − 1) Q(− 5 , 7);

R(6 , 7) iyo S(8 , 19)
R(0 , 7) iyo S(− 8 , 1)
R(− 6 , 2) , S(− 6 , − 9)
R(− 4 , 3) , S(2 , − 9)
R(4 , 5) , S(6 , − 9)

Baraha B, Q, R iyo S waxay yihiin geesaha afar-gees. Sheeg shaxanku inuu yahay barbarroole, koor, laydi, ama intaa midnaba.

b) B(2 , 0), Q(9 , 1),
t) B(0 , 0), Q(4 , 3),
j) B(− 5 , − 1) , Q(− 1 , − 7),
x) B(− 5 , 1), Q(2 , − 3),

R(11 , 6) , S(4 , 4)
R(14 , 2) , S(12 , 6)
R(8 , − 1), S(5 , 5)
R(7 , 2) , S(1 , 6)

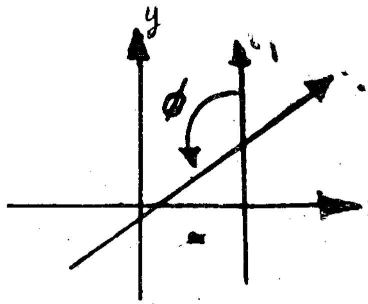
4. Adoo adeegsanaya jidka lagu soo saaro tiirada tus in barahani A(8 , 6), B(4 , 6) C(2 , 5) ay yihiin geesaha saddexagal qumman.

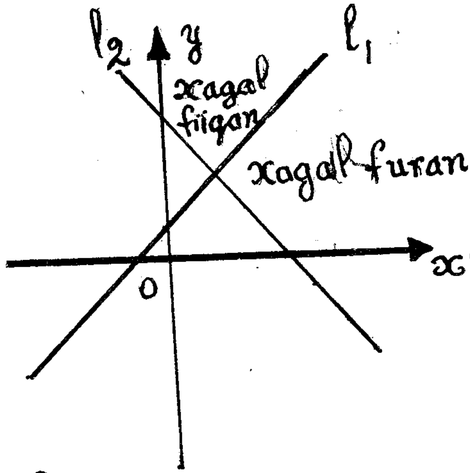
5. Tus in saddexdan barood A(− 3 , 4) , B(3 , 2) iyo C(6 , 1) ay xarriiqda wadaag yihiin.

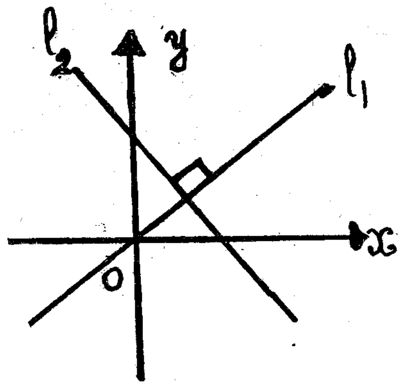
6. Tus in saddexagalka geesihiisu yihiin barahan (0 , 0), (− b , a) , (a , b) uu yahay saddexagalka qumman.

XAGASHA U DHAXAYSA LABA XARRIIQOOD:

Haddii ay laba xarriiqood isgooyaan waxay sameeyaan afar xaglood. Xagal kasta oo afarta ka midihi waxay noqon kartaa 90° ama labada xarriiqood waxay sameeyaan laba xaglood oo fiiqan oo isle'eg iyo laba xaglood oo furan oo isle'eg.

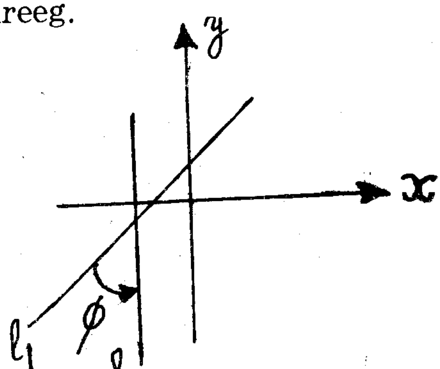


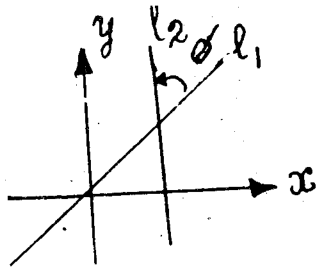




Qeex:

Xagasha u dhexaysa laba xarriiqood oo isgooya waxa tahay xagasha ϕ, ee dhinac bilowgeedu yahay L_1 dhinac dhammaadkeedu yahay L_2 marka loo cabbirro lid saacad-wareeg.



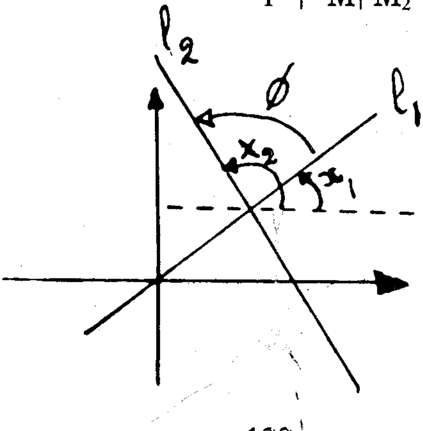


Haddii laba xarriiqood ay yihiin barbarro waxaa muuqata'inayna isgooynin. Marka xagasha u dhexaysaa waa eber. Labada xarriiqood ee isku qotoma xagasha u dhexaysa waa 90°. Haddaba xagasha u dhexaysa laba xarriiqood waxay mar kasta u dhexaysa 0° iyo 180°. Sidaasi waxay tahay $0° \leq \phi \leq 180°$. Sidee baa loo soo saaraa xagasha u dhexaysa labada xarriiqood ee isgooya?

A r a g t i i n :

Haddii L_1 iyo L_2 ay yihiin laba xarriiqood (iskuma qotomaan) oo isgooya oo tiirooyinkooduna yihiin M_1 iyo M_2, xagasha u dhexaysa labada xarriiqood waxa ina siiya jidkan.

$$\tan \phi = \frac{M_2 - M_1}{1 + M_1 M_2}$$



Caddayn:

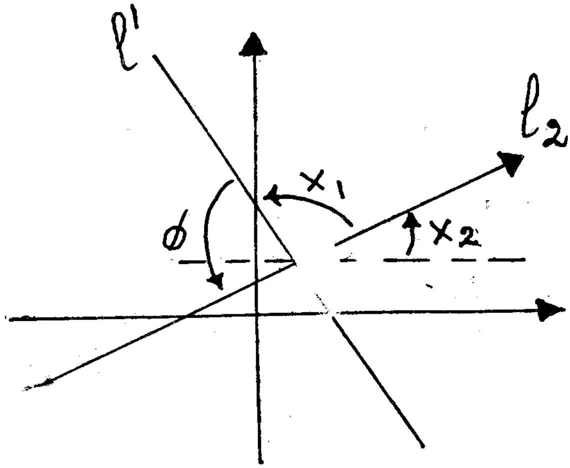
a) $\alpha_2 > \alpha_1$ adoo shaxanka adeegsanaya waxaad aragtaa in $\phi = \alpha_2 - \alpha_1$.
Markaa $\tan \phi = \tan (\alpha_2 - \alpha_1)$. Kolkaa inagoo isticmaalayaasha midaal trignoomatari oo la yaqaanno,

$$\tan \phi = (\alpha_2 - \alpha_1) = \frac{\tan \alpha_2 - \tan \alpha_1}{1 + \tan \alpha_1 \tan \alpha_2}$$

Kolkaa $\tan \alpha_1 = m_1$, $\tan \alpha_2 = M_2$.

Markaa $\tan \phi = \dfrac{M_2 - M_1}{1 + M_1 M_2}$.

b) Adoo adeegsanaya shaxankan:
$\phi = 180° - (\alpha_1 - \alpha_2) = 180° + \alpha_2 - \alpha_1$.
Kolkaa $\tan \phi = \tan [180° + (\alpha_2 - \alpha_1)]$ kolkaa sidii (a) oo kale.



$$\text{Tan } \phi = \tan (\alpha_2 - \alpha_1)$$

$$= \frac{\tan \alpha_2 - \tan \alpha_1}{1 + \tan \alpha_1 \tan \alpha_2} = \frac{M_2 - M_1}{1 + M_1 M_2}$$

Marka xagasha u dhexaysa L_1 iyo L_2 waxa mar kasta

agu helaa Tan $\phi = \frac{M_2 - M_1}{1 + M_1 M_2}$. Haddii $\frac{M_2 - M_1}{1 + M_1 M_2}$ ay tahay tiro taban.

Kolkaa ϕ wuxuu yahay (a) xagal furan, taas oo ah:

$$90^\circ < \phi < 180^\circ$$

Haddii $\frac{M_2 - M_1}{1 + M_1 M_2}$ ay tahay tiro togan.

Kolkaa ϕ wuxuu yahay (b) xagal fiiqan, sidaasi waxay tahay in:

$$0^\circ < \phi < 90^\circ$$

Tusaale 1:

a) Soo saar taanjanka xagasha ϕ ee ka bilaabta xarriiqda L_1 ee marta baraha $(-3, -1)$ iyo $(1, 15)$ ilaa xarriiqda L_2 ee marta baraha $(-4, 6)$ iyo $(-1, 5)$.

Furfuris:

tiirada xarriiqda L_1, $M_1 = \frac{15 - (-1)}{1 - (-3)} = \frac{16}{4} = 4$

tiirada xarriiqda L_2, $M_2 = \frac{5 - 6}{-1 - (-4)} = \frac{1}{3}$. Kolkaa

$$\tan \phi = \frac{M_2 - M_1}{1 + M_1 M_2} = \frac{-1/3 - 4}{1 + 4(-1/3)} = \frac{-4\ 1/3}{-1/3} = \frac{13}{}$$

b) Soo saar tanjanka xagasha Θ ee u dhexaysa L_2 iyo L_1.

Furfuris:

tiirada xarriiqda L_2 , $M_1 = -\frac{1}{3}$ tiiarada xarriiqda

$L_1 M_2 = 4$. Kolkaa $\tan \Theta = \frac{4 - (-1/3)}{1 + -1/3\,(4)} = -13.$

$$\text{Tan } \Theta = -\tan \phi = -13$$

Tusaale 2:

Xagasha u dhexaysa labada xarriiqood ee L_1 iyo L_2 waa $45°$ Haddii tiirada xarrjiqda L_1 ay tahay $M_1 = \frac{2}{3}$ soo saar tiiarada xarriiqda L_2 oo ah M_2.

Furfuris:

$$\text{Tan } 45° = \frac{M_2 - M_1}{1 + M_1 M_2}$$

$$1 = \frac{M_2 - \frac{2}{3}}{1 + \frac{2}{3} M_2}.$$

Kolkaa $M_2 = 5$.

Layli:

1. Soo saar tanjannada xagalaha gudaha ee saddexagalka geesihiisu yihiin $A(-3, -2)$, $B(2, 5)$ $C(4, 2)$.

   Jaw. $\left[\tan A = \frac{25}{63} ;\ \tan B = \frac{29}{11} ;\ \tan C = \frac{29}{2}\right]$.

2. Xagasha u dhexaysa labada xarriiqood ee mid maro baraha $(-4, 5)$ iyo $(3, y)$ midka kalana maro baraha $(-2, 4)$ iyo $(9, 1)$ waa $135°$ Soo saar qiimaha y.

   Jaw. $y = 9$.

3. Soo saar tiirada xarriiqda la samaysa xagal ah $45°$ xarriiqda marta barahan $(2, -1)$ iyo $(5, 3)$. (Jaw. $M_2 = -7$).

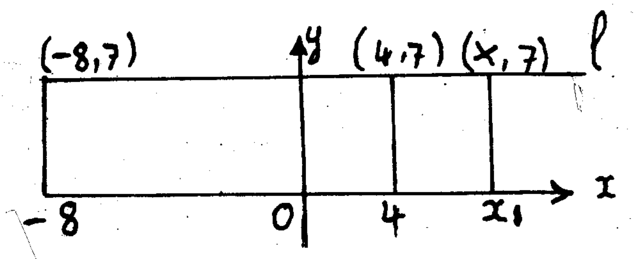
4. Soo saar isle'egta xarriiqda marta $(2, 5)$ ee la samaysa xagal ah $45°$ xarriiqda isle'egteedu tahay $x - 3y + 6 = 0$.
   Jaw. $2x - y + 1 = 0$.

5. Soo saar xagasha fiiqan ee u dhexaysa labada xarriiqood ee mid marto baraha $(-1, -4)$ iyo $(9, 1)$ ta kalana marto baraha $(1, 5)$ iyo $(5, -1)$.
   Jaw. $8°$.

XARRIIQ TOOSAN:

Hadda ka horrow waxaan soo aragnay in tiirada xarriiqda aan qotomin ee marta laba barood (x_1, y_1) iyo (x_2, y_2) ay tahay $M = \frac{y_2 - y_1}{x_2 - x_1}$. Imikana waxaan rabnaa inaan hello isle'egta xarriiqda toosan ee isku xirta labada barood ama isle'egta xarriiq.kasta oo toosan.

U qaado in xarriiqda ku sawiran shaxanka ay la barbarro tahay dhidibka — x. Kolkaa waxaan aragnaa in bar kasta oo ku taal xarriiqda la barbarro ah dhidib — x ay leedahay kulan — y oo ah 7. Bar kasta oo ku taal xarriiqda la barbarro ah dhidib — x waxay u jirtaa dhidibka — x 7, halbeeg. Markaa xarriiqdaasi waxay dhidibka — x u jirtaa 7 halbeeg. Kolkaa isle'egta xarriiqdaasi waa y = 7. Guud ahaan haddii xarriiqda L ay la barbarro tahay dhidibka — x una jirto b halbeeg, isle'egta xarriiqdaasi waa y = b.



Sidaas oo kale haddii xarriiqi ay la barbarro tahay dhidibka — y una jrto «a» halbeeg dhidibka — x, isle'egta xarriiqdaasi waa x = a. Markaa haddii tiirada xarriiqda la barbarro ah dhidibka — x tahay eber, isle'egta xarriiqdaasi waa y = b. Haddii xarriiqdu la barbarro tahay dhidibka — y waxaan naqaannaa in tiiradeedu ayna qeexnayn. Laakiin sle'egta xarriiqdaasi waa x = a.

Tusaale 1:

Qor isle'egta xarriiqda marta barta (— 3 , 6) ee tiiradeedu tahay eber.

Furfuris:

y = b.

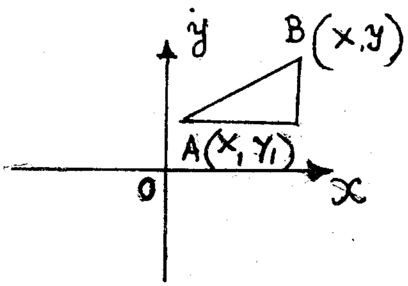
Tusaale 2:

Qor isle'egta xarriiqda tiiradeedu ayna qeexnayn ee marta barta (11 , 9).

Furfuris:

$x = 11$. Haddaba, haddaan ka gudubno xarriiqda la barbarro ah dhidibyada sidee loo soo saara isle'egta xarriiqda aan la barbarro ahayn labada dhidib midnaba? Haddii xarriiqda L ee aan taagneyn ay marto $A(x_1, y_1)$, oo bartan kale $B(x, y)$ ay taallo xarriiqda, kolkaa tiirada xarriiqdani waa $M = \dfrac{y - y_1}{x - x_1}$.

Haddii sansaanta $M = \dfrac{y - y_1}{x - x_1}$ loo qoro sida sansaan-



tan $y - y_1 = M(x - x)$ waxay noqonaysaa isle'eg. Waxaana la yiraa saansaankan $y - y_1 = M(x - x_1)$ saansaanka bar-tiiro ee isle'egta xarriiq toosan.

Tusaale 3:

Soo saar isle'egta xarriiqda marta bartan (11, 15) tiiradeeduna tahay 2.

Furfuris:

$$\frac{y - 15}{x - 11} = 2 \text{ ama } y - 15 = 2(x - 11),$$
$$y - 2x + 7 = 0.$$

Saansaanka bar-tiiro ee isle'egta xarriiqda toosan ee marta barta (x_1, y_1) waa $y - y_1 = M(x - x_1)$. Haddaba haddii xarriiqda marta bar kale oo ah (x_2, y_2) tiiradeedu waa $M = \frac{y_2 - y_1}{x_2 - x_1}$. Kolkaa haddaan $\frac{y_2 - y_1}{x_2 - x_1}$ ku

beddello M waxaan helaynaa $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Isle'egta waxa la yiraa saansaanka laba-barood ee sle'egta xarriiqda toosan.

Tusaale 4:

Soo saar isle'egta xarriiqda marta baraha $(-7, -3)$ iyo $(-1, -9)$.

Furfuris:

Adoo isticmaalaaya saansaanka laba barood ee isle'egta xarriiqda toosan iyo labada barood mid ahaan. Soo saar isle'egta la rabo?

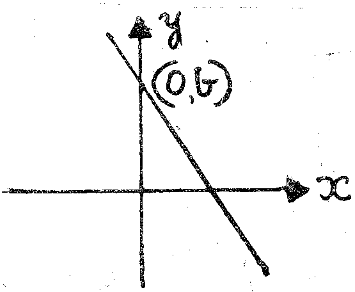
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-3) = \frac{9 - (-3)}{-1 - (-7)}(x - (-7))$$

$$y + 3 = \frac{12}{6}(x + 7)$$

$$y + 3 = 2(x + 7)$$
$$y - 2x - 11 = 0$$

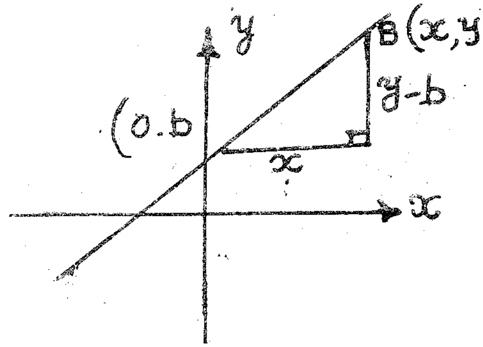
Baraha uu ka gooyo garaafku dhidibyada waxaa la yiraa Tikraarro. Barta u ka gooyo garaafku dhidibka – y waa tikraarka y. Barta u ka gooyo garaafku dhidibka x waa tikraarka x. Markaa xarriiq kasta oo aan la barbarro ahayn dhidibka — y wuxuu dhidibka — y. Ka gooyaa bar sida (0 , b). Tirada b waxa la yiraa tikraarka y ee xarriiqda. Saansaanka bar-tiirada ee xarriiqda marta barta (x_1, y_1) waa $y - y_1 = M(x - x_1)$.



Haddii Tikraarka y u yahay b, isle'egta kor ku qorani waxay noqonaysaa $y - b = M(x - 0)$ ama $y = Mx + b$. Isle'egta $y = Mx + b$ waxaa la yiraa saansaanka tiirotikraar ee isle'egta xarriiqda toosan. Sidani waa si gaar ah oo lagu keenay saansaanka bar-tiiro. Waayo tikraarka y waa barta (0 , b).

Caddayn kale:

Barta ay xarriiqdu ka goyso dhidibka — y u qaado inay tahay (0 , b). Haddaba haddii xagal janjeerku yahay α, tiirada $M = \tan \alpha = \dfrac{y - b}{x}$ ama $Mx = y - b$ markaa $y = Mx + b$.



Tusaale 5:

Soo saar isle'egta xarriiqda tikraarka y yahay -3, tiiradeeduna tahay $\frac{5}{6}$.

Furfuris:

Adoo adeegsanaaya saansaanka tiiro-tikraar ee u

$y = Mx + b$ waxaad heli in $y = \frac{5}{6}x - 3$

Saansaanka tikraar:

U qaado tikraarrada xarriiqdu inay yihiin (a , 0). Adoo isticmaalaaya saansaanka laba-barood ee isle'egta xarriiq isle'egtu waa:

$y - b = \frac{-b}{a - 0}(x - 0)$ ama $y - b = \frac{-bx}{a}$ ama

$$ay + bx = ab$$

U qaybi tibix kasta a b. Markaa waxaan helayna

$$\frac{y}{b} + \frac{x}{a} = 1$$

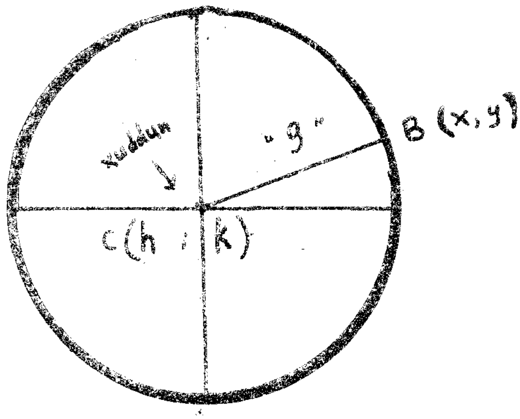
Isle'egta waxaa la yiraa saansaanka tikraarka ee isle'egta xarriiq.

Caddayn kale:

Shaxankan waxaan ka aragnaa in saddexagalka B R A iyo saddexagalka C O A ay isku eg yihiin.

$$\text{Kolkaa } \frac{RB}{OC} = \frac{RA}{OA} \text{ ama } \frac{y}{b} = \frac{a - x}{a} \text{ ama } \frac{y}{b} = \frac{1 - x}{a}$$

$$\text{ama } \frac{y}{b} + \frac{x}{a} = 1.$$



Tusaale 6:

Soo saar isle'egta xarriiqda tikraarkeeda y u yahay — 3. Tikraarkeeda x u yahay — 7.

Furfuris:

$$\frac{y}{b} + \frac{x}{a} = 1$$

$$\frac{y}{-3} + \frac{x}{-7} = 1.$$

Ogow:

Saansaankani waa saansaan gaar ahaaneed oo ah laba-barood isle'egta xarriiqda leedahay. Labada barood waa: (0 , b) iyo (a , 0).

SAANSAANKA GUUD AHAANEED EE ISLE'EGTA XARRIIQDA TOOSAN:

Isle'egta saansaankeedu yahay $AX + BY + C = 0$ ee A iyo B ayna labaduba ahayn eber waxa weeye isle'eg heerkeedu yahay 1 oo leh labada doorsoome, ee x iyo y.

A r a g t i i n :

Xarriiq kasta oo ku taal sallax waxay leedahay isle'-egta saansaankeedu yahay $AX + BY + C = 0$ ee A iyo B labaduba ahayn eber.

C a d d a y n :

Xarriiq kasta oo toosan waxa loo qori karaa sida:

1. $y = b$ isle'egta xarriiqda la barbarro ah dhidibka x.

2. $x = a$ isle'egta xarriiqda la barbarro ah dhidibka y.

3. $y - y_1 = M(x - x_1)$ ama $y - mx + (Mx_1 - y_1) = 0$ isle'egta xarriiqda aanaan la barbarro ahayn labada dhidib midnaba.

Kolkaa isle'egta (1) : $A = 0$, $B = 1$, $C = -b$

» (2) : $A = 1$, $B = 0$, $C = -a$

» (3) : $A = -M$, $B = 1$, $C = Mx_1 - y_1$.

Kolkaa xarriiq kasta oo toosani waxay tahay sida saansaankan $Ax + By + C = 0$ ee A iyo B labadaba ahayn eber.

A r a g t i i n 2 :

Isle'eg kasta oo saansaankeedu yahay $Ax + By + C = 0$ oo A iyo B ayna labaduba eber ahayn waa isle'egta xarriiq toosan.

Caddayn:

Mar haddii A iyo B ayna labaduba noqon karin eber, waxaan haysanaa saddex xaaladood.

Sida 1. Haddii $A \neq 0$, $B \neq 0$, isle'egteena waxaan u qori karnaa sidan: $y = \frac{-A}{B} x - \frac{C}{B}$.

Isle'egtaasi waxay u qoran tahay sidan saansaanta tiir-tikraarka ee isle'egta xarriiq oo ah $y = Mx + b$.

Kolkaa $M = \frac{-A}{B}$, $b = \frac{-C}{B}$.

Kolkaa $Ax + By + C = 0$ waa isle'eg xarriiqeed.
Sida 2. Haddii $A = 0$, $B \neq 0$, kolkaa isle'egtu waxay noqonaysaa sida: $y = \frac{-C}{B}$.

Kolkaa $y = \frac{-C}{B}$ waa isle'egta xarriiqda la barbarro ah dhidibka $- x$.

Sida 3. Haddii $B = 0$, $A \neq 0$. Markaa isle'egta $Ax + By + C = 0$ waxay noqonaysaa $Ax + C = 0$ ama $x = -\frac{C}{A}$.

Kolkaa $x = -\frac{C}{A}$ waa isle'egta xarriiqda barbarro la ah dhidibka y. Markaa xaalad kasta, waa isle'egta xarriiqeed $Ax + By + C = 0$ A, B iyo C ay qiime kasta qaataan, laakiin A iyo B aayna labadoodu eber wada ahayn.

Layli:

1. Soo saar isle'egta xarriiqda haddii lagu siiyo:

b) $M = 4$, (-5)
Jaw. $y = 4x + 17$.

t) $M = \frac{-5}{6}$, $(3, -4)$

Jaw. $6y + 5x + 9 = 0$.

j) $(8, 1)$, $(1, 2)$

Jaw. $7y + x - 15 = 0$.

x) $M = 0$, $(-2, 9)$

Jaw. $y = 9$.

kh) $(-1, 7)$, $(7, -2)$

Jaw. $8y + 9x - 17 = 0$.

d) $M = \frac{1}{5}$, Tikraarka $x = 5$

Jaw. $5y - x + 5 = 0$

r) $M = -7$, Tikraarka $y = 0$.

Jaw. $y = -7x$.

c) Tikraarka $x = 8$, Tikraarka $y = -9$.

Jaw. $8y + 9x - 72 = 0$.

2. Soo saar tiiarada iyo tikraarka y ee xarriiqda
$x + 7y + 5 = 0$.

Jaw. $M = -\frac{1}{7}$, tik. $y = -\frac{5}{7}$.

3. Soo saar labada tikraar ee xarriiqda:

$x + 4y - 7 = 0$. Jaw. tik. $x = 7$, tik. $y = \frac{4}{7}$.

Soo saar isle'egta xarriiqda marta barta $(-3, 8)$ lana barbarro ah xarriiqda $7x + 2y + 9 = 0$. Jaw. $7x - 2x - 55 = 0$.

6. Tus in ay $\frac{A}{A^1} = \frac{B}{B^1}$ haddii labada xarriiqood ee

$Ax + By + C = 0$ $A^1x + B^1y + C^1 = 0$ ay barbarro yihiin. Haddii ay isku qotomaana inay

$AA^1 + BB^1 = 0$.

7. Soo saar isle'egta xarriiqda ah qotome badhaha xrriijinta isku xirta labada barood $(7, 4)$ iyo $(-1, -2)$.

Jaw. $4x + 3y - 15 = 0$.

8. Soo saar isle'egta xarriiqda marta $(2, -3)$ ee xagal janjeerkeedu yahay $60°$.
Jaw. $\sqrt{3}x - y - 3 - 2\sqrt{3} = 0$.

9. Soo diir isle'egta xarriiqda marta labadan barood (x_1, y_1) iyo (x_2, y_2).

10. Soo diir isle'egta xarriiqda tikraaradeedu yihiin $(a, 0)$ iyo $(4, 2)$.

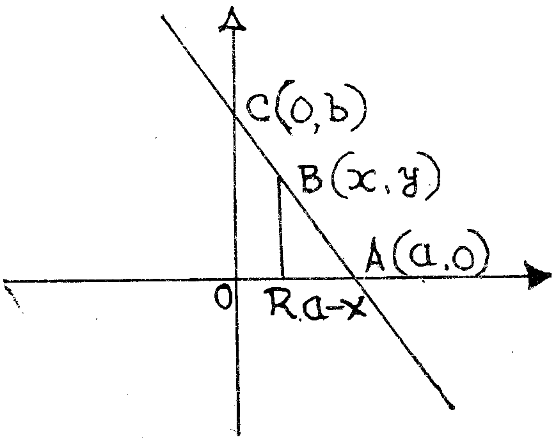
11. Soo diir isle'egta xarriiqda marta $B_1(x_1, y_1)$ haddii tiirada xarriiqdu tahay M.

12. Soo diir isle'egta xarriiqda tiiradeedu tahay M, tikraarkeedu – y na yahay $(0, b)$.

GOOBO:

Qeex:

Goobadu waa ururka dhammaan baraha sallaxa ee la ogyahay u wada jira bar maguuraan ah. Fogaanta la ogyahay waxa la yiraa gacanka goobada astadiisuna waa r. bar maguuraan kana waxa la yiraa xuddunta goobada. U qaado in barta C(h , k) ay tahay xuddunta goobada gacankeedu yahay «r». Marka barta B(x , y) waxay ka mid tahay goobada haddii iyo haddii qura $\sqrt{BC^2} = r$. Kolkaa haddaad isticmaashid jidka fogaanta:



$$|BC| = \sqrt{(x-h)^2 + (y-k)^2} = r \quad (1).$$

Isle'egta (1) waa isle'egta goobada.
(a) Haddii la labo jibbaaro labada dhinac ee isle'egta (1). waxaan helaynaa in $(x-h)^2 + (y-k)^2 = r^2$ ———(2)

Markaa baraha kulammadeedu raaligeliyaan isle'egta (1) Kulammadeedu way raaligeliyaan isle'egta (2) Kolkaa bar kasta oo ka mid ah baraha goobada xudduntecdu tahay C(h , k), gacankeeduna yahay «r» way raaligeliyaan isle'egta (2). Markaa aan labo Jibbaarro isle'egta (1) ee aan hello isle'egta (2) waxa jira baro kulammadoodu raaligeliyaan isle'egta (2) laakiin aanay raaligelin isle'egta (1). Sidaas darteed waa inaan xagga kalana ka fiiriraa oo aan niraa bar kasta oo kulammadeeda x iyo y ay raaligeliyaan isle'egta (2) Kulammadeedu way raaligeliyaan isle'egta (1) T. a. roganta hawraata hore waa run. (b) haddii x iyo y ay raaligeliyaan isle'gtan $(x-h)^2+(y-k)^2=g$ ——— (2) oon qaadanno xidid jibbaarka labada dhinac ee isle'egta (2), kolkaa x iyo y way raaligeliyaan:

$$\sqrt{(x-h)^2+(y-k)^2}=r \text{ ama } \sqrt{(x-h)^2+(y-k)^2}=-r$$

Laakiin $\sqrt{(x-h)^2+(y-k)^2}$ way togan tahay, $-r$ na wuu taban yahay.

Sidaas darteed $\sqrt{(x-h)^2+(y-k)^2}=-r$ malaha furfuris dhab ah. Markaa barta $B(x, y)$ way raaligelisaa isle'egtan $(x-h)^2+(y-k)^2=g^2$ haddii iyo haddii qura ay raaligeliso isle'egta $\sqrt{(x-h)^2+(y-k)^2}=g$. Laakiin barta $B(x, y)$ way raaligelisaa:

$$\sqrt{(x-b)^2+(y-k)^2}=r.$$

Haddii iyo haddii qudha ay bartu ka mid tahay goobada xudunteedu tahay $C(h, k)$, gacankeeduna yahay r.

Markaa waxaan tusnay in $(x-h)^2+(y-k)^2=r^2$ ay tahay isle'egta goobada xuddunteedu tahay $C(h, k)$, gacankeeduna yahay r. Haddii xuddunta goobadu ay tahay unugga isl'egta goobadu waxay noqonaysaa $x^2+y^2=r^2$.

Aragtiin:

Isle'eg kasta oo saansaankeedu yahay:

$$x^2+y^2+Dx+Ey+F=0$$

waxaa weeye garaafka goobada xudduntεedu tahay

$$C\left(\frac{-d}{2}, \frac{-E}{2}\right), \text{ gacankeeduna yahay } r=\frac{1}{2}\sqrt{D^2+E^2-4F}$$

Gaddayn:

Waxa la ina siiyay. $x^2 + y^2 + Dx + Ey + F = 0$. Haddii aan u qoranno isle'egta sidan:

$$x^2 + Dx + y^2 + Ey + F = 0$$

oon dhammaynno laba jibbaarka, waxaan helaynaa:

$$x^2 + Dx + \frac{D^2}{4} + y^2 + Ey + \frac{E^2}{4} = \frac{D^2}{4} + \frac{E^2}{4} - F$$

$$\text{ama } \left(x + \frac{D}{2}\right)^2 + \left(y + \frac{E}{2}\right)^2 = \frac{D^2 + E^2 - 4F}{4} \qquad (1)$$

Markaa haddii aan garab dhigno beegalka isle'egta goobada oo ah $(x - h)^2 + (y - k)^2 = r^2$ waxaan ogaanaynaa in isle'egta (1) ay ka joogto goobo xuddunteedu tahay $C\left(-\frac{D}{2}, -\frac{E}{2}\right)$, gacankeeduna yahay:

$$g = \frac{1}{2}\sqrt{D^2 + E^2 - 4F}$$

1) **Kolkaa haddii $D^2 + E^2 - 4F > 0$ goobadu waa dhab.**

2) **Haddii $D^2 + E^2 - 4F < 0$, goobadu ma jirto.**

3) **Haddii $D^2 + E^2 - 4F = 0$, gacanka goobadu waa eber; isle'egteeduna waa barta: $\left(-\frac{D}{2}, -\frac{E}{2}\right)$**

Tusaale 1:

Soo saar isle'egta goobada xuddunteedu tahay barta $(-2,3)$, gacankeeduna yahay 4.

Furfuris:

$(x-h)^2 + (y-k)^2 = r^2$ laakiin $C(h,k) = C(-2,3)$, $y = 4$. Markaa $(x+2)^2 + y-3)^2 = 4^2$ ama
$x^2 + y^2 + 4x - 6y - 3 = 0$

Tusaale 2:

Haddii $x^2 + y^2 - 3x + 5y - 14 = 0$ ay tahay isle'eg goobo waxaad soo saartaa kulammada xuddunta goobada iyo gacankeeda adoo isticmaalaya (b) habka dhammaynta laba jibbaarka iyo (t) jidka.

Furfuris:

b) $$x^2 - 3x + \frac{9}{4} + y^2 + 5y + \frac{25}{4} = 14 + \frac{9}{4} + \frac{25}{4}$$

$$\text{ama} \quad \left(x - \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{90}{4}$$

$$\text{Xuddun}: \ C\left(\frac{3}{2}, -\frac{5}{2}\right) \quad \text{Gacan}: \ r = 3\sqrt{\frac{10}{2}}$$

t) $$h = -\frac{D}{2} = \frac{3}{2}, \ k = \frac{E}{2} = -\frac{5}{2}$$

$$\text{Xuddun}: \ C\left(\frac{3}{2}, -\frac{5}{2}\right)$$

$$\text{Gacan} = G = \frac{1}{2}\sqrt{D^2 + E^2 - 4F}$$

$$= \frac{1}{2}\sqrt{9 + 25 + 56} = \frac{1}{2}\sqrt{90}$$

$$= \frac{3\sqrt{10}}{2}$$

Tusaale 3:

Soo saar isle'egta goobada dhexroorkeedu yahay xarriijinta isku xirta labadan barood ee ah $(5, -1)$ iyo $(-3, 7)$.

Furfuris:

Soo saar kulammada bar badhtanka xarriijinta, taas oo ah xuddunta goobada.

$$h = \frac{5 - 3}{2} = 1 \;,\; k = -\frac{1 + 7}{2} = 3$$

kokaa C $(1, 3)$ waa xuddunta, gacankuna waa

$$G = \sqrt{(5 - 1)^2 + (-1 - 3)^2}$$
$$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$$

Kolkaa $(x - 1)^2 + (y - 3)^2 = 32$

Ama $x^2 + y^2 - 2x - 6y - 22 = 0$
waa isle'gta goobada.

Tusaale 4:

Soo saar isle'egta goobada marta barahan :

$(5, 3)$, $(6, 2)$ iyo $(3, -1)$.

F u r f u r i s :

Waxaan naqaannaa in isle'egta guud ahaaneed ee goobadu ay tahay,

$(x-h)^2 + (y-k)^2 = r^2$...(1) iyo

$x^2 + y^2 + Dx + Ey + F = 0$ ··· (2).

Ma naqaanno qiimaha ma doorsoomayaasha D, E, iyo F. Ku beddelo kulammada baraha ay marto goobadu X iyo Y adoo isticmaalaya isl'egta (2)
Kolkaa waxaan helaynaa :

(1) $25 + 9 + 5D + 3E + F = 0$

(2) $36 + 4 + 6D + 2E + F = 0$

(3) $9 + 1 + 3D - E + F = 0$

U furfuro isle'egyadaas sidaad u furfuri jirtay habadiska isle'egyada toosan adoo marna qaadanayaa (1) iyo (2) marna (1) iyo (3). Kolkaa labada isle'eg ee ka soo baxa u furfuro sidii kuwii hore. Markaas $D = -8$, $E = -2$, $F = 12$. Ku beddel qiimaha D, F iyo E isle'egta (2) ee goobada, kolkaa isle'egta goobadu waxay noqonaysaa :

$$x^2 + y^2 - 8x - 2y + 12 = 0$$

L a y l i s :

1. Soo saar isle'egta goobada :

   b) Xuddunteedu tahay (2, −5) martana barta (−3,2).

   t) Xuddunteedu tahay (−3,4) martana barta (4,2).

2. Soo saar xuddunta iyo gacanka gooboooyinka isle'egyadoodu yihiin kuwa soo socda :

   b) $x^2 + y^2 - 8x - 6y + 9 = 0$

   t) $4x^2 + 4y^2 + 16x - 12y - 7 = 0$

j) $x^2 + y^2 - 1 = 0$

x) $x^2 + y^2 + x - 10y + 18 = 0$

kh) $3x^2 + 3y^2 - 2x - 12y + 11 = 0$

3. Soo saar isle'egta goobada dhexroorkeedu yahay xarriijinta isku xirta labadan barood (2 , B) iyo (− 3, − 1).

4. Soo saar isle'egta goobadu marto saddexdan

b) (4,5), (3,2), iyo (1,—4)	b) $x^2 + y^2 + 7x - 5y - 44 = 0$
t) (8,—2), (6,2), iyo 3,—7)	t) $x^2 + y^2 - 6x + 4y - 12 = 0$
j) (1,1), (1,3), iyo (9,2)	j) $8x^2 + 8y^2 - 79x - 32y + 95 = 0$
x) (—4, —3), (—1, —7) iyo (0,0)	x) $x^2 + y^2 + x + 7y = 0$
kh) (1,2), (3,1), iyo (—3,—1)	kh) $x^2 + y^2 - x + 3y - 10 = 0$

5. Soo saar isle'egta goobada xuddunteedu tahay (−4,2) taabteheeduna yahay xarriiqda $3x + 4y - 16 = 0$.
Jaw. $x^2 + y^2 + 8x - 4y + 4 = 0$

6. Soo saar dheerarka taabtaha ka yimaada barta B (x, y) ee goobada isle'egteedu tahay $(x - h)^2 + (y - k)^2 = r^2$

$\sqrt{(x - h)^2 + (y - k)^2 - g^2} = T$

7. Soo saar isle'egta goobada marta (−2,1), taabteheeduna yahay xarriiqda $3x - 2y - 6 = 0$ martana barta (4,3).

Jaw. $7x^2 + 7y^2 + 4x - 82y + 55 = 0$.

8. Soo saar isle'egta goobada la xuddun ah goobada isle'egteedu tahay $x^2 + y^2 - 3x + 4y - 10 = 0$ martana barta (−3,0)

9. Soo saar isle'egta goobada ku dhex meeraan, saddexagalka xarriiqyada sameeyay ay yihiin :

$L_1 : 4x - 3y - 65 = 0 \quad L_2 : 7x - 24y + 55 = 0$

$L_3 : 3x + 4y - 5 = 0$

Jaw. $x^2 + y^2 - 20x + 75 = 0$

10. Soo saar isle'egta goobada taabteheedu yahay dhidibka — X lana xuddun ah goobada isle'egteedu tahay

$2x^2 + 2y^2 - 11x + 6y - 8 = 0$

Jaw. $\left(x - \frac{11}{4}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{9}{4}.$

11. Soo saar isle'egta goobada meeraysa saddexagalka xarriiqyada sameeyaa yihiin.

$L_1 : x + y = 8,$

$L_2 : 2x + y = 22$

$L_3 : 3x + y = 22$

Jaw. $x^2 + y^2 - 6x + 4y - 12 = 0$

12. Soo saar isle'egta goobada marta baraha (2,3) iyo (−1,1) ee xuddunteeduna ku taal xarriiqda

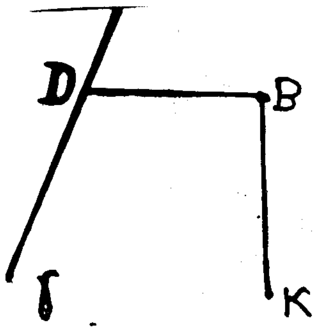
$x - 3y - 11 = 0$

Qeex: **Saab**

Saab waa ururka baraha in u wada jira xarriiq iyo bar maguuraan ah. Xarriiqda waxa la yiraa **Jeedshe**, bartana **Kulmis**.

Haddii L ay tahay xarriiq maguuraan ah; K na ay tahay bar maguuraan ah; kolkaa B waxay ka mid tahay Saabka haddii

$$|BD| = |BK|$$



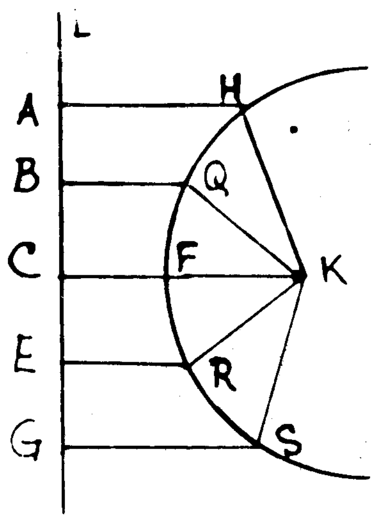
Qeexda saabka haddii aan u eegno si joomatari ahaaneed waxan helaynaa sawirka hoose oo kale:

Kolkaa haddii aan raacno qeexda waxan arkaynaa

in AH = HK, BQ = QK,

CF = FK, ER = EK

GS = SK.

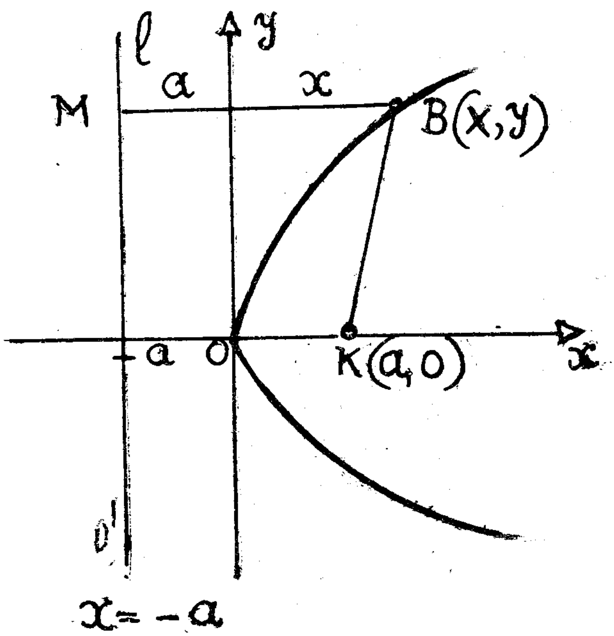


Qeexda saabka waxa la inagu siiyay si joomatari ahaaneed. Markaas si loo helo isle'egta Saabka waa inaan isticmaalnaa habdhiska kulammada. Ka soo qaad in geeska saabku uu ku yaallo ugugga; jeedshuhuna yahay xarriiqda $X = -a$.

U qaado in barta $B(x, y)$ ay ka mid tahay Saabka. Mar haddii $KB = BM$ innagoo adeegsanayna jidka fogaanta, waxaan helaynaa in

$$KB = \sqrt{(x-a)^2 + (y-0)^2}$$

$$BM = x + a.$$



Kolkaa $KB = BM = \sqrt{(x-a)^2 + (y-0)^2} = x + a$
Haddii aan labo jibbaarro labada dhinaca waxaan helaynaa in

$$(x-a)^2 + y^2 = (x+a)^2.$$

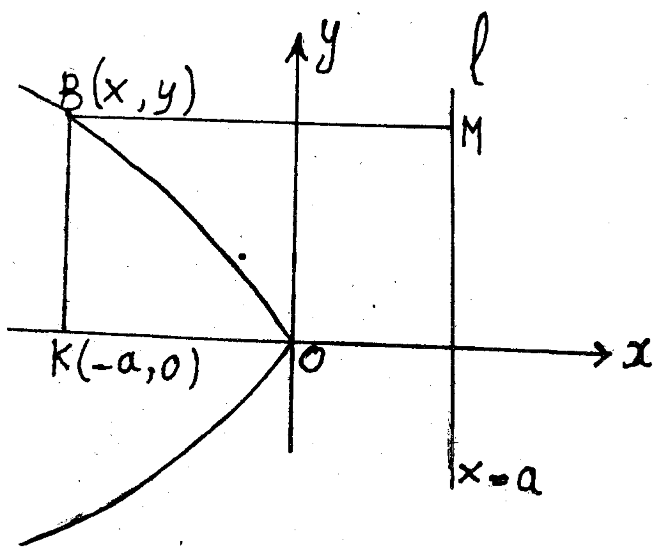
Ka bixi labada dhinacba. Markaa,

$x^2 - 2ax + a^2 = x^2 + 2ax + a^2$

Markaan fududaynno waxaan helaynaa in
$y^2 = 4ax$.

Kolkaa $y^2 = 4ax$ waxa weeye isle'egta Saabka.

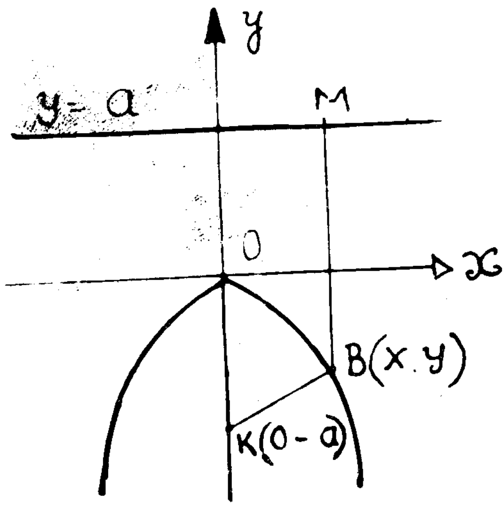
Waxaan ka aragnaa saansaanka isle'egta in Saabku ku wanqaran yahay dhidibka $-x$. Barta uu saabku ka jaro dhidibka wanqarka waxa weeye geeska saabka. Marka uu saansaanka isle'egta saabku yahay $y^2 = 4ax$, saabku wuxuu midigta ka xigaa jeedshaha. Kolkaa saabku wuxuu u furan yahay midigta. Haddii uu Kulmisku bidixda ka xigo jeedshaha isle'egta saabka saansaankeedu waa $y^2 = -4ax$. Markaa saabku wuxuu u furan yahay bidixda sida shaxankan :

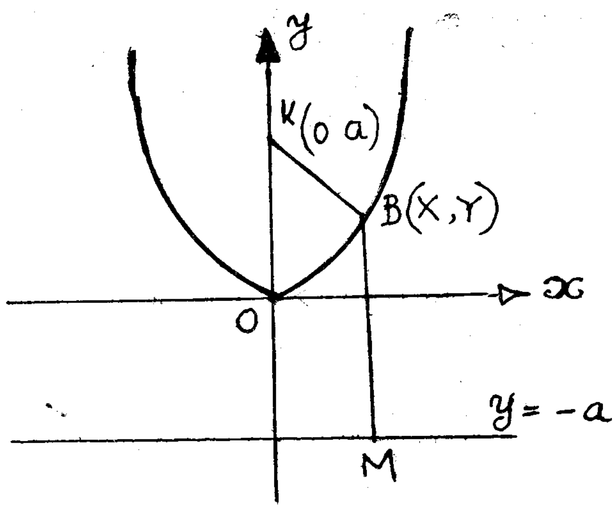


Haddii kulmisku yaallo dhidibka $-y$ saansaanka isle'egta saabka wuxuu yahay $x^2 = +4ay$.

Summaddu waxay ku tusaysaa kolba xagga uu kulmisku ka xigo jeedshaha.

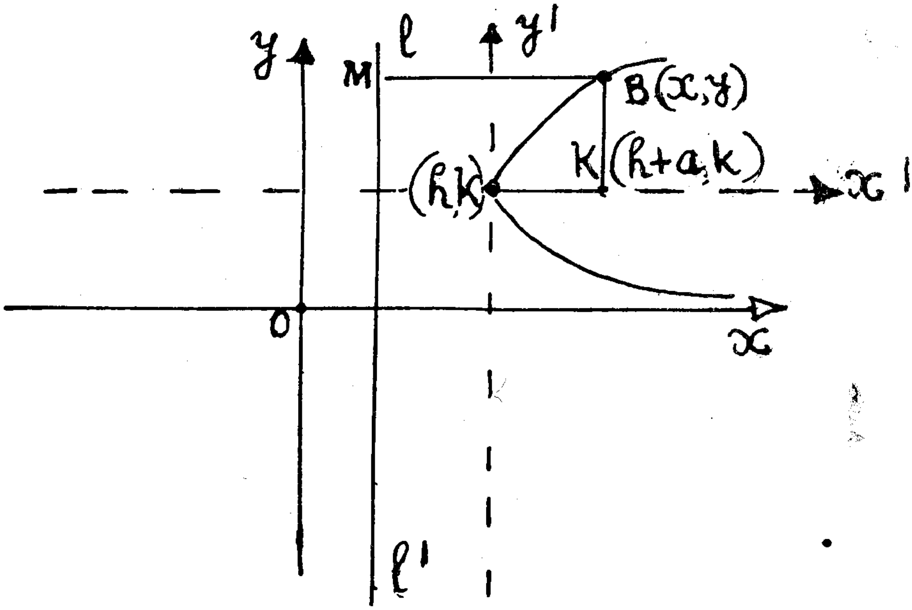
Tixgeli Shaxankan:





Ilaa hadda geeska saabku wuxuu ku yaallay unugga. Haddaba ka soo qaad in geeska saabku yahay barta (h,k) oo ku taalla xarriiq la barbarro ah dhidibka – X, kulmiskuna xagga midigta ka xigo geeska fogaan ah «a».

Isle'egta jeedshaha la barbarro ah dhidibka — Y una jira kulmiska fogaan ah «2a» waa $x = h - a$ ama $x - h + a = 0$.



U qaado in $B(x, y)$ ay tahay bar ka mid ah saabka, mar haddii $BK = BM$.

Kolka $\sqrt{(x-h-a)^2 + (y-k)^2} = x-h+a$

Ama $y^2 - 2ky + k^2 = 4ax - 4ah$

Ama $(y-k)^2 = 4a\ (x-l)$

Sidaas oo kale sansaannada kale waxay yihiin:

$(y-k)^2 = -\ 4a\ (x-h)$

$(x-h)^2 = 4a\ (y-k)$

$(x-h)^2 = -\ 4a\ (y-k)$

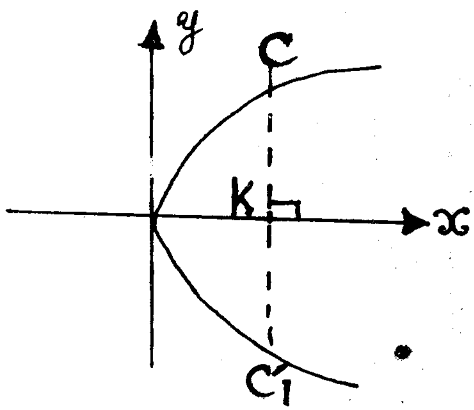
Markaa haddii la kala bixiyo isle'egyada saansaankoodu waxay noqonayaan :

$x = ay^2 + by + C$

$y = ax^2 + bx + C.$

Ogow:

Boqonka mara kulmiska ee ku qotoma labada dhidib kolba kii kulmisku yaallo waxa la yiraa **Taab.**



Dhererka taabku 4a waa horgalaha tibixda heerka kowaad. Waxa kale oo uu la mid yahay fogaanta u dhexaysa kulmiska iyo jeedshaha.

Tusaale 1:

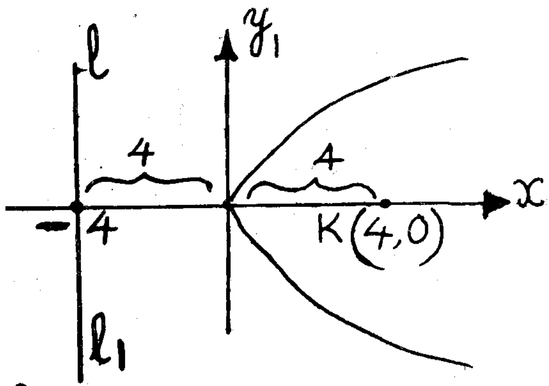
Soo saar isle'egta Saabka kulmiskiisu yahay (4,0), jeedshihiisuna yahay $x = -4$.

Furfuris:

Waxaan naqaannaa in saabkaas oo kale leeyahay isle'eg saansaankeedu yahay $y^2 = 4ax$.

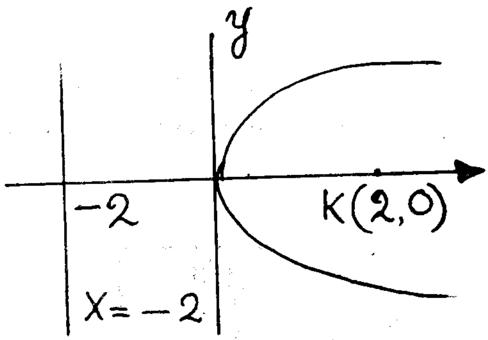
Markaa $a = 4.$

Markaa isle'egtu waa $y^2 = 16x.$



Tusaale 2:

Soo saar kulmiska iyo jeedshaha saabka $y^2 = 8x$, waa-shir garaafka.



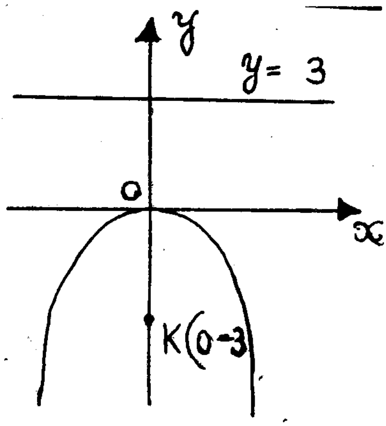
Furfuris:

$$y^2 = 4a\,x$$
$$4a = 8$$
$$a = 2.$$

Kolkaa kulmisku waa (2,0) jeedshuhuna waa $x = -2.$

Tusaale 3:

Soo saar kulmiska iyo jeedsha saabka $y^2 = -12x$. washir garaafkiisa.



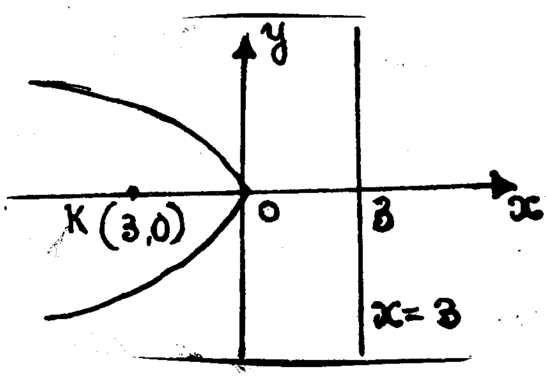
Furfuris:

$y^2 = -4ax$
$-4a = -12$
$a = 3$

Kolkaa kulmisku waa $(-3,0)$ jeedshuhuna waa $x = 3$.

Tusaale 4:

Soo saar kulmiska iyo jeedshaha saabka $x^2 = -12y$ washir garaafka.



Furfuris:

$$x^2 = -4ay$$
$$-4a = -12$$
$$a = 3$$

Kolkaa kulmisku waa $(0, -3)$ jeedshuhuna waa $y = 3$.

Tusaale 5:

Soo saar kulmiska. jeedshaha iyo dherarka taabka Saabka $3y^2 = 8x$ ama $y^2 = \frac{8}{3}x$.

Furfuris:

$$y^2 = 4ax$$

$$4a = \frac{8}{3}$$

$$a = \frac{2}{3}$$

Kulmis : $\left(\frac{2}{3}, 0\right)$

Jeedsha:

$x = -\frac{2}{3}$, dhererka taabka waa $4a$ ama $\frac{8}{3}$.

Tusaale 6:

Soo saar isle'egta saabka mara barta $(4,5)$ ee dhidibkiisu la barbarro yahay dhidibka y.

Geeskiisuna yahay $(2,3)$.

F u r f u r i s :

Waxaan naqaannaa in $(x-h)^2 = 4a\ (y-k)$.
Kolkaa $(x-2)^2 = 4a\ (y-3)$.
Mar haddii barta (4,5) ay ka mid tahay saabka, waa inay raaligelisaa isle'egta.

$$(4-2)^2 = 4a\ (5-3)$$

$$a = \frac{1}{2}$$

Isle'egtu waa $(x-2)^2 = 2\ (y-3)$

ama $x^2 - 4x - 2y + 10 = 0$.

L a y l i :

1. Soo saar kulammada kulmiska, dhererka taabka, iyo isle'egta jeedshaha saababka soo socda. Washir garaafyadooda.

Jaw.

b) $y^2 = 6x$ $\qquad \left(\frac{3}{2}, 0\right);\ 6;\ x + \frac{3}{2} = 0$

t) $x^2 = 8y$ $\qquad (0,2);\ 8;\ y + 2 = 0$

j) $3y^2 = -4x$ $\qquad \left(-\frac{1}{3}, 0\right);\ \frac{4}{3};\ x - \frac{1}{3} = 0$

2. Soo saar isle'egta saab kasta haddii :
   b) Kulmisku yahay (3,0) jeedshuhuna yahay $x + 3 = 0$.
   (Jaw. $y^2 - 12x = 0$)

   t) K (0,6); jeedshuhuna waa dhidibka $-y$.
   (Jaw. $x^2 - 12y + 36$)

   j) Geesku yahay unugga, jeedshuhuna yahay dhidibka $-x$.

x) Geesku yahay unugga, jeedshuhuna yahay dhidibka $-x$, marana barta $(-3,6)$.

Jaw. $y^2 = -12x$).

3. Soo saar isle'egta Tubta barta socota ee in u wada jirta barta $(-2,3)$ iyo xarriiqda, $x + 6 = 0$

(Jaw. $y^2 - 6y - 8x - 23 = 0$).

4. Ku soo celi isle'egyadan saababka saansaan beegal, soona saar kulammada (b) geeska (t) kulmisyada (j) iyo dhererka taababka (x) iyo isle'egyada jeedsheyaasha.

b) $y^2 - 4y + 6x - 8 = 0$ Jaw. b) $(2,2)$; t) $\left(\frac{1}{2}, 2\right)$;

j) 6 x) $x - \frac{7}{2} = 0$

t) $3x^2 - 9x - 5y - 2 = 0$ Jaw. b) $\left(\frac{3}{2}, -\frac{7}{4}\right)$

t) $\left(\frac{2}{2}, -\frac{4}{3}\right)$ j) $\frac{5}{3}$

j) $y^2 - 4y - 6x + 13 = 0$ Jaw. b) $\left(\frac{3}{2}, 2\right)$

t) $(3,2,)$ j) 6

x) $x = 0$

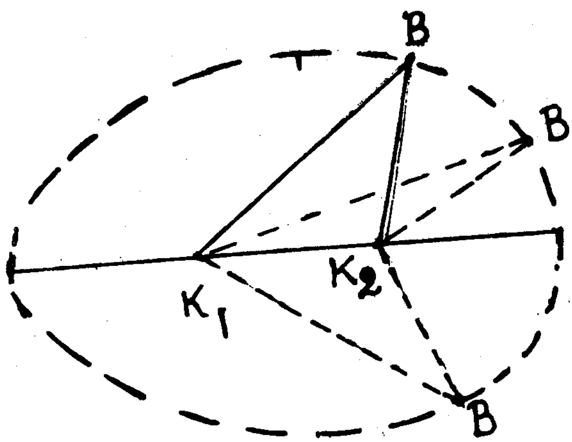
6. Soo saar isle'egta saabka dhidibkiisu taagan yahay ee uu marna barahan $(4,5)$, $(-2, 11)$ iyo $(-4, 21)$

Jaw. $x^2 - 4x - 2y + 10 = 0$.

7. Haddii qaanso saabeed jooggeedu yahay 25 m., fadhigeeduna yahay 40 m. Soo saar joogga meelaha ka mid ah qaansada ee 8 m. u jira xuddunta fadhiga. Jaw. 21.

QABAAL.

Waxaad qaadataa dun. Ku xir dunta labadeeda daraf labo musbaar oo ka dhidban sallax. Ku qabo dunta qalin wareejinaya sida shaxanka ku muujisan. Kolkaa waxa samaysmaya xood. Kolkaa, mar kasta wadarta fogaanta $B K_1$ iyo $B K_2$ waxay la mid tahay dhererka dunta. Sax ma tahay haddii aad u qortid sidan: $B K_1 + B K_2 = K_1 K_2$?



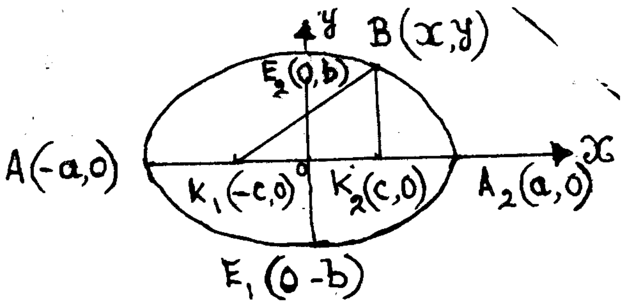
Qeex:

Qabaalku waa tubta bar socota sallax oo wadarta fogaanta ay u jirtaa laba dhibcood oo maguuraan ah ay tahay madoorsoome. Labada dhibcood oo maguuraanka ah waxa la yiraa kulmisyo, midiina waa kulmis. Fogaanta $B K_1$ iyo $B K_2$ waa gacannada kulmiska B. U qaado in labada barood ee maguuraanka ahi yihiin K_1 (C,0) iyo K_2 (– C,0), wadarta madoorsoomaha ahina tahay 2a.

U qaado in barta B(x, y) ay ka mid tahay qabaalka. Kolkaa innagoo raacayna qeexda waxan helay naa in $K_1 . B + B K_2 = 2a$.

Mar haddii wadarta laba dhinac ee saddexagal ay ka weyn tahay dhinaca saddexaad waxan ku soo koobi karnaa in

(b) $K_1 B + B K_2 > 2C$ (t) $a > C$.



Kolkaa, waxan had iyo jeer qaadanaynaa in ay $a > C$. Kolkaa dhibicda B (x,y) waxay qabaalka ka mid noqon kartaa haddii iyo haddii qura ay $K_1 B + B K_2 = 2a$.

Haddaba innagoo isticmaalayna jidka fogaanta waxan helaynaa in

$$K_1 B = \sqrt{(x + C)^2 + (y - 0)^2}$$

$$B K_2 = \sqrt{(x - C)^2 + (y - 0)^2}$$

Kolkaa $$\sqrt{(x + C)^2 + (y - 0)^2} + \sqrt{(x - C) + (y - 0)^2} = 2a$$

amä $$\sqrt{(x + c)^2 + (y - 0)^2} = 2a - \sqrt{(x - c)^2 + (y - 0)^2}$$

Laba jibbaar labada dhinac ee isle'egta. Markaa waxan helaynaa in

$$(x + c)^2 + y^2 = 4a^2 - 4a \sqrt{(x - c)^2 + y^2} + x^2 - 2cx + c^2 + y^2$$

$$\therefore \quad 2cx = 4a^2 - 4a \sqrt{(x - c)^2 + y^2} - 2cx$$

Fududee:

$$4a\sqrt{(x-c)^2+y^2} = 4a^2 - 4cx.$$

ama $$a\sqrt{(x-c)^2+y^2} = a^2 - cx.$$

Laba jibbaar oo fududee :

$$(a^2-c^2)\,x^2 + a^2\,y^2 = a^2\,(a^2-c^2)$$

U qaybi $a^2\,(a^2-c^2)$ labada dhinacba. Markaa isle'egtu waxay noqonaysaa

$$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$$

Kolkaa saansaanka beeggal ee isle'egta qabaalku waa

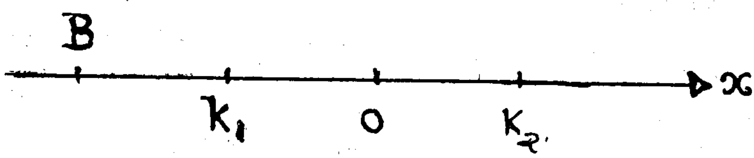
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ama } b^2x^2 + a^2y^2 = a^2b^2$$

Xarriiqda marta labada kulmis K_1 iyo K_2 waxa la yiraa **Dhidib Weyne.** Xarriiqda marta E_1 iyo E_2 ee ah qotome-badhaha xarriijinta K_1K_2 waxa la yiraa **Dhidib Yare.** Baraha A_1 iyo A_2 waxa weeye geesaha qabaalka. Kulammada barahaasi waa $(-a, 0)$ iyo $(a, 0)$. Dhererka dhidib weynuhu waa $2a$, ka dhidib yaruhuna waa $2b$.

Haddaba, haddii $a > b$, dhidib weynaha qabaalku waa dhidibka $-x$. Kolkaa isle'egta qabaalku waa

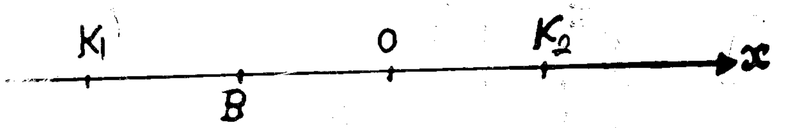
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ka soo qaad in barta B ay ka mid tahay qabaalka ay kaga taallana dhidibka $-x$ meel xarriijinta K_1K_2 dibedda ka ah, sida shaxankan.



(b) Markaa $K_2B + BK_1 = K_2K_1$ (t) $a > c$

Laakiin haddii B ay ka mid tahay qabaalka ay taallana xarriijinta $K_2 K_1$ sida :



(b) Markaa $K_2 B + B K_1 = K_2 K_1$ (t) $a = c$.

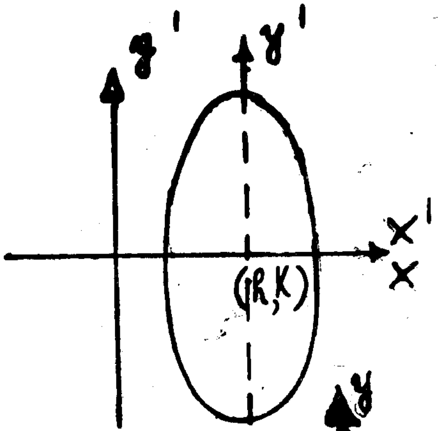
kolkaa garaafka qabaalka ee $a = c$ wuxuu yahay ururka dhammaan baraha xarriijinta $K_2 K_1$.
unugga.

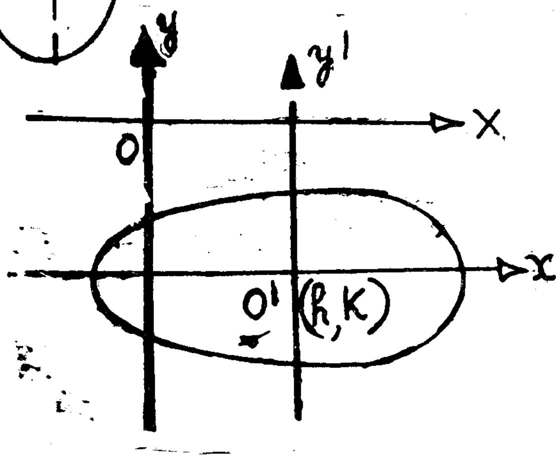
Haddii a < ma jiraan baro-baro raalligeliya qeexda qabaalka, had iyo jeer waxan qaadannaa in ay $a > c$. Markaa waxa wanqara qabaalka dhidibka $-x$ iyo dhidibka $-y$. Ilaa hadda waxan barannay marka xuddunta qabaalka ku taallo unugga. Kolkaa haddii xuddunta ay tahay (h, k) oo dhidib weynuhuna la barbarro yahay dhidibka $-x$ waxaa la tusi karaa in saansaanka isle'egta qabaalku yahay:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad (a > b)$$

Haddii dhidib weynuhu la barbarro yahay dhidibka $-y$, saansaanka isle'egta qabaalku waa

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \quad (a > b)$$





Tusaale 1:

Soo saar isle'egta qabaalka ee kulamisyadiisu yihiin (3 , 0) iyo (– 3 , 0), geesihiisuna (5 , 0) iyo (– 5 , 0) (Xusuusnow in $b^2 = a^2 - c^2$)

Furfuris:

Saansaanka isle'gta qabaalka kulmisyadiisuna yaallaan dhidibka – x waa $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(a > b)$

geesihu hadday yihiin (5,0) iyo (– 5 , 0) waxaan naqaanaa in $a = 5$

kulamisyadu waa (3,0) iyo (− 3 , 0) kolkaa c = 3.

Markaa $b^2 = a^2 - c^2 = 25 - 9 = 16$

Haddaba isle'egtu waa ETAOIN

Haddaba isle'egtu waa $\frac{x^2}{25} + \frac{y^2}{16} = 1.$

T u s a a l e 2 :

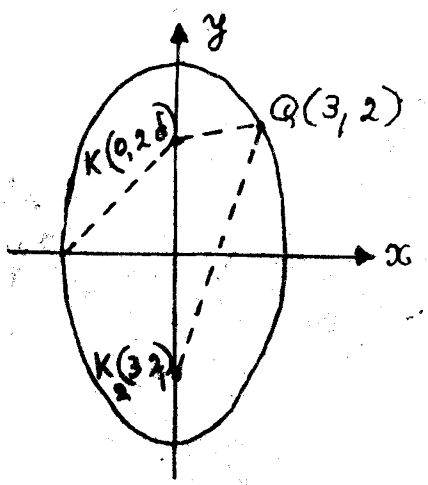
Soo saar isle'egta qabaalka mara barta Q (3,2), kulmisyadiisuna yihiin (0,2) iyo (0,−2).

F u r f u r i s :

Mar haddii kulmisyadu yihiin (0,2) iyo (0,−2) waxa muuqata in dhidib weynaha qabaalku yaallo dhidibka −y. Kolkaa saansaanka isle'egta qabaalku waxay tahay:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad (a > b).$$

Waxaan naqaannaa in wadarta gacannada kulmisyada barta Q ay tahay 2a. Taas oo ah
$QK_1 + QK_2 = 2a$
Adoo isticmaalaya jidka fogaanta.
$QK_1 = 3, \; QK_2 = 5.$



Kolkaa $3 + 5 = 2a$, $a = 4$

Waxa kale oo naqaannaa in $b^2 = a^2 - c^2$

kolkaa $b^2 = 16 - 4 = 12$

kolkaa isle'egta la rabo waa $\frac{x^2}{12} + \frac{y^2}{16} = 1$

Tusaale 4:

Haddii lagu siiyo qabaalka isle'egtiisu tahay

$$\frac{x^2}{49} + \frac{y^2}{33} = 1$$

Soo saar (b) kulmisyadiisa (t) geesihiisa (j) iyo dhererka dhidib yaraha. (x) Washir garaafka.

Furfuris:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 49$, $b^2 = 33$ iyo $a^2 = b^2 + c^2$

Kolkaa $c^2 = a^2 - b^2 = 49 - 33 = 16$

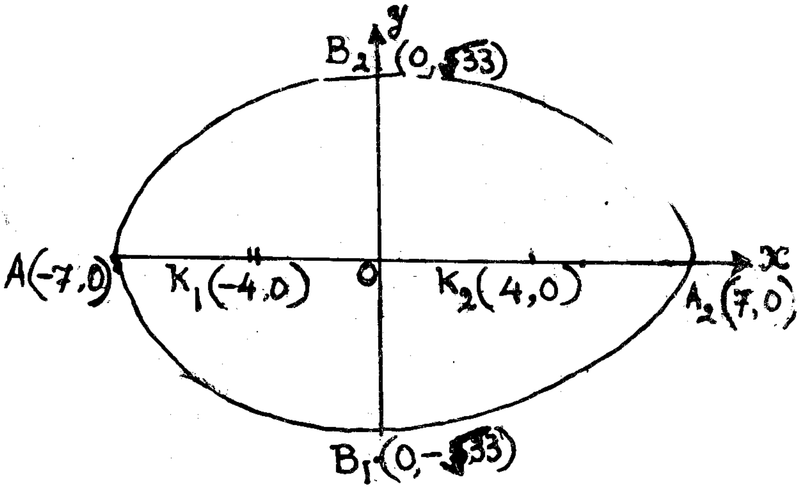
$a = \pm 7$; $b = \pm \sqrt{33}$; $c = \pm 4$.

b) Kulmisyo : $K_1 (-4,0)$ iyo $K_2 (4,0)$.

t) geeso : $A_1 (-7,0)$ iyo $A_2 (7,0)$.

j) dhererka dhidib yaraha $= 2\sqrt{33}$.

x) garaaf.



Tusaale 5:

Haddii lagu siiyo qabaal isle'egtiisu tahay $4x^2 + 9y^2 - 48x + 72y + 144 = 0$, soo saar xudduntiisa, geesihiisa, iyo kumisyadiisa.

Furfuris:

Isle'egta lagu siiyay waxaad u qortaa sidan :

$$\frac{(x-h)}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Dhammee laba jibbaarka isle'egta lagu siiyay:

$$4(x - 12x + 36) + 9(y^2 + 8y + 16) = 144$$

$$\left(\frac{x-6}{36}\right)^2 + \left(\frac{y+4}{16}\right)^2 = 1$$

Kolkaa xuddunta qabaalku waa (6, − 4).
$a = 6$; $b = 4$; $c^2 = a^2 - b^2 = 36 - 16 = 20$

$$c = \pm 2\sqrt{5}.$$

Kulmisyo: $(6, + 2\sqrt{5})$

$$(6, - 2\sqrt{5}).$$

Geesaha : (0, − 4) iyo (12, − 4).

Tusaale 6:

Soo saar isle'egta qabaalka mara (6, 4), xudduntiisuna tahay (1,2), kulmisna yahay (6,2).

Furfuris:

Isticmaal isle'egtan: $$\frac{(x-1)^2}{a^2} + \frac{(y-2)^2}{b^2} = 2$$

mar haddii barta (6,4) ay ka mid tahay qabaalka waa in ay raalligelisaa isle'egta.

$$\frac{(4-1)^2}{a^2} + \frac{(16-2)^2}{b^2} = 1$$

$$\frac{9}{a^2} + \frac{16}{b^2} = 1$$

Mar haddii $c = 6 - 1 = 5$, $b^2 = a^2 - c^2 = a^2 - 25$

Kolkaa $\frac{9}{a^2} + \frac{16}{a^2-25} = 1$. Raadi a^2.

$$a^2 = 45 \; ; \; b^2 = 20.$$

Kolkaa isle'egtu waa $$\frac{(x-1)^2}{45} + \frac{(y-2)^2}{20} = 1.$$

L a y l i :

1. Adoo isticmaalaya qeexda qabaalka soo diir isle'egta qabaalka kulmisyadiisu yihiin K_1 $(c,0)$ iyo K_2 $(-c,0)$ geesihiisuna yihiin A_1 $(a,0)$ iyo A_2 $(-a,0)$.

2. Soo saar isle'egta qabaalka kulmisyadiisu yihiin $(0,5)$ iyo $(0,-5)$, baro dhammaadka dhidib yaruhuna yihiin $(7,0)$ iyo $(-7,0)$.

   Jaw. $\frac{x^2}{49} + \frac{y^2}{74} = 1.$

3. Soo saar isle'egta qabaalka kulmisyadiisu yihiin $(1,0)$ iyo $(-1,0)$, geesihiisuna $(9,0)$ iyo $(-9,0)$.

   Jaw. $\frac{x^2}{81} + \frac{y^2}{80} = 1$

4. Soo saar isle'egta qabaalka mara barta $\left(\sqrt{\frac{7}{2}}, 11\right)$ kulsimyadiisuna yihiin $(0,8)$ iyo $(0,-8)$.

   Jaw. $\frac{x^2}{64} + \frac{y^2}{128} = 1$

5. Soo saar isle'egta qabaalka baro dhammaadka dhidib weynihiisu yihiin $(7,0)$ iyo $(-7,0)$ kuwa dhidib yarihiisuna yihiin $(0,5)$ iyo $(0,-5)$.

   Jaw. $\frac{x^2}{49} + \frac{y^2}{25} = 1$

6. Haddii lagu siiyo isle'egta qabaalka $\frac{x^2}{1} + \frac{y^2}{3} = 1$

soo saar (b) dhererka dhidib yaraha, (t) kulmisyada, (j) iyo geesaha.

Jaw. (b) 2 (t) $(0, \sqrt{2})$, $(0, -\sqrt{2})$.

(j) $(0, \sqrt{3})$, $(0, -\sqrt{3})$.

7. Haddii lagu siiyo isle'eg'ta qabaalka oo ah $x^2 + 7y^2 = 7$, soo saar (b) dhererka dhidib yaraha (t) kulmisyada (j) iyo geesaha.

Jaw. (b) 2, (t) $(\sqrt{6},0)$, $(-\sqrt{6},0)$;

(j) $(\sqrt{7},0)$, $(-\sqrt{7},0)$.

8. Soo saar isle'egta qabaalka geesihiisu yihiin $(-7,9)$ iyo $(-7,1)$, baro dhammaadka dhidib yaruhuna yihiin $(-9,5)$ iyo $(-5,5)$.

Jaw. $$\frac{(y-5)^2}{20} + \frac{(x+7)^2}{4} = 1$$

9. Soo saar isle'egta qabaalka mara (5,8) ee geesihiisuna yihiin (7,5) iyo (– 13,5).

Jaw. $$\frac{(x+3)^2}{100} + \frac{(y-5)^2}{25} = 1.$$

BOSA...

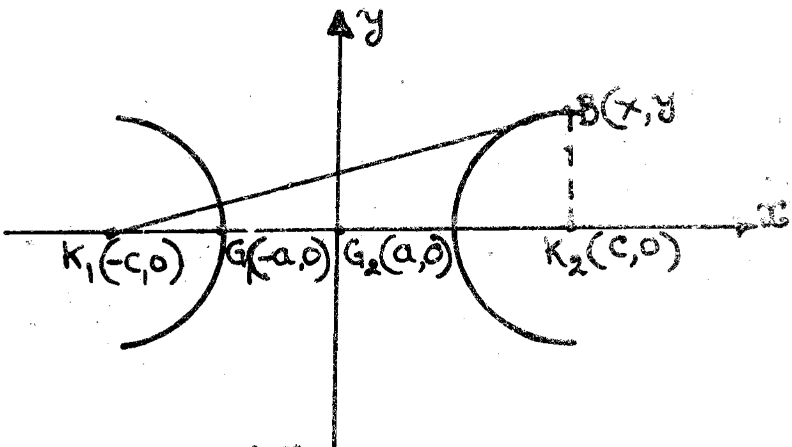
...e x :

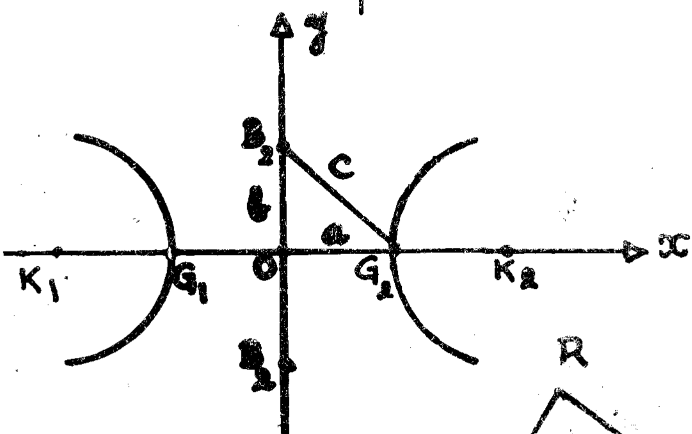
Labasaabku waa ururka dhammaan baraha sallax ee faraq fogaanta ay jiraan laba barood oo maguuraan ah, oo sallaxa ku yaal, ay tahay madoorsoome. Dhibcaha maguuraanka ah waxa la yiraa **Kulmisyo.**

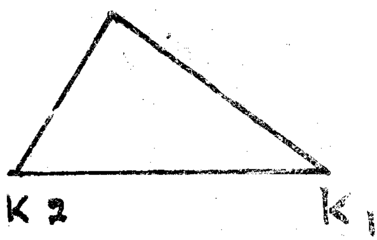
Shaxankan hoos ku sawiran wuxu ku tusayaa haddii K_1 iyo K_2 ay yihiin kulmisyo barta R ay tahay bar ka mid ah labasaabka, in $|K_1R| - |RK_2|$ ay tahay **Madoorsoome Togan.** Kolkaa waxan oran karnaa barta R waxay baraha labasaabka ka mid noqon kartaa haddii iyo haddii qudha ←

(b) $|K_1R| - |RK_2|$

ama (t) $|RK_2| - |K_1R|$ ay la mid tahay madoorsoome togan oo la ogyahay, 2 a.







Ka soo qaad in barta B (x,y) ee shaxanka (b) ay ka mid tahay Tubta. Kolkaa $K_1 B - B K_2 = 2a$ Ama

$$\sqrt{(x+c)^2 + (y-0)^2} - \sqrt{(x-c)^2 + (y-0)^2} = 2a$$

$$\sqrt{(x+c)^2 + (y-o)^2} = 2a + \sqrt{(x-c)^2 + (y-o)^2}$$

Innaga oo laba jibbaaraynа labada dhinac, siina fududaynayna waxan helaynaa $cx - a^2 = a\sqrt{x-c)^2 + (y-o)}$ laba jibbaar haddana labada dhinac siina fududee :

$$(c^2 - a^2)\, x^2 - a^2 y^2 = a^2 (c^2 - a^2)$$

U qaybi labada dhinacba $a^2 (c^2 - a^2)$.

Kolkaa isle'egta labasaabku waxay noqonaysaa :

$$\frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1.$$

Mar haddii $c > a$, kolkaa $c^2 - a^2$ way togan tahay. U qoro in $c^2 - a^2 = b^2$. Kolkaa waxan haysanaa isle'egta saansnaakeeda beegal tahay $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (1)

Kolkaa isle'egta (1) waxa weeye isle'egta labasaabka xudduntiisu tahay unugga, kulmisyadiisuna ay ku yaallaan dhidibka $-x$. Haddii kulmisyadu ay yihiin (0,c) iyo (0, $-$ c) saansaanka beeggal ee labasaabka waxay noqonaysaa $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ (2).

Labasaabku wuxu ku wanqaran yahay dhidibka $-x$ iyo ka y iyo unugga, waayo isle'egtu ismabeddesho haddii x lagu beddelo $-x$, iyo haddii y lagu beddelo $-y$ ama x iyo y lagu beddelo $-x$ iyo $-y$ sida ay u kala horreeyaan. Xarriiqda marta labada kulmis waxa la yiraa **Dhidib Wadaaje.** Qotomaha kala badhana waxa la yiraa **Dhidib Xisti.** Kolkaa shaxanka (b) dhidib wadaajuhu waa $Q_1 Q_2$, dhererkiisuna waa 2 a. Dhidib Xistiguna waa $B_1 B_2$, dhererkiisuna waa 2 b. Haddaba sidee lagu gartaa midka dhidib wadaajaha ah iyo ka dhidib xistiga ah haddii la ina siiyo isle'egta labasaabka ? Haddii tibixda y^2 ay togan tahay dhidib wadaajuhu waa dhidibka — y. Haddii tibixda x ay taban tahayna dhidib — x yaa dhidib xistiga ah.

Haddaba haddii xuddunta labasaabku ay ka duwan tahay unugga oo ay tahay (h, k) oo dhidib wadaajuhu uu la barbarro yahay dhidibka — x, saansaanka beeggal ee

isle'egta labasaabku waa $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ (1).

Haddii dhidib wadaajuhu la barbarro yahay dhidibka —y,

isle'egta labasaabku waa $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ (2).

Kolkaa saansaanka guud ahaaneed ee isle'egta labasaabka dhidibbihiisu la barbarro yihiin dhidibka — x iyo ka y waa $A x^2 - B y^2 + D X + E Y + F = 0$. Taasoo A iyo B ay isku waafaqaan Summadda.

Tusaale 1:

Soo saar isle'egta labasaabka kulmisyadiisu yihiin (5,0) iyo (— 5,0), geesihiisuna (3,0) iyo (— 3,0).

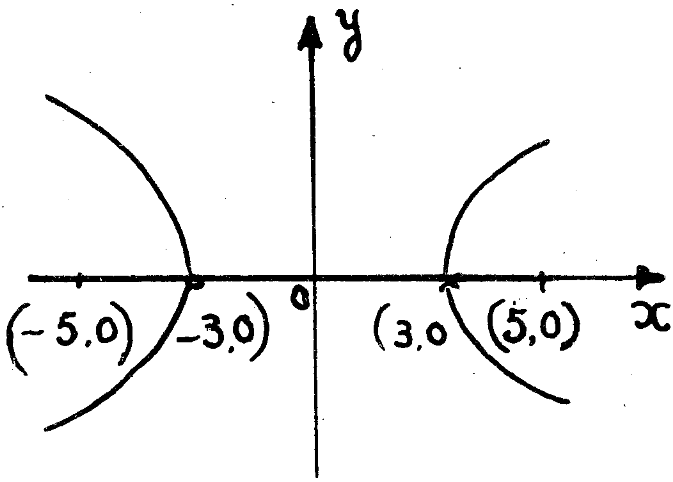
Furfuris:

Saansaanka beeggal ee isle'egta labasaabka kulmisyadiisu ku yaallaan dhidibka — x waa $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Kolkaa a = 3; c = 5

Kolkaa $b^2 = c^2 - a^2 = 25 - 9 = 16$

Kolkaa isle'egta labasaabku waa $\frac{y^2}{16} =$



Tusaale

Soo saar isle'egta labasaabka mara barta (10,3) geesimiisuna yihiin (8,0) iyo (− 8,0).

Furfuris:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{ama } b^2 x^2 - a^2 y^2 = a^2 b^2.$$

Mar haddii geesuhu yihiin (8,0) iyo (8 − 8,0) ∴ a = 8.
Kolkaa b^2 (100) − 64 (9) = 64 b^2,

laakiin labasaabku wuxuu maraa dhibicda, (10,3).

Kolkaa $b^2 x^2 - 8^2 y^2 = 8^2 b^2$

ama $36 b^2 = 576 \therefore b^2 = 16$

isle'egtu markaa waxa weeye $\frac{x^2}{64} - \frac{y^2}{16} = 1.$

Tusaale 3:

Haddii lagu siiyo labasaabka isle'egtiisu tahay

$$\frac{x^2}{64} - \frac{y^2}{36} = 1$$

Soo saar kulammada kulmisyada iyo kuwa geesaha. Sheeg dhidib Wadaajaha iyo dhidib Xistiga

Furfuris:

Haddii saansaanka isle'egta labasaabka tahay

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \text{ ———— } (2)$$

Dhidib wadaajuhu waa dhidibka — y, kulmisyaduna waxay ku yaallaan dhidibka — y. Haddii saansaanka isle'egtu tahay :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ ———— } (1)$$

Dhidib wadaajuhu waa dhidibka — x, kulmisyadiisuna waxay ku yaallaan dhidibka — x. Kolkaa imminka saansaanka isle'egteenu waa sida ka (2). Kolkaa dhidib wadaajuhu waa dhidibka — x. Dhidib xistiguna waa dhidib — y. Markaa, saansaanka isle'egteenu waa sidan :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Kolkaa $a^2 = 64$; $b^2 = 36$; $c^2 = 64 + 36$; $c = \sqrt{100} = \pm 10$
Kulmisyo: $(-8,0)$ iyo $(8,0)$. Geeso: $(-10,0)$ iyo $(10,0)$.

Tusaale 4:

Soo saar isle'egta labasaabka kulmisyadiisu yihiin (0,10) iyo (0, − 6.

Furfuris:

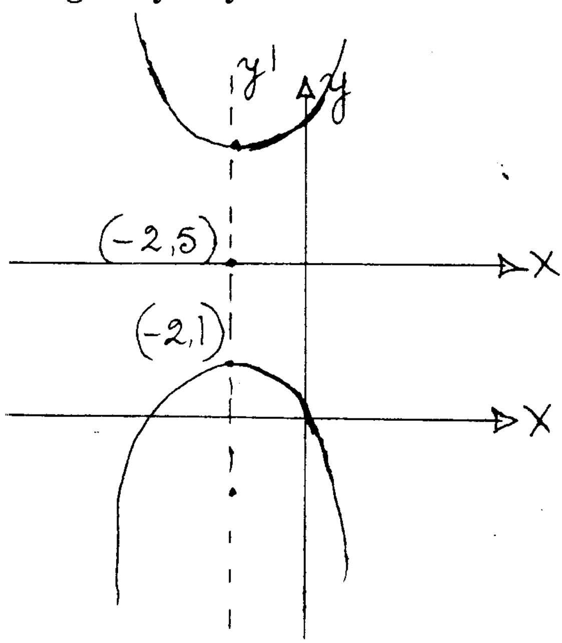
$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Kolkaa $a = 6$; $c = 10$; $b^2 = 100 - 36 = 64$ kolkaa isle'-

egtu waa $\frac{y^2}{6^2} - \frac{x^2}{8^2} = 1$ ama $\frac{y^2}{36} - \frac{x^2}{64} = 1$

Tusaale 5:

Soo saar isle'egta labasaabka xudduntiisu tahay (− 2,5), geesihiisuna yihiin (− 2,9) iyo (− 2,1), dhererka dhidib xistiguna yahay 6.



Furfuris:

Haddaan u eegno dhidbaha cusub ee ah x′ y′, saanaasnka isle'egta waa

$$\frac{y'^2}{a^2} - \frac{x'^2}{b^2} = 1. \text{ Kolkaa } a = 4 ; b = \frac{6}{2} = 3 .$$

Markaa isle'egtu waa $\frac{y'^2}{16} - \frac{x'^2}{9} = 1.$

Laakiin $x' = x + 2$; $y' = y - 5$.
Kolkaa isle'egta labasaabkani waa

$$\frac{(y-5)^2}{16} - \frac{(x+2)^2}{9} = 1.$$

Layli:

1. Soo saar isle'egta labasaabka kulmisyadiisu yihiin (0,8) iyo (−0,8) geesihiisuna yihiin (0,2) iyo (0,−2).

   Jaw. $\frac{y^2}{4} - \frac{x^2}{60} = 1.$

2. Soo saar isle'egta labasaabka kulmisyadiisu yihiin (0,8) iyo (−8,0), baro dhammaadka dhidib xistiguna yihiin (0,4) iyo (0,−4).

   Jaw. $\frac{x^2}{48} - \frac{y^2}{16} = 1.$

3. Soo saar isle'egta labasaabka mara barta (5,4) ee geesihiisuna yihiin (3,0) iyo (−3,0)

   Jaw. $\frac{x^2}{9} - \frac{y^2}{9} = 1.$

4. Haddii lagu siiyo labasaabka isle'egtiisu tahay

$$\frac{y^2}{1} - \frac{x^2}{2} = 1.$$

b) Soo saar dhererka dhidib wadaajaha iyo xistiga.

t) Kulammada kulmisyada.

j) Kulammada geesaha.

Jaw. (b) 2; $2\sqrt{2}$. (t) $(0, \sqrt{3})$ iyo $(0, -\sqrt{3})$; (j) (0,1) iyo (0,−1).

Haddii lagu siiyo labasaabka isle'egtiisu tahay $3x^2 - 8y^2 = 24$, soo saar

b) Dhererka dhidib wadaajaha iyo xistiga.

t) Kulmisyada. j) Geesaha.

Jaw. (b) $2\sqrt{8}$; $2\sqrt{3}$. (t) $(\sqrt{11}, 0)$ iyo $(-\sqrt{11}, 0)$. (j) $(\sqrt{8}, 0)$ iyo $(-\sqrt{8}, 0)$.

6. Soo saar isle'egta labasaabka geesihiisu ku yaal-laan (− 2, − 4) iyo (− 2,8), ee dhererka dhidib wadaajihiisuna yahay 14.

Jaw. $$\frac{(y-2)^2}{36} - \frac{(x+2)^2}{49} = 1.$$

7. *U beddel isle'egtan* $16x^2 - 4y^2 + 64x + 4y + 20 = 0$ *saansaan beeggal. Ma Qabaalbaa, ma* Saabbaa, mise waa labasaab ? Soo saar

b) Kulmisyadiisa, t) Geesihiisa,

j) iyo xudduntiisa.

Jaw. (b) labasaab. (t) (− 2,1) iyo (− 2,9)

(j) $(-2.5 + \sqrt{20})$ iyo $(-2,5 - \sqrt{20})$

(x) (− 2,5).

CUTUB 3

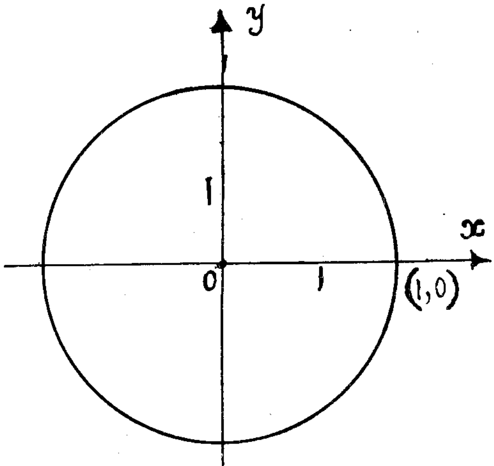
TIRIGNOOMETERI

Inta aanan u gelin falaqaynta fansaarrada tirignoo-meteri bal aan naqtiino astaamaha goobo.

GOOBOOYIN.

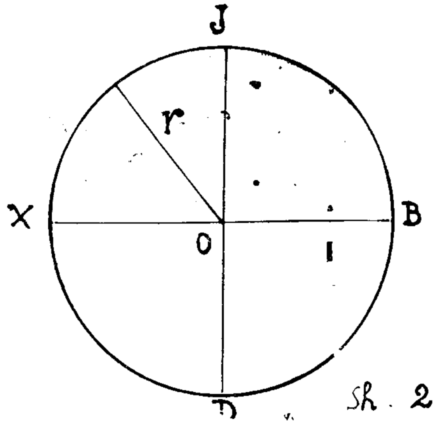
Qeex: Goob halbeeg.

Goob halbeeg waa goobada gacankeedu yahay hal-beeg.



sh.1

Goobo halbeeg xudduunteedu ku taal [illegible] kaartis. Bal u fiirso goobada Sh. 2 ee gacankeedu yahay r.



Sh. 2

Meeriska goobo waxa lagu helaa jidkan. Meeris $= 2\,\pi \times$ Gacan. Markaa, meeriska goobadani waa $2\,\pi r$.

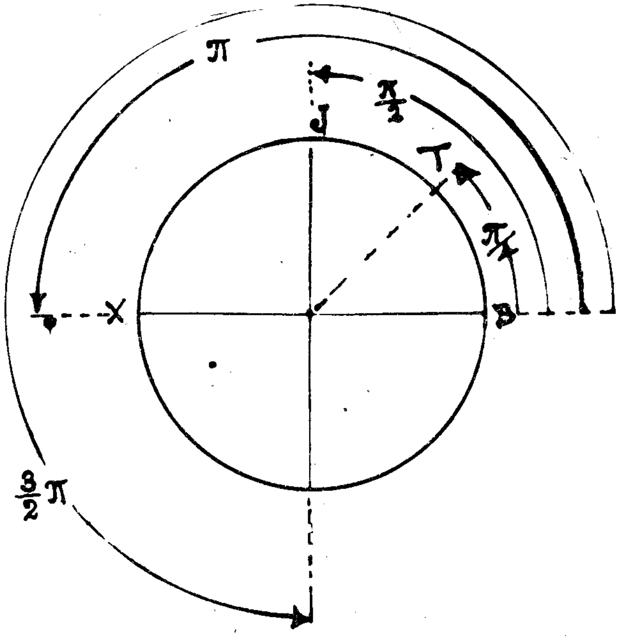
Haddii qaansada BT tahay $\frac{1}{8}$ ka meeriska, markaa dhererka $BT = \frac{1}{8}(2\,\pi r) = \frac{\pi}{4}$. Sidoo kale

$$BT = \frac{1}{4}(2\,\pi r) = \frac{\pi r}{2}$$

$$BX = \frac{1}{2}(2\,\pi r) = \pi r$$

$$BD = \frac{3}{4}(2\,\pi r) = \frac{3\,\pi r}{2}$$

Haddii goobada shaxanka 2aad, goobo halbeeg tahay t. a. haddii $r = 1$. Markaa $BJ = \frac{\pi}{2}$. $BX = \frac{3}{2}\pi$. $BD = \frac{3}{2}\pi$.



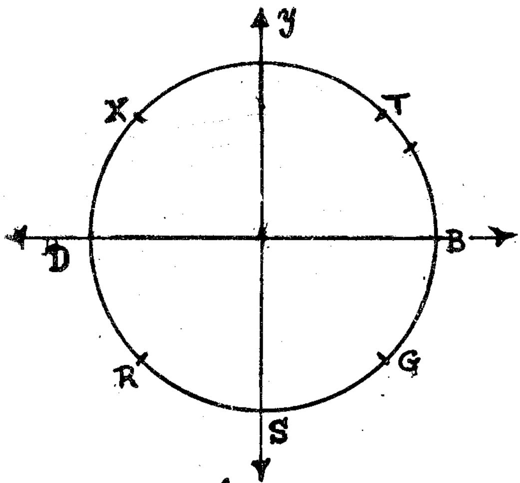
sh. 2

Ogow:

BT waxa loo akhriyaa «qaanso BT» waxayna tahay fogaanta B iyo T ay isku jiraan marka B laga bilaabo ee meeriska goobada loo maro lid saacad wareeg. Had iyo jeer waxa loo qaataa in dhererka qaansadu togan yahay marka lid saacad wareeg loo cabbiro, in uuna taban yahay marka saacad wareeg loo cabbiro.

Tusaale 1:

Haddii shaxanka 4aad, goobadu ay tahay goobo halbeeg, soo saar dhererrada qaansooyinkan BT, BJ, BX, BD, BS, BR, BG, dhammaan waxa loo cabbiray lid saacad wareeg. Baruhu meeriska 8 qaybood oo isle'eg bay u qaybiyaan.



sh. 4

Furfuris:

Dhererka meerisku waa $= 2\pi r = 2\pi x\ 1 = 2\pi$

$$BT = \frac{1}{8}(2\pi) = \frac{\pi}{4}$$

$$BJ = \frac{2}{8}(2\pi) = \frac{\pi}{2}$$

$$BX = \frac{3}{8}(2\pi) = \frac{3\pi}{4}$$

$$BD = \frac{4}{8}(2\pi) = \pi$$

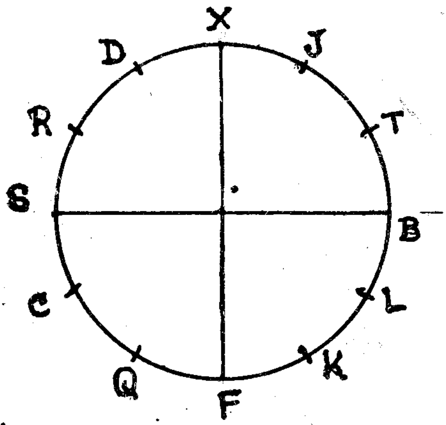
$$BR = \frac{5}{8}(2\pi) = \frac{5\pi}{4}$$

$$BS = \frac{6}{8}(2\pi) = \frac{3\pi}{2}$$

$$BG = \frac{7}{8}(2\pi) = \frac{7\pi}{4}$$

Tusaale 2:

Shaxanka Saad wuxu muujinayaa goobo halbeeg. Baruhu meeriska waxay u qaybiyaan 12 qaanso oo is-le'eg, haddaba raadi dhererka qaansooyinka soo socda. Dhammaan waxa loo cabbiray lid saacad wareeg.
BT, BJ, BX, BD, BR, BS, BC, BQ, BF, BK, iyo BL.



Furfuris:

Meeriska goobo halbeeggu $= 2\pi \times 1 = 2\pi$

$$BT = \frac{1}{12}(2\pi) = \frac{\pi}{6}$$

$$BJ = \frac{2}{12}(2\pi) = \frac{\pi}{3}$$

$$BX = \frac{3}{12}(2\pi) = \frac{\pi}{2}$$

$$BD = \frac{4}{12}(2\pi) = \frac{2\pi}{3}$$

$$BR = \frac{5}{12}(2\pi) = \frac{5\pi}{6}$$

$$BS = \frac{6}{12}(2\pi) = \pi$$

$$BC = \frac{7}{12}(2\pi) = \frac{7\pi}{6}$$

$$BQ = \frac{8}{12}(2\pi) = \frac{4\pi}{3}$$

$$BF = \frac{9}{12}(2\pi) = \frac{3\pi}{2}$$

$$BK = \frac{10}{12}(2\pi) = \frac{5\pi}{3}$$

$$BL = \frac{11}{12}(2\pi) = \frac{11\pi}{6}$$

Tusaale 3:

Adoo isticmaalaya shaxanka 5aad, raadi dhererka qaansooyinkan haddii ay u cabbiran yihiin saacad wareeg BL, BF, BQ, BS, BX, BJ iyo BT.

Furfuris:

$$BL = -\frac{1}{12}(2\pi) = -\frac{\pi}{6}$$

$$BF = -\frac{3}{12}(2\pi) = -\frac{\pi}{2}$$

$$BQ = -\frac{4}{12}(2\pi) = -\frac{2\pi}{3}$$

$$BS = -\frac{6}{12}(2\pi) = -\pi$$

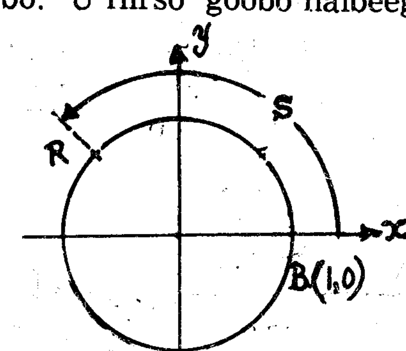
$$BX = -\frac{9}{12}(2\pi) = -\frac{3\pi}{2}$$

$$BJ = -\frac{10}{12}(2\pi) = -\frac{5\pi}{3}$$

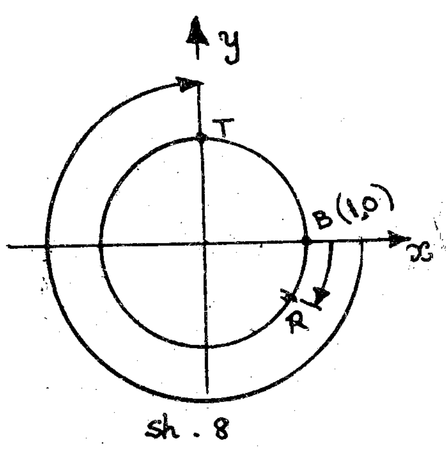
$$BT = -\frac{11}{12}(2\pi) = -\frac{11\pi}{6}$$

FANSAAR GOOBO

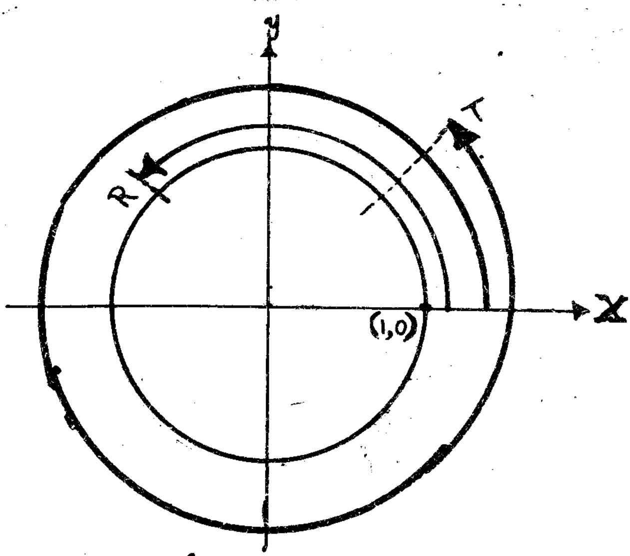
Hadda, bal aan dhisno fansaar la xiriira dhererka qaansooyinka goobo. U fiirso goobo halbeegga shaxanka hoose.



Ka dhig xuddunta goobada unugga habdhiska kulammada laydi. Hadda, isle'egta goobadu waa $x^2 + y^2 = 1$. Ka soo qaad in B ay ku taal isgoyska goobada iyo dhidibka – x togan. Markaa kulammada B waa (1,0). Ka soo qaad in S tahay tiro maangal ah, marka aan S halbeeg ka soconno B inaga oo meeriska raacayna waxan gaari karraa bar kale oo meeriska ku taal, ka soo qaad inay tahay R (sh. 6). Marka aan dhererka qaansooyinka ka hadlayno, had iyo jeer bar bilawgeenna waxaan u qaadannaa barta (1,0), lid saacad wareeg waxan u qaadannaa jiho togan, saacad wareegna mid taban. Haddii S ka weyn tahay meeriska, socodkeennii waan wadaynaa ilaa aan jarayno fogaan ah S halbeeg. Shaxanka 7aad, BR waxay u taagan tahay fogaan ah $\frac{3}{4}\pi$, BT-na fogaan ah $2\pi + \frac{\pi}{4}$ ama $\frac{9}{4}\pi$. Shaxanka 8aad, BR waxay u taagan tahay fogaan ah $\left(-\frac{\pi}{4}\right)$, BT-na fogaan ah $-\frac{3\pi}{2}$.



sh. 8



Sh. 7

Hadda waxan aragnay in tiro kasta oo maangal ah S aan u heli karro barta R oo fogaanta ay B u jirtaa tahay S marka meeriska la maro.

Hubaal, taasi waa isku aaddin ama xiriir min tirada maangaka ah S ilaa barta R (S→R). Haddaba, ururka {(S, R)}, S tahay tiro maangal ah, R-na bar ku taal goobada $x^2 + y^2 = 1$, ma yahay fansaar min ururka tirooyinka maangal ah ilaa bar ku taal meeriska goobo halbeegga. Bal labadii su'aalood ee fansaarka lagu garan jiray aan isweydiinno :

Haddii S tahay tiro maangal ah, ma jiraan laba barood oo meeriska ku yaal oo S halbeeg u wada jira barta (1,0), marka meeriska laga cabbiro π. Ma jirtaa tiro maangal ah S, oo aan cabbirayn fogaanta ay bari u jirto (1,0) ? Jawaabta labadaa su'aaloodba waa maya. Markaa ururku waa fansaar. Fansaarkaa waxa la yiraa **Fansaar Goobo.** W = {(S, R) | S tahay tiro maangal ah, R-na bar ku taal goobada $x^2 + y^2 = 1$}. Horaadka W waa ururka dhammaan tirooyinka maangalka ah.

Bar kasta R, oo meeriska goobo halbeeg ku taal waxay leedahay kulammada R ay yihiin (x, y). Markaa W waxan u qori karraa sidan: W = { (S, (x, y)) | S tahay tiro maangal ah, (x, y) kulammada bar ku taal goobada $x^2 + y^2 = 1$}.

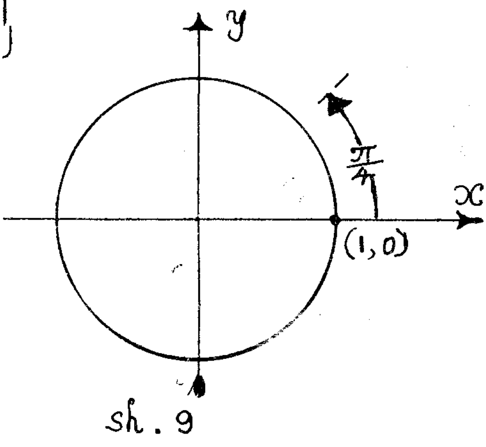
Ogow:

Danbeedku maaha dhammaan lammaaneyaasha horsan ee tirooyinka maangalka ah ee waa lammaaneyaasha horsan ee raalligeliya isle'egta $x^2 + y^2 = 1$. Fansaarkani muxuu kaga duwan yahay kuwii aan ku soo aragnay cutubkii xiriir iyo fansaar ?

Hadda, bal aan eegno tusaale ku saabsan sida loo soo saaro (x, y), marka S lagu siiyo. Ogow marka aan soo saarayno qiimaha W (S), waxa aan helaynaa in uu yahay kulammada bar ku taal meeriska goobo halbeeg oo fogaanta ay u jirto barta (1,0) tahay S halbeeg oo laga cabbiray meeriska. Markaa, waxan u baahannahay in aan naqaanno joomatariga goobo halbeeg iyo jidka fogaanta, $D^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Tusaale 1:

Raadi $W\left(\frac{\pi}{4}\right)$



sh. 9

Shaxanka 9aad wuxu muujinayaa goobo halbeeg iyo qaansada dhererkeedu yahay $\frac{\pi}{4}$. Haddaba, mar haddii (x, y) ay kala badho qaansada min (1,0) ilaa (0,1) $\left(\frac{\pi}{4} = \frac{1}{2} x \frac{\pi}{2}\right)$. Markaa waxan leennahay $x = y$. Waliba waxan naqaan in $x^2 + y^2 = 1$, $x^2 + y^2 = 1. \Rightarrow x^2 + x^2 = 1$ ama :

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

Sidoo kale, $y = \pm \frac{1}{\sqrt{2}}$ waayo $x = y$.

Laakiin x iyo y labaduba waxay ku yaallaan waaxda 1aad, oo way togan yihiin. Markaa jawaabta la inaga rabaa

waa $x = \frac{1}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$

$$W\left(\frac{\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

Ogow:

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Qiimayaasha $W\left(\frac{3\pi}{4}\right)$, $W\left(\frac{5\pi}{4}\right)$ iyo

$W\left(\frac{7\pi}{4}\right)$ waxa lagu soo saari karaa wanqarka.

NOQTIIN KU SAABSAN WANQARKA.

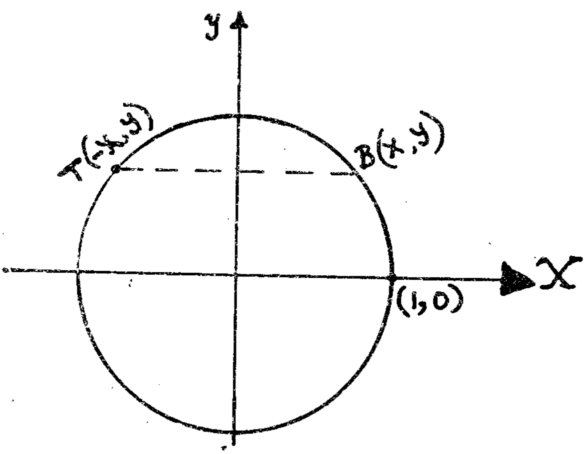
Qeex 1:

Barta B waxay ku wanqaran tahay xarriiqda L haddii ay jirto bar kale B, oo uu L yahay qotome badhaha xarriiqda BB. Markaa, B waxa la yiraa **Noqodka** B ee L. Sidoo kale B waa noqodka B ee L.

Qeex 2:

Barta M waxay ku wanqaran tahay barta kale ee N haddii ay jirto barta M oo ay N tahay bar bartamaha xarriijinta MM. Markaa, M waa noqodka M oo loo eegay N, M-na waa noqodka M.

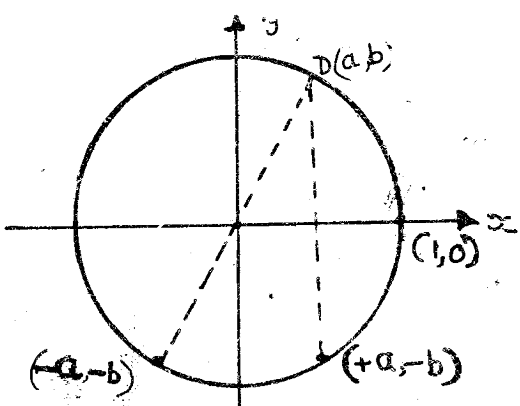
Bal aan tixgelinno baraha meeriska goobo halbeeg (eeg shaxanka 10)



Haddii barta B (x, y) ay goobada ka mid tahay, waxa jirta bar kale, T (– x, y) oo isla goobada ka mid ah. Raadi bar bartanka BT. Ma ku taal dhidibka – y ? Tirada BT eber ma tahay ? Xarriijinta BT ma ku qotontaa dhidibka – y ? Jawaabta dhammaan su'aalahaasi waa haa. Markaa, waxan oran karraa bar kasta oo goobo halbeeg way ku wanqaran tahay dhidibka – y. Waliba, haddii kulammada bar ku taal goobo halbeeg, ay yihiin (x, y), kulammada bar noqodkeeda dhidibka – y waa (– x, y).

Sidoo kale, waxan helaynaa in bar kasta oo goobo halbeeggu ku wanqaran tahay dhidibka – x iyo unugga. Haddii D(a, b) ay ku taal goobo halbeegga, bar noqodka D ee dhidibka – x waa barta (a, – b). Bar noqodka D ee unugga waa (– a, – b).

(eeg shaxanka 11).



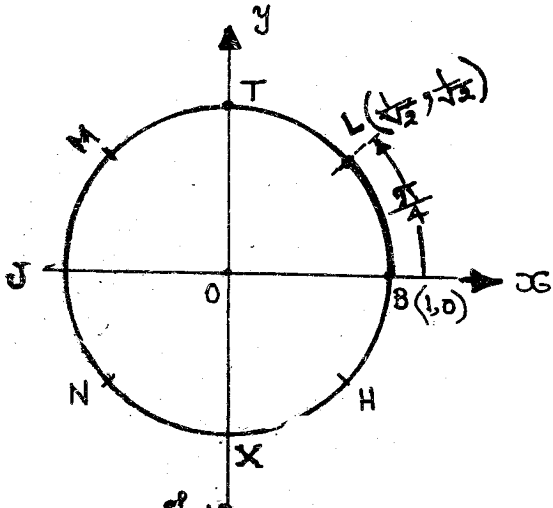
sh. 11

Tusaale 2:

Raadi $W\left(\frac{3\pi}{4}\right)$, $W\left(\frac{5\pi}{4}\right)$ iyo $W\left(\frac{7\pi}{4}\right)$.

Shaxanka 12aad wuxu muujinayaa.

$\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$ iyo $\frac{7\pi}{4}$.



Sh.12

U fiirso L, M, N iyo H in ay yihiin baro dhammaad-yada qaansooyinka dhererradoodu yihiin $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$ iyo $\frac{7\pi}{4}$ siday u kala horreeyaan. Waliba waa baro badh-tamaha qaansooyinka BT, TJ, JX iyo XB siday u kala horreeyaan. Haddaba, ma oran karraa LT = TM, BL = BH, LO = ON ? Waayo ? Haddaba, waxa cad in M tahay noqodka L ee dhidibka – y. H-na noqodka L ee dhidibka – x. N-na noqodka L ee unugga O. Markaa kulammada M, N iyo H waa $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$$\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \text{ iyo } \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right).$$

Siday u kala horreeyaan, haddaba,

$$W\left(\frac{3\pi}{4}\right) = \left(\frac{1 -}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right),$$

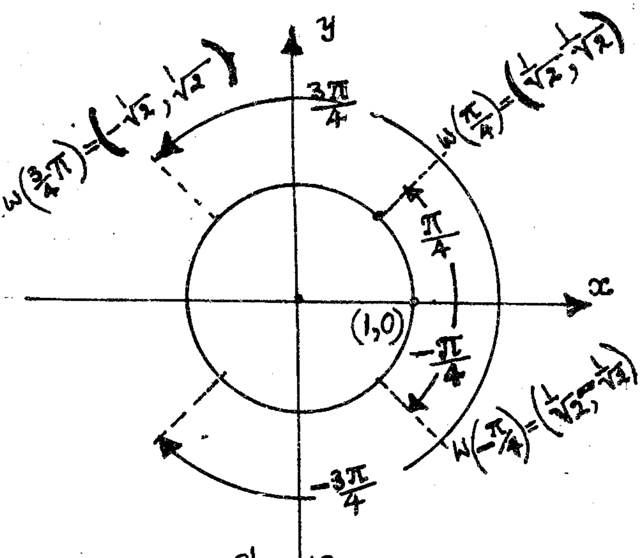
$$W\left(\frac{5\pi}{4}\right) = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$W\left(\frac{7\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

Tusaale 3:

Raadi $W\left(\frac{-\pi}{4}\right)$ iyo $W\left(\frac{3\pi}{4}\right)$.

(eeg shåxanka 13)

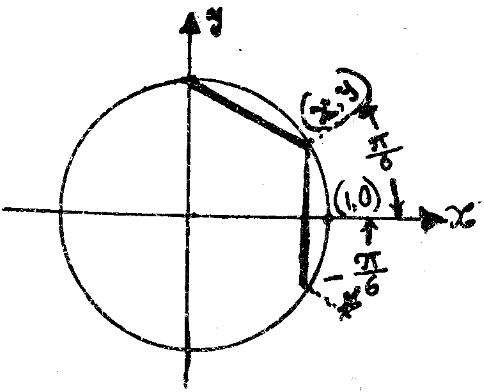


Sh. 13

Waxa shaxanka ka muuqda in $\frac{\pi}{4}$ iyo $\frac{-\pi}{4}$ ay ka mid walba tahay noqodka ta kale ee dhidibka $-x$. Sidoo kale $\frac{3\pi}{4}$ iyo $\frac{-3\pi}{4}$, mid walba waa noqodka ta kale ee dhidibka $-x$, markaa, $W\left(\frac{-\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$.

$$W\left(\frac{-3\pi}{4}\right) = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

Guud ahaan, haddii $W(\Theta) = (a, b)$, markaa $W(-\Theta) = (a, -b)$.



Sh. 14

Tusaale:

Raadi $W\left(\frac{\pi}{6}\right)$.

Shaxanka 14 ayaa muujinaya. Barta (x, y) waa $W\left(\frac{\pi}{6}\right)$. Wanqarku wuxuu inoo sheegi karaa $W\left(\frac{-\pi}{6}\right)$.

Dhererka qaansada min (x, y) ilaa $(0, 1)$ waa $\frac{\pi}{6} + \frac{\pi}{6} =$ $\frac{2\pi}{6} = \frac{\pi}{3}$. Dhererka sh. 14 qaansada min (x, y) ilaa $(0, 1)$ waa $\frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$. Joomatariga waxan ka baranay in qaansooyinka isle'egki ay sameeyaan boqonno isle'eg. Markaa fogaanta min (x, y) ilaa $(x - y)$ waxay isle'eg tahay fogaanta min (x, y) ilaa $(0,1)$ hadda isticmaal jidkaa fogaanta.

$$(x - 0)^2 + (y - 1)^2 = (x - x)^2 + (- y - y)^2$$
$$x^2 + y^2 - 2y + 1 = 0 + 4y^2 .$$

Laakiin, $x^2 + y^2 = 1$

$$\therefore \quad 1 - 2y + 1 = 4y^2$$

$$0 = 4y^2 - 2y + 2$$
$$0 = 2y^2 - y + 1$$
$$0 = (2y - 1)(y + 1).$$

Haddaba $y = \frac{1}{2}$ ama $y = - 1$.

Mar haddii (x, y) ay ku taallo waaxda 1aad, markaa kulanka y waa $\frac{1}{2}$, mar haddii $y = \frac{1}{2}$, markaa $x^2 + y^2 = 1$

$$\Longrightarrow x^2 + \left(\frac{1}{2}\right)^2 = 1.$$

$$\therefore \quad x^2 = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\because\ x = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$$

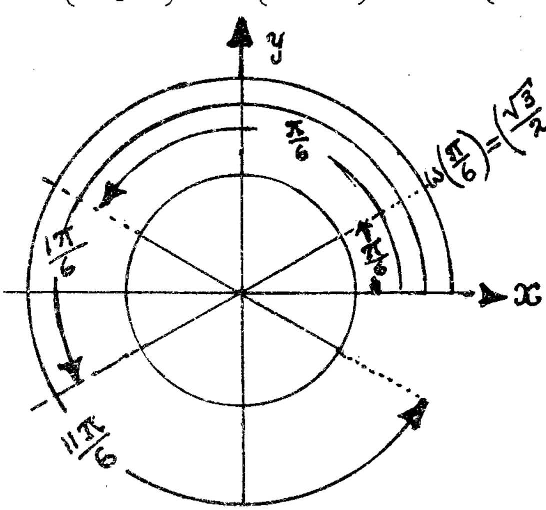
Mar haddii (x, y) ay ku taallo waaxda 1aad $x = + \sqrt{\frac{3}{2}}$.

Haddaba $W\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}, \sqrt{\frac{3}{2}}\right)$.

Marka aan isticmaalno wanqarka waxaynu si dhib yar u soo saari $W\left(\frac{5\pi}{6}\right)$, $W\left(\frac{7\pi}{6}\right)$ iyo $W\left(\frac{11\pi}{6}\right)$.

Tusaale:

Raadi $W\left(\frac{5\pi}{6}\right)$, $W\left(\frac{7\pi}{6}\right)$ iyo $W\left(\frac{11\pi}{6}\right)$.



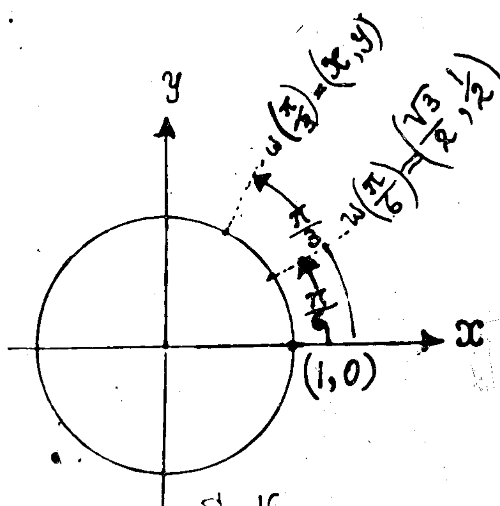
sh. 15

Waxa shaxanka ka cad in $W\left(\frac{\pi 5}{6}\right)$ ay tahay noqodka $W\left(\frac{\pi}{6}\right)$ ee dhidibka $-y$; $W\left(\frac{7\pi}{6}\right)$ waa noqodka $W\left(\frac{\pi}{6}\right)$ ee unugga; sidoo kale $W\left(\frac{11\pi}{6}\right)$ waa noqodka $W\left(\frac{\pi}{6}\right)$ ee dhidibka $-x$. Markaa $W\left(\frac{5\pi}{6}\right) = \left(-\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$, $W\left(\frac{7\pi}{6}\right) = \left(-\sqrt{\frac{3}{2}}, -\frac{1}{2}\right)$

$W\left(\frac{11\pi}{6}\right) = \left(\sqrt{\frac{3}{2}}, -\frac{1}{2}\right)$.

Tusaale:

Raadi $W\left(\frac{\pi}{3}\right)$.



sh 16

Shaxanka 16 wuxu tusayaa goobo halbeeg, lammaanaha horsan (x, y) waa $\left(\frac{\pi}{3}\right)$. Mar haddii $\frac{\pi}{3} = \frac{\pi}{6}$

$+\frac{\pi}{6}$, waxa hubaal ah in qaansada min (1,0) ilaa

$\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$ ay le'eg tahay qaansada min $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$

ilaa (x, y). Markaa boqonka min $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$ ilaa (x,y)

wuxu le'eg yahay boqonka min (1,0) ilaa $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$,

marka aan jidka fogaanta la kaashanno waxanu heli in

$$\left(x - \sqrt{\frac{3}{2}}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \left(1 - \frac{\sqrt{3}}{2}\right)^2 + \left(0 - \frac{1}{2}\right)^2.$$

$$\therefore \quad x^2 - 2\sqrt{3} + \frac{3}{4} + y^2 - y + \frac{1}{4} = 1 - \sqrt{3} + \frac{3}{4} + \frac{1}{4}$$

$$\therefore \quad x^2 + y^2 - x\sqrt{3} + \frac{3}{4} + \frac{1}{4} = 1 - \sqrt{3} + \frac{3}{4} + \frac{1}{4}$$

Mar haddii $x^2 + y^2 = 1$,

$$1 - x\sqrt{3} + 1 - y = 1 - \sqrt{3} + 1$$

$$\therefore \quad -y - x\sqrt{3} = -\sqrt{3}$$

$$y = -\sqrt{3x} + \sqrt{3}$$

$$y = -\sqrt{3}(x - 1)$$

$$y = +\sqrt{3}(1 - x)$$

$\therefore$ mar haddii $x^2 + y^2 = 1$

$$x^2 + [\sqrt{3}(1 - x)]^2 = 1$$

$$x^2 + 3(1 - 2x + x^2) = 1$$

$$x^2 + 3 - 6x + 3x^2 = 1$$

$$4x^2 - 6x + 2 = 0$$

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - 2x - x + 1 = 0$$

$$2x(x - 1) - 1(x - 1) = 0$$

$$(2x - 1)(x - 1) = 0$$

$\therefore$ $2x - 1 = 0$ ama $x - 1 = 0$

$\therefore$ $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$,

$x - 1 = 0 \Rightarrow x = 1.$

Laakiin, haddii $x = 1$, markaa $y = \sqrt{3}(1 - x) = 3(0) = 0$, bartuna waxay ku taal dhidibka $- x$. Markaa qiimaha la rabaa waa $x = \frac{1}{2}$. Haddii $x = \frac{1}{2}$, markaa

$$y = \sqrt{3}\left(1 - \frac{1}{2}\right) = \sqrt{3} \times \frac{1}{2} = \sqrt{\frac{3}{2}}.$$

Markaa $W\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \sqrt{\frac{3}{2}}\right)$

Layli:

1. Haddii ay bari ku wareegayso goobo gacankeedu yahay 1 sm. Ku soo saar fogaanahan sintimitirka ugu dhow.

b) hal wareeg   t) $\frac{2}{3}$ wareeg

j) $2\frac{1}{2}$ wareeg   x) $3\frac{1}{3}$ wareeg

kh) $5\frac{1}{2}$ wareeg.

2. Raadi mid kastoo soo socota :

b) $W\ (2\pi)$ t) $W\ (0)$

j) $W\left(\frac{2\pi}{3}\right)$ x) $W\left(\frac{3\pi}{4}\right)$

kh) $W\left(\frac{5\pi}{6}\right)$ d) $W\left(\frac{7\pi}{6}\right)$

r) $W\left(\frac{5\pi}{4}\right)$ s) $W\ (-3\pi)$

sh) $W\left(-\frac{3\pi}{4}\right)$ dh) $W\ (-2\pi)$

c) $W\ (-5\pi)$ q) $W\ (7\pi)$

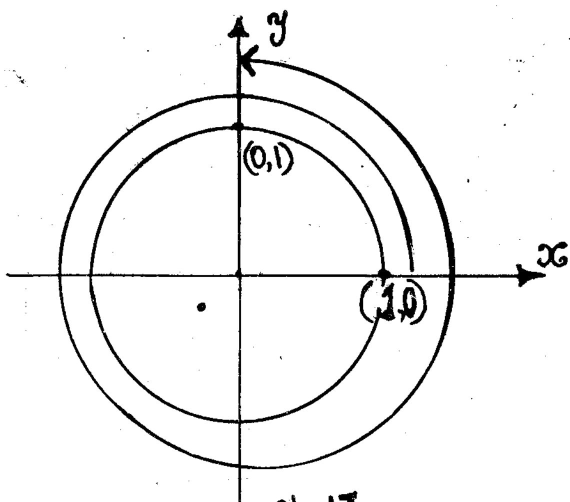
k) $W\left(\frac{9\pi}{2}\right)$ l) $W\ (-\pi)$

m) $W\left(\frac{-13\pi}{4}\right)$

3. Raadi x mar kasta oo soo socda;

Tusaale:

$W(x) = (0,1) \qquad 2\pi < x < 3\pi$



sh. 17

Barta (0,1) waa isgoyska dhidibka – y ee togan iyo halbeeg. Markaa fogaanta min (1,0) ilaa (0,1) oo laga cabbiray meeriska waxay noqon kartaa $\frac{1}{4}$ meeriska oo ah $\frac{2\pi}{4} = \frac{\pi}{2}$. Waxa kale oy noqon kartaa 1 wareeg oo min (1,0) ilaa (1,0) am 2π oo loo qeeyay $\frac{\pi}{2}$. Waxa kale oy noqon kartaa 2,3,4,5, iwm oo wareeg oo loo qeeyay $\frac{\pi}{2}$.

t. A., waxay noqon kartaa $2\pi + \frac{\pi}{2}$, $2\ (2\pi) + \frac{\pi}{2}$; $3\ (2\pi) + \frac{\pi}{2}$; $4\ (2\pi) + \frac{\pi}{2}$; $5\ (2\pi) + \frac{\pi}{2}$; iwm. U fiirso xannibaadda su'aasha u socota, x way ka weyn tahay ka yar tahay, 3π. Markaa x waa $2\pi + \frac{\pi}{2}$ oo ah $\frac{5\pi}{2}$.

b) $W\ (x) = (1,0)$ Haddii $0 \angle x \angle \frac{\pi}{2}$

t) $W\ (x) = \left(-\frac{1}{2}, \frac{3}{2}\right)$ » $\frac{\pi}{2} \angle x \angle \pi$

j) $W\ (x) = (0, -1)$ » $0 \angle x \angle 2\pi$

x) $W\ (x) = \left(-\frac{2}{2}, -\frac{2}{2}\right)$ » $\pi \angle x \angle 2\pi$

kh) $W\ (x) = \left(\frac{2}{2}, \frac{2}{2}\right)$ » $2\pi \angle x \angle \frac{5\pi}{2}$

d) $W\ (x) = (0,1)$ » $3\pi \angle x \angle 4\pi$

r) $W\ (x) = \left(\frac{3}{2}, -\frac{1}{2}\right)$ » $-\frac{\pi}{2} \angle x \angle 0$

s) $W\ (x) = \left(-\frac{2}{2}, \frac{2}{2}\right)$ » $-\frac{3\pi}{2} \angle x \angle -\frac{\pi}{2}$

sh) $W\ (x) = \left(-\frac{3}{3}, -\frac{2}{2}\right)$ » $\frac{5\pi}{2} \angle x \angle \frac{7\pi}{2}$

dh) $W(x) = (1,0)$

4. **Tus in**

b) $W\left(\frac{5\pi}{2}\right) = W\left(\frac{\pi}{2}\right)$

t) $W(5\pi) = (\pi)$

j) $W\left(\frac{9\pi}{4}\right) = W\left(\frac{\pi}{4}\right)$

x) $W\left(-\frac{\pi}{2}\right) = W\left(\frac{3\pi}{2}\right)$

kh) $W(-\pi) = (\pi)$

d) $W\left(\frac{7\pi}{2}\right) = W\left(-\frac{\pi}{2}\right)$

r) $W\left(\frac{\pi}{6}\right) = W\left(\frac{25\pi}{6}\right)$

s) $W(2\pi) = W(4\pi)$

sh) $W(2\pi) = W(-2\pi)$

dh) $W(3\pi) = W(-3\pi)$

KALGALID

Fansaarkaa aan soo qeexnay, k.a., fansaar goobo, wuxuu leeyahay sifo u gaar ah oo laga garto fansaarrada tibxaale ee aan horay u soo sheegnay. Sifadaa waxa la yiraa **Kalgalid.**

CUTUB III

Waxan ognahay, in horaadka fansaarkeenu yahay ururka dhammaan tirooyinka maangalka ah, iyo in dhambeedkiisu yahay ururka lammaanayaasha horsan (x, y) ee tirooyinka maangalka ah ee $x^2 + y^2 = 1$.

T.a. $H(w) = \{(a|a) \in$ ururka tirooyinka maangalka ah$\}$, $D(w) = \{(x, y)\ x, y \in$ ururka tirooyinka maangalka ah, $x^2 + y^2 = 1\}$. Hadda bal an qeexno kalgalid.

Qeex :

Fandaarka $F(x)$ ee horaadkeedu yahay urur tirooyin maangal ah, waxa la yiraa **way kalgashaa**, kalkeeduna waa q haddii:

1. $F(x + q) = F(x)$, x waa kutirsane kasta oo horaadka.

2. $q \neq 0$.

3. q waa tirada maangalka ah ee ugu yar ee rumaysa xaaladda 1aad.

Qeexdan sare, horaadka waxan ku koobnay inuu noqdo urur tirooyin maangal ah, laakiin taasi khasab maaha, inkastoo ay fududahay.

Xaaladdo 2aad.

$q \neq 0$. Haddii $q \neq 0$, markaa fansaar kasta $F(x)$, xaaladda 1aad way raalligelin, t.a., $F(x + 0) = F(x)$, waayo $x = x + 0$ marka x tahay maangal. Markaa fansaar kasta kalgal buu noqon. Laakiin ma rabno in aan fansaar kasta ku sheegno kalgal. Haddaba q waa in uuna le-egkaan eber, t.a. $q \neq 0$.

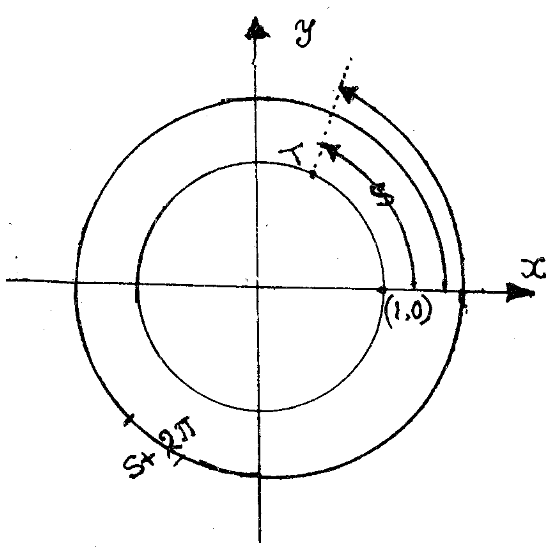
q waa tirada maangalka ah ee togan ee ugu yar ee rumaysaa $F(x + q) = F(x)$. Haddaba, bal ka warran $F(x + 2q)$, $F(x + 3q)$,....... $F(x + nq)$.

U fiirso $F(x + 2q) = F_1(x + q)_2 = F(x)_3$
Sidoo kale,

$$\begin{aligned} F(x+nq) &= F(x+(n-1)+q) = F(x+(n-1)q) \\ &= F(+(n-2)q)+q) = F(x+(n-2)q) \\ &= F(x+(n-3)q)+q) = F(x+(n-3)q) \\ &= F(x+(n-(n-1)q) = F(x+q) = F(x) \end{aligned}$$

Haddaba, haddii $F(x + q) = F(x)$, markaa dhufsane kasta oo q isna sidaas oo kale ayuu sameynayaa. Haddaba, si aan u dooranno mid aan ula baxno **kal** waa in aan qaadannaa ka ugu yar ee togan.

Imika bal aan u soo noqonno fansaarkeennii W. Ma yahay fansaar kalgala? Waa imisa kalkiisu, t.a., waa imisa q-diisu?



Qaansooyinka S, $2\pi + S$, $4\pi + S$, $6\pi + S$,..... isla bar bay ku dhammaadaan, taaso oo ah T. Haddii T tahay barta kulamadeedu yihiin (a, b), raadi W(s), W(s + 4π), W(6π + s)? Mid walba waxay le'eg tahay (a, b).

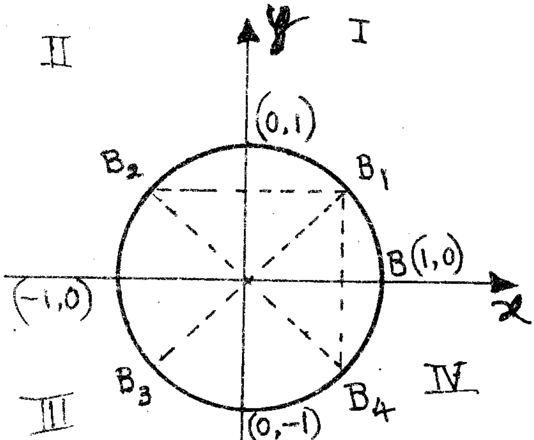
Markaa
$$W(s + 2\pi) = W(s)$$
$$W(s + 4\pi) = W(s)$$
$$W(s + 6\pi) = W(s)$$
$$W(s + 8\pi) = W(s)$$
$$W(s + 2n\pi) = W(s)$$

Markaa waxa cad in kalka fansaarku yahay 2π. Guud ahaan, $W(x + 2\pi) = W(x)$ ama $W(x + 2n\pi) = W(x)$, n waa abyoone.

Habkaa waxa la yiraa **xeerka u celinta,** waayo fansaar kasta oo qaanso dhererkii la doono leh waxa loo celin karaa fansaar qaanso dhererkeedu u dhexeeyo 0 iyo 2π.

$$W\left(\frac{5\pi}{2}\right) = W\left(\frac{\pi}{2} + 2\pi\right) = W\left(\frac{\pi}{2}\right)$$

$$W\left(\frac{\pi}{2}\right) = W\left(2\pi - \frac{\pi}{2}\right) = W\left(\frac{3\pi}{2}\right)$$

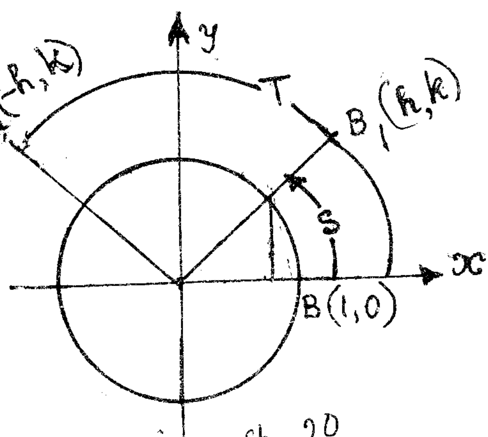


sh. 19

Ku celinta Waaxda 1aad :

Shaxanka 19aad wuxu muujinayaa goobo halbeeg ay ku jiraan baraha B_1, B_2, B_3 iyo B_4 oo ku kala yaal waaxda I, II, III iyo IV siday u kala horreeyaan. B_1 waa bar dhammaadka qaansada BB_1, sidoo kale B_2, B_3 iyo B_4 waa baro dhammaadyada qaansooyinka BB_2, BB_3 iyo BB_4 siday u kala horreeyaan.

Haddaba, ka soo qaad in kulammada barta B_1, ay yihiin (h, k), kuwa B_2 ay yihiin $(-h, k)$, iyo in dhererka qaansada BB_1, u yahay S, ka BB_2 u yahay T. Markaa $W(s) = (h, k)$, $W(t) = (-h; k)$.



Sh. 20

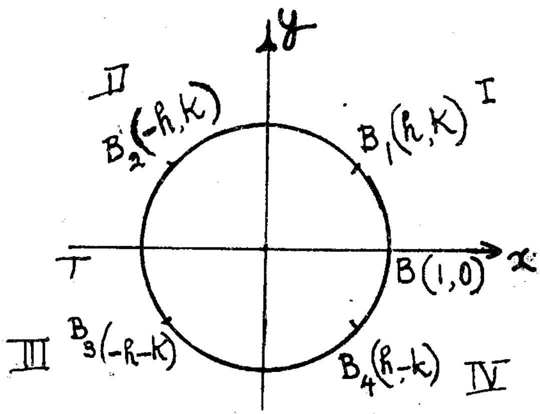
U fiirso, qaansada BB_1, waxay le'eg tahay qaansada B_2D. Laakiin $BB_2 = BD - B_2D$. Haddaba, ma oran karnaa $BB_2 = BD - BB_1$? Laakiin, $BB_1 = S$, $BB_2 = T$, $BD = \pi$, waayo? Markaa $T = \pi - S$. Haddaba, $W(\pi - S) = (-h, k)$, waayo?.

Guud ahaan, haddii T tahay tiro maangal ah oo waaxda II, S-na tahay tiro maangal ah oo waaxda I, isla markaa haddii $T = \pi - S$, markaa:

$$W(s) = (h, k) \longrightarrow W(t) = (-h, k).$$

OGOW: B_1 waa noqodka B_2 ee dhidibka $-y$.

U fiirso shaxanka 21aad.



B_2 waa noqodka B_1 ee dhidibka $-y$, B_3 waa noqodka B_1 ee unugga, B_3-na waa noqodka B_1 ee dhidibka $-x$. Haddaba ma oran karnaa qaansooyinka BB_1, B_2T, TB_3 iyo B_4B way isle'eg yihiin? Waayo? Haddaba

$$BB_2 = \pi - BB_1$$
$$BB_3 = \pi + BB_1$$
$$BB_4 = 2\pi - BB_1$$

Haddii $BB_1 = S$, $BB_2 = T$, $BB_3 = J$, $BB_4 = D$, markaa $T = \pi - S$, $J = \pi + S$, $D = 2\pi - S$. Markaa, haddii $W(s) = (h, k)$, waxan helaynaa in

$$W(\pi - A) = (-h, k), \quad W(\pi + A) = (-h, -k),$$
$$W(2\pi - A) = (h, -k)$$

Guud ahaan, haddii S tahay tiro maangal ah, A-na tahay tirada maangalka ah ee la xiriira ee waaxda I, isla markaa $W(A) = (h, k)$, kolkaa

1) S waaxda I, $W(s) = (h, k)$
2) S waaxda II, $W(s) = (-h, k)$
3) S waaxda III, $W(s) = (-h, -k)$
4) S waaxda IV, $W(s) = (h, -k)$

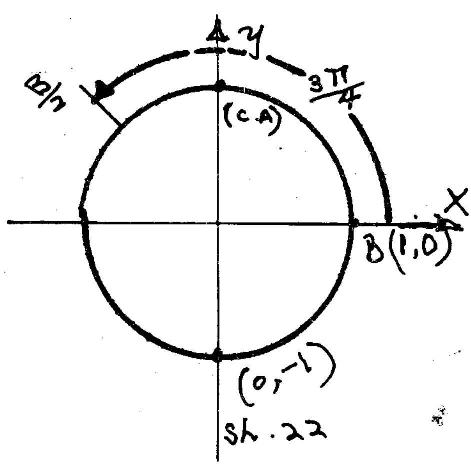
Ogow :

1. Haddii B_1 tahay bar ku taal waaxda kowaad, A-na yahay fogaanta min (0,1), ilaa B oo laga cabbiray meeriskan, markaa tirooyinka A la xiriirta ee waaxda II, III IV waa fogaanta min (0, 1) ilaa, noqodka B_1 ee dhidibka $-y$, noqodka B_1 ee unugga, noqodka B_1 ee dhidibka $-x$. siday u kala horreeyaan.

2. Haddii S tahay tiro waaxda I, markaa tirooyinka S la xiriira ee waaxda II, III iyo IV waa $\pi - S$, $\pi + S$ iyo $2\pi - S$ siday u kala horreeyaan markaa waxa cad in

$$W(\pi - s) = (-h, k), \quad W(\pi + s) = (-h, -k),$$

$$W(2\pi - s) = (h, -k).$$



Tusaale 1:

Raadi tirada maangalka ah ee waaxda I ee la xiriirta $\frac{3\pi}{4}$.

$\frac{3\pi}{4}$ waxay ku dhacdaa waaxda II waana fogaanta min (1, 0) ilaa B_2 oo laga cabbiray meeriska, sida u shaxanka 22 tusayo, ka soo qaad in tirada la xiriirta ee waaxda laad ay tahay x.

$$\therefore \quad \frac{3\pi}{4} = \pi - x, \Longrightarrow x = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$$

Tirada la xiriirta $\frac{3\pi}{4}$ waa $\frac{\pi}{4}$.

Tusaale 2:

Raadi $W\left(\frac{3\pi}{4}\right)$.

Furfuris :

Mar haddii $\frac{3\pi}{4}$ ay la xiriirto $\frac{\pi}{4}$, t.a., $\frac{\pi}{4} = \pi - \frac{3\pi}{4}$

waan heli karraa $\frac{3\pi}{4}$ waayo $W\left(\frac{\pi}{4}\right) = \left(\frac{+1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$\therefore \quad W\left(\frac{3\pi}{4}\right) = \left(\frac{-1}{\sqrt{2}} \ \frac{1}{\sqrt{2}}\right).$$

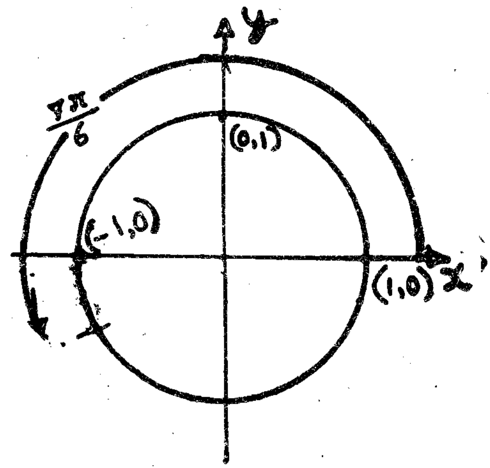
Tusaale 3:

Raadi $W\left(\frac{31\pi}{6}\right)$

Furfuris :

$W\left(\frac{31\pi}{6}\right) = W\left(\frac{7\pi}{6} + 4\pi\right) = W\left(\frac{7\pi}{6}\right)$ xeerka u celinta.

Haddaba, siday u Sh. 23aad muujinayo, $\frac{7\pi}{6}$ waxay taal waaxda III.



Sh. 23

Ka soo qaad in tirada la xiriirta ee waaxda ay tahay x.

$$\therefore \quad \frac{7\pi}{6} = \pi + x, \qquad \therefore \quad x = \frac{7\pi}{6} - \pi = \frac{1\pi}{6}.$$

Laakiin $\quad W\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right).$

Haddaba $\quad W\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right).$

$$\therefore \quad W\left(\frac{31\pi}{6}\right) = W\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right).$$

Layli :

1. Raadi mid kasta oo soo socota, adoo la kaashanaya xeerka u celinta mar allaale markii loo baahdo.

b) $W\left(\frac{11\pi}{2}\right)$

t) $W\left(\frac{23\pi}{4}\right)$

j) $W\left(\frac{25\pi}{6}\right)$

x) $W\left(\frac{3\pi}{2}\right)$

kh) $W\left(\frac{9\pi}{4}\right)$

d) $W(3\pi)$

r) $W\left(-\frac{\pi}{3}\right)$

s) $W\left(-\frac{2\pi}{3}\right)$

sh) $W\left(-\frac{3\pi}{2}\right)$

dh) $W(-2\pi)$

Dhammaystir tusahan haddii $W(s) = (h, k)$.

S	O	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
h					
k					

SAYN IYO KOSAYN

Waxan ognahay in fansaarka W, u horaadkiisa yahay ururka R ee tirooyinka maangalka ah, dambeedkiisuna ururka lammaanayaasha horsan (x, y) ee $X^2 + Y^2 = 1$. Waxa jira lammaanayn lagama maarmaan ah, ka isku aaddinta kutirsaneyaasha R iyo xubnaha lammanayaasha horsan ee (x, y), isku aaddintaas waa fansaarro ka mid ah fansaarrada tirignoomatari.

Qeex :

Haddii $x, y \in R$, $X^2 + Y^2 = 1$, oo $W(s) = (x, y)$ markaa x waa Kosaynka s, y-na waa Saynka S.

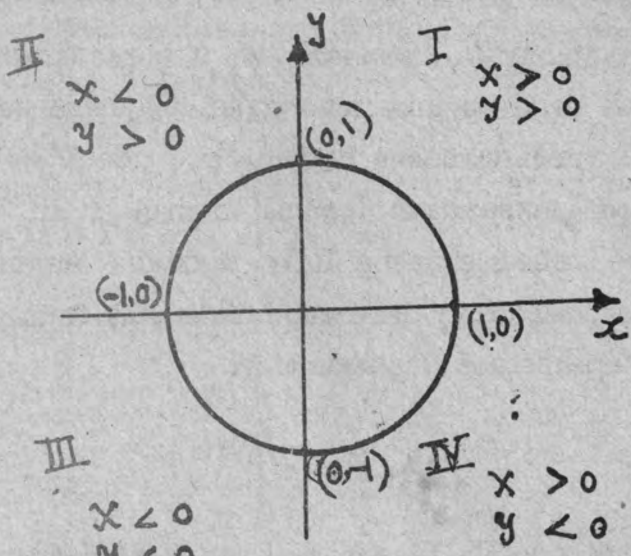
Qormo ahaan

$$x = \cos s$$
$$y = \sin s$$

Hadda, magacyo ayaan u bixinay xubnihii kutirsaneyaasha dambeedka W. Taas oo ah waxanu bixnay kosayn S, xubinta hore ee barta (x, y) marka S ay tahay fogaanta laga cabbiray meeriska ee min (1, 0) ilaa (x, y). Xubinta dambe ee (x, y)-na waxan u bixinay sayn S. Markan $W = \{s, (x, y)\}$ waxan u qori karnaa

$$W = \{s, (\cos S, \sin S)\}$$

Bal u fiirso summadda $x = \sin S$ iyo $y = \cos S$, ee waa x kasta.



Sh. 24 (b)

Waaxda I, x iyo y ama cos S iyo sin S soo labaduba way togan yihiin. **Waaxda II**, cos S wuu taban yahay sin S-na wuxu togan yahay. **Waaxda III**, cos S iyo sin S labaduba way taban yihiin. **Waaxda IV**, cos S wuu togan yahay sin S wuu taban taban yahay.

Tusaha hoose ayaa markii oo dhan soo gaabinaya.

Waaxda I

$x = \cos S > 0$
$y = \sin S > 0$

Waaxda II

$x = \cos S < 0$
$y = \sin S > 0$

Waaxda III

$x = \cos S < 0$
$y = \sin S < 0$

Waaxda IV

$x = \cos S > 0$
$y = \sin S < 0$

Qeex :

Haddii S e (ururka tirooyinka maangalka ah)

$$\text{Kosayn} = \{ (s, x) \mid x = \cos S\}$$
$$\text{Sayn} = \{ (s, y) \mid y = \sin S\}$$

Siday qeexdani sheegayso, kosaynku waa xiriir min dhererka qaansada S, ilaa xubinta hore ama kulanka $-x$ ee bar dhammaadka qaansada. Sidoo kale saynku waa xiriir min dhererka qaansada S ilaa kulanka $-y$ ee bar dhammaadka qaansada.

Ogow :

S waa tiro maangal ah. X iyo Y waa tirooyin maangal ah oo $|X| \leq |$, $|Y| \leq |$.

Hadda, waxad caddayn kartaa in kosaynku iyo saynku labaduba ay yihiin fansaarro horaadkoodu yahay ururka tirooyinka maangalka ah, dambeedkooduna yahay gaaliska, $\{m \mid m \in R, \mid m \mid \leq 1\}$.

Mar haddii cos S iyo sin S ay yihiin xubnaha kutir saneyaasha dambeedka W, markaa fansaarka saynka iyo fansaarka kosaynku labaduba way kalgalaan, kalkooduna waa 2π.

Haddaba:

1. ....$\cos(s + 2n\pi) = \cos S$.....n waa abyoone.
2. ....$\sin(s + 2n\pi) = \sin S$....n waa abyoone.

Mar haddii $(x, y) = (\cos S, \sin S)$, aanan naqaano sida loo soo saaro $W(s)$, waan soo saari karnaa cos S iyo sin S.

Tusaale :

Raadi $\cos\frac{\pi}{4}$ iyo $\sin\frac{\pi}{4}$.

Furfuris :

Waxan ognahay in $W\left(\frac{\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

Markaa, $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

Qiimayaasha W(s) ee aan ilaa hadda soo sarnay wa-xay ku yaallaan tusaha hoose.

U fiirso $0 \leq s \leq 2\pi$.

S	W(s)	Cos S	Sin S	S	W(s)	Cos S	Sin S
0	(1, 0)	1	0	$\frac{5\pi}{6}$	$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	π	$(-1, 0)$	-1	0
$\frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{7\pi}{6}$	$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\frac{\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{5\pi}{4}$	$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

S	W(s)	Cos S	Sin S	S	W(s)	Cos S	Sin S
$\frac{\pi}{2}$	$(0,\ 1)$	0	1	$\frac{4\pi}{3}$	$\left(-\frac{1}{2},\ -\frac{\sqrt{3}}{2}\right)$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$\frac{2\pi}{3}$	$\left(-\frac{1}{2},\ \frac{\sqrt{3}}{2}\right)$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{3\pi}{2}$	$(0,\ -1)$	0	-1
$\frac{3\pi}{4}$	$\left(-\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right)$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{5\pi}{3}$	$\left(\frac{1}{2},\ -\frac{\sqrt{3}}{2}\right)$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
				$\frac{7\pi}{4}$	$\left(\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
				$\frac{11\pi}{6}$	$\left(\frac{\sqrt{3}}{2},\ -\frac{1}{2}\right)$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
				2π	$\{1,\ 0\}$	1	0

OGOW :

1. $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

2. Marka aan la kaashanno isleegyada 1 iyo 2, iyo tusaahan, waxan heli karnaa kosaynka ama saynka tirooyinka $(s + 2n\pi)$ marka ay S tahay qiima tusaha ku jira, n-na tahay abyoone.

Tusaale :

Raadi $\cos\frac{9\pi}{2}$ iyo $\sin\frac{9\pi}{2}$.

Furfuris :

$\frac{9\pi}{2}$ wuxu le'eg yahay $4\pi + \frac{\pi}{2}$.

$$\text{Markaa } \cos\left(\frac{9\pi}{2}\right) = \cos\left(4\pi + \frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0.$$

$$\sin\left(\frac{9\pi}{2}\right) = \sin\left(4\pi + \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1.$$

Haddii $W(s) = (x, y)$, barta (x, y) waxay ka mid tahay barta goobo halbeegga, markaa $x^2 + y^2 = 1$. Laakiin $x = \cos S$, $y = \sin S$.

ARAGTIIN **I**

Haddii $S \in R$,

$$\cos^2 S + \sin^2 S = 1 \qquad (3)$$

U fiirso $\cos^2 S$ waa si kale oo lo qoro $(\cos S)^2$. Sidoo kale, $(\sin S)^2$ waxa loo qoraa $\sin^2 S$.

Haddaba,

$$\sin S = \begin{cases} \sqrt{1 - \cos^2 S} & \text{Waaxda I iyo II} \\ -\sqrt{1 - \cos^2 S} & \text{Waaxda III iyo IV} \end{cases}$$

$$\cos S = \begin{cases} \sqrt{1 - \sin^2 S} & \text{Waaxda I iyo IV} \\ -\sqrt{1 - \sin^2 S} & \text{Waaxda II iyo III} \end{cases}$$

Haddii sin S ama cos S aan naqaan, iyo waaxda bar dhammaadka qaansadu ay ku dhacdo, waan soo saari karna ka kale.

Tusaale :

Haddii $\cos S = -\frac{3}{5}$, $\pi < S < \frac{\quad}{2}$, raadi sin S.

Saynku wuu taban yahay waaxda III, markaa:

$$\text{Sin } S = -\sqrt{1 - \cos^2 S}$$

$$= -\sqrt{1 - \left(-\frac{3}{5}\right)^2}$$

$$= -\sqrt{1 - \frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

Waxan ognahay in marka

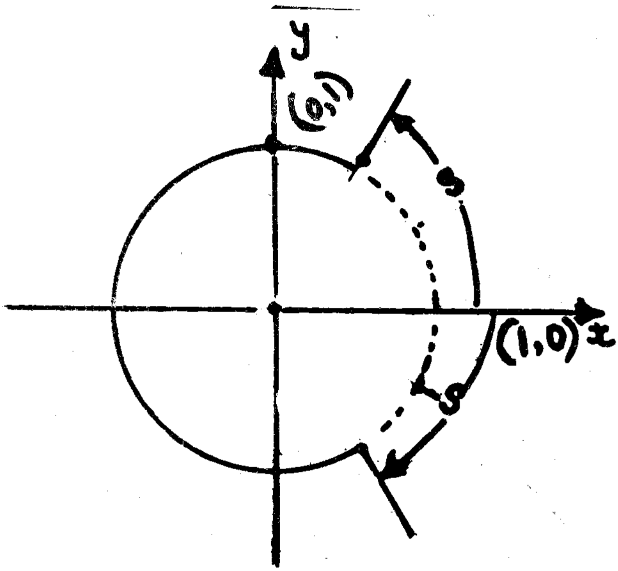
$$W(\Theta) = (x, y), \quad W(-\Theta) = (x, -y).$$

ARAGTIIN

S kasta oo kutirsane R ah,

$$\mathbf{\cos(-S) = \cos S}$$

$$\mathbf{\sin(-S) = -\sin S}$$



Fansaar kasta F, oo horaadkeedu yahay $D \leq R$, haddii $F(-s) = (F(s)$, F waxa la yiraa **fansaar dhaban ah;** haddii $F(-s) = -F(s)$, F waxa la yiraa **fansaar fisi ah.**

Layli :

Layliyada 1 - 9, waxad la kaashataa tusaha iyo isleegyada (1) iyo (2).

1) $\cos \dfrac{9\pi}{4}$

2) $\sin \dfrac{9\pi}{4}$

3) $\cos \left(- \dfrac{8\pi}{3} \right)$

4) $\sin \left(\dfrac{15\pi}{6} \right)$

5) $\sin\left(-\frac{5\pi}{3}\right)$

6) $\cos\left(\frac{15\pi}{6}\right)$

7) $\sin\left(-\frac{11\pi}{2}\right)$

8) $\cos\left(-\frac{11\pi}{2}\right)$

9) $\sin\left(\frac{25\pi}{4}\right)$

10) Haddii $\sin S = \frac{1}{3}$, $\cos S > 0$, raadi cos S.

11) Haddii $\sin \Theta = -\frac{2}{3}$, $\cos \Theta < 0$, raadi $\cos \Theta$.

12) Haddii $\cos A = \frac{12}{13}$, $\sin A > 0$, raadi sin A.

13) Haddii $\cos B = \frac{5}{13}$, $\sin B < \cos B$, raadi sin B.

JIDADKA KU CELINTA WAAXDA KOOWAAD EE SAYNKA IYO KOSAYNKA

Waxan ognahay haddii S tahay tiro maangal ah ee waaxda koowaad, markaa tirooyinka la xiriira ee waaxda II, III iyo IV ay yihiin $(\pi - s)$, $(\pi + s)$ iyo $(2\pi - s)$ siday u kala horreeyaan. Waliba haddii $W(s) = (h, k)$

markaa $W(\pi - s) = (-h, k)$, $W(\pi + s) = (-h, -k)$, $W(2\pi - s) = (h, -k)$.

Markaa, $\cos S = h$, $\sin S = k$. Haddaba waa imisa $\cos(\pi - s)$, $\cos(\pi + s)$ iyo $\cos(2\pi - s)$?. Sidoo kale u raadi $\sin(\pi - s)$, $\sin(\pi + s)$ iyo $\sin(2\pi - s)$. Xiriiryada hoos ku qoran ma gaari karnaa?

b) $\cos(\pi - s) = -\cos S$
t) $\cos(\pi + s) = -\cos S$
j) $\cos(2\pi - s) = \cos S$

iyo

1) $\sin(\pi - s) = \sin S$
2) $\sin(\pi + s) = -\sin S$
3) $\sin(2\pi - s) = -\sin S$

Saddexda hore waxay ka mid yihiin jidadka ku celinta waaxda koowaad ee Kosaynka; saddexda dambena waxay ka mid yihiin jidadka ku celinta waaxda koowaad ee Saynka.

Tusaale 1:

Raadi $\cos \dfrac{3\pi}{4}$.

Furfuris :

$\dfrac{3\pi}{4}$ way ka weyn tahay $\dfrac{\pi}{2}$ wayna ka yar tahay π, markaa waxay ku dhacaysaa waaxda II. Markaa waxa loo qori karraa sansaanka $(\pi - s)$

U fiirso in $\dfrac{3\pi}{4}$ la mid tahay $\left(\pi - \dfrac{\pi}{4}\right)$. Markaa haddii aan la kaashanno jidka ku celinta waaxda I ee kosaynka waxan heleynaa $\cos\left(\frac{3\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right)$

$$= -\cos\frac{\pi}{4}.$$

Laakiin $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}.$

$$\therefore \quad \cos\left(\frac{3\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}.$$

Tusaale 2:

Raadi $\sin\frac{29\pi}{4}$.

Furfuris :

$$\sin\frac{29\pi}{4} = \sin\left(7\pi + \frac{\pi}{4}\right)$$

$$= \sin\left(6\pi + \frac{5\pi}{4}\right)$$

$$= \sin\frac{5\pi}{4}.$$

Laakiin $\pi < \frac{5\pi}{4} < \frac{3\pi}{2}$, markaa waxay ku dhacaysaa waaxda III, waxana loo qori karraa sansaanka $(\pi + s)$.

$$\therefore \quad \frac{5\pi}{4} = \left(\pi + \frac{\pi}{4}\right)$$

Laakiin $\sin(\pi + s) = -\sin S$.

$$\text{Markaa } \sin\left(\frac{29\pi}{4}\right) = \sin\frac{5\pi}{4}$$

$$= -\sin\frac{\pi}{4}$$

$$= -\frac{\sqrt{2}}{2}.$$

Tusaale 1:

Raadi $\sin\left(-\frac{22\pi}{3}\right)$.

Furfuris :

$$\sin\left(-\frac{22\pi}{3}\right) = -\sin\left(\frac{22\pi}{3}\right)$$

$\sin\frac{22\pi}{3}$ waxay le'eg tahay $\left(6\pi + \frac{4\pi}{3}\right)$

$$\therefore \sin\left(\frac{22\pi}{3}\right) = \sin\left(6\pi + \frac{4\pi}{3}\right) = \sin\frac{4\pi}{3}$$

Laakiin $\pi < \frac{4\pi}{3} < \frac{3\pi}{2}$ oo waxay ku dhacdaa waaxda III, waxana loo qori karaa sansaanka $(\pi + s)$. Haddaba $\frac{4\pi}{3} = \pi + \frac{\pi}{3}$.

$$\text{Markaa} \quad \sin\frac{4\pi}{3} = \sin\left(\pi + \frac{\pi}{3}\right) = -\sin\frac{\pi}{3}.$$

$$\therefore \quad \sin\left(-\frac{22\pi}{3}\right) = -\sin\left(\frac{22\pi}{3}\right).$$

$$= -\sin\frac{4\pi}{3}$$

$$= -\left\{\sin\left(\pi + \frac{\pi}{3}\right)\right\}$$

$$= -\left(-\sin\frac{\pi}{3}\right)$$

$$= +\sin\frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$

Layli :

Adoo la kaashanaya tusaha hoose, ka shaqee layliyada soo socda :

A	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin A	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

1) $\sin \frac{2\pi}{3}$

2) $\cos \frac{2\pi}{3}$

3) $\sin \frac{3\pi}{4}$

4) $\sin \frac{4\pi}{3}$

5) $\cos \frac{11\pi}{6}$

6) $\sin \frac{7\pi}{4}$

7) $\cos \frac{5\pi}{6}$

8) $\sin \frac{5\pi}{6}$

9) $\cos \frac{7\pi}{4}$

10) $\sin \frac{11\pi}{6}$

11) $\cos \frac{4\pi}{3}$

12) $\sin \frac{13\pi}{6}$

13) $\cos \frac{13\pi}{4}$

14) $\sin \frac{10\pi}{4}$

15) $\cos \frac{11\pi}{4}$

Isla tusihii adoo isticmaalaya, raadi:

1. $\sin \left(-\frac{19\pi}{4}\right)$

2. $\cos \frac{25\pi}{4}$

3. $\sin \left(-\frac{29\pi}{6}\right)$

4. $\cos \frac{35\pi}{6}$

5. $\sin \left(-\frac{11\pi}{3}\right)$

6. $\sin \frac{23\pi}{6}$

7. $\cos \frac{27\pi}{4}$

8. $\sin \frac{162\pi}{3}$

9. $\cos \left(- \frac{55\pi}{6} \right)$

10. $\sin \frac{23\pi}{2}$

FANSAARRADA KALE EE GOOBO

Inagoo la kaashanayna fansaarrada saynka iyo ko-saynka waxan qeexi karnaa fansaarro kale oo goobo.

Qeex :

Ka soo qaad in $S \in R$ (R = ururka tirooyinka maan-galka ah).

1) Taanjenka S oo loo qoro tan S waa $\frac{\sin S}{\cos S}$

t.a., $\tan S = \frac{\sin S}{\cos S} \left\{ s \neq \frac{\pi}{2} + k\pi, \text{k waa abyoone} \right\}$

2) Siikanka S oo loo qoro sec S waa $\frac{1}{\cos S}$ t.a.,

$\sec S = \frac{1}{\cos S} \left\{ s \neq \frac{\pi}{2} + k\pi, \text{ k waa abyoone} \right\}$

3) Kosiikanka S oo loo qoro csc S waa $\frac{1}{\sin S}$ t.a.,

$$\csc S = \frac{1}{\sin S} \quad (s \neq k\pi, \ k \text{ waa abyoone})$$

4) Kotaanjanka S oo loo qoro cot S waa $\frac{\cos S}{\sin S}$ t.a.,

$$\cot S = \frac{\cos S}{\sin S} \quad (s \neq k\pi, \ k \text{ waa abyoone})$$

Hadda haddii qiimaha fansaarradaa mid ahaan lagu siiyo iyo waaxda S ay ku taal, waad soo saari kartaa qiimaha kuwa kale.

Tusaale 1:

Haddii $\sin S = \frac{3}{5}, \ \frac{\pi}{2} \leq S \leq \pi$ raadi:

cos S, tan S, sec S, csc S iyo cot S.

Furfuris :

Waxan naqaan in haddii $\frac{\pi}{2} \leq S \leq \pi$.

$$\cos S = -\sqrt{1 - \sin^2 S}$$

Markaa $\cos S = -\sqrt{1 - \left(\frac{3}{5}\right)^2} = -\frac{4}{5}$

$$\cot S = \frac{\cos S}{\sin S} = \frac{-\frac{5}{5}}{-\frac{3}{4}} = \frac{4}{3}$$

$$\sec S = \frac{1}{\cos S} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$$

$$\csc S = \frac{1}{\sin S} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$$

Hadda, bal fansaarradaa mid walba goonidiisa aan u falanqayno.

Qeex :

Haddii $S \in R$ (tiro maangal), $S \neq \frac{\pi}{2} + k\pi$, k-na yahay abyoone, markaa taanjant $= \{(s, t) \mid t = \tan S\}$.

Mar haddii $\tan S = \frac{\sin S}{\cos S}$, markaa horaadka fansaarku waa R (ururka tirooyinka maangalka ah) ee $\cos S \neq 0$, t.a., H (taanjant) $= \{S \mid S \in R, \cos S \neq 0\}$.

OGOW: $\cos S = 0$ marka $S = \frac{\pi}{2} + k\pi$, kna- yahay abyoone. Dambeedka taanjanku waa ururka dhammaan tirooyinka maangalka ah. Kalka taanjantku waa π. Bal fiirso in

$$\tan S = \frac{\sin S}{\cos S} = \frac{-\sin (S+\pi)}{-\cos (S+\pi)} = \tan (s+\pi)$$

Markaa, S waa kalka taanjanka.
Waliba taanjanku waa fansaar kisi ah waayo.

$$\text{Tan } (-s) = \frac{\sin (-s)}{\cos (-s)} = \frac{-\sin S}{\cos S} = -\tan S$$

Qeex :

Haddii $S \in R$, $S \neq k\pi$, k-na yahay abyoone markaa kotaanjant $= \{(s\ u) \mid u = \cot S\}$. Mar haddii

$\cot S = \frac{\cos S}{\sin S}$, horaadka fansaarka kotanjanku waa

ururka dhammaan tirooyinka maangalka ah ee aan sin S le'ekayn eber, t.a., S ayna ahayn sansaanka $k\pi$, marka k yahay abyoone dambeedka fansaarka kotaanjinku waa ururka dhammaan tirooyinka maangalka ah.

H (kotaanjant) $= \{S \mid S \in R,\ S \neq k\pi$, k waa abyoone$\}$

D (kotaanjant) $= \{U \mid U \in R\}$

Kalka kotaanjanku waa π, waayo

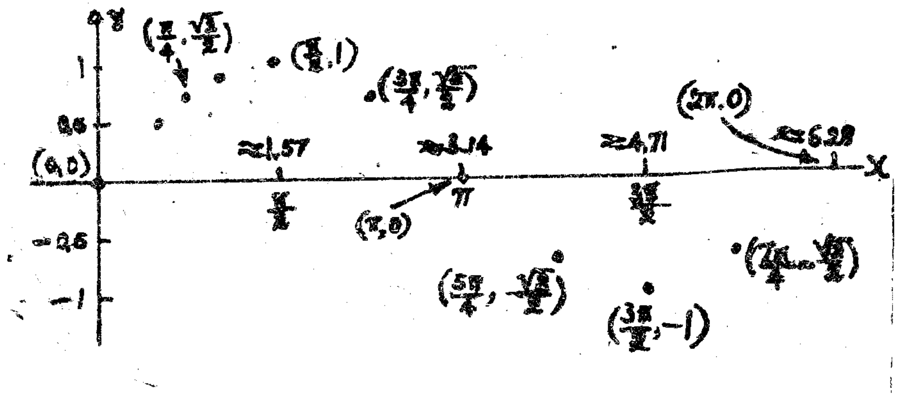
$$\cot S = \frac{\cos S}{\sin S} = \frac{-\cos (\pi + S)}{-\sin (\pi + S)} = \cot (\pi + S).$$

Kootaanjanku waa fansaar kisi ah waayo

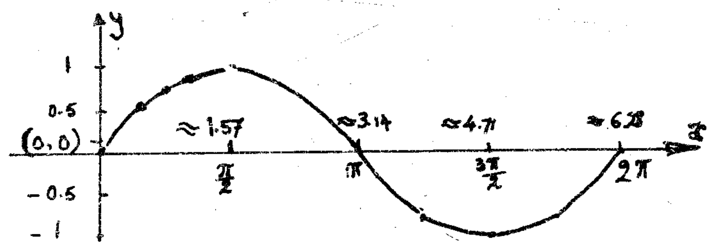
$$\cot (-S) = \frac{\cos (-S)}{\text{Sin } (-S)} = \frac{\cos S}{-\sin S} = -\cot S$$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{2}{3\pi}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

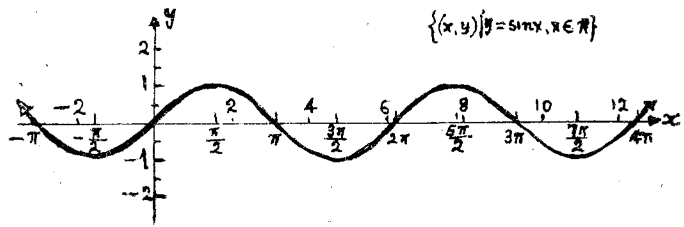
Markaa aad barahaas dhigtid sallaxa Kaartis waxad heli shaxanka 25.



Haddii aad u qaadanno in saynku sansaar is haysta yahay, t.a., in garaafkiisu lahayn daollo, oo aan isku xirno baraha waxaynu heli garaafka shaxanka 26.



Mar haddii sin $(x + 2\pi) = \sin x$, garaafka saynka waa ka shaxanka 26, oo lagu celiyay geralis kasta oo dhererkiisu yahay 2π. Markaa guud ahaan, garaafk asaynka waa ka ku muujisan shaxanka 27.



GARAAFFADA FANSAARRADA KALE EE GOOBO

Garaafyada fansaarrada $y = \tan x$, $y = \cot x$, $y = \sec x$ iyo $y = \csc x$ uma eka kuwa saynka ama kosaynka laakiin iyaga qudhoodu waxa ka muuqata kal-[illegible]lidda.

Garaafyada $y = \tan x$ iyo $y = \cot x$ muuqoode [illegible] eg. Hadda, bal aan eegno garaafka $y = \tan x$. U [illegible] [illegible]n kalka tanjanku yahay π.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Tan x	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$\frac{\sqrt{3}}{3}$	0

U fiirso, $\tan x$ ma qeexna marka $x = \frac{\pi}{2}$ waayo

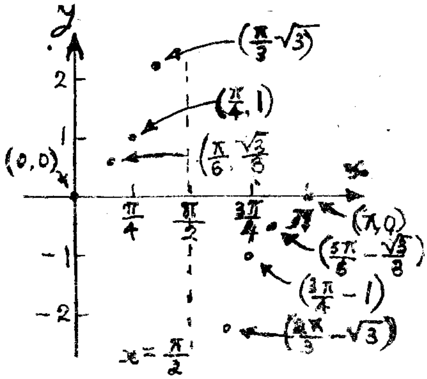
$\tan \frac{\pi}{2} = \frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}}$. Laakiin $\sin \frac{\pi}{2} = 1$. $\cos \frac{\pi}{2} = 0$. Mar-

kaa $\tan \frac{\pi}{2} = \frac{1}{0}$. Haddaba ma jirto bar garaafka ku taal

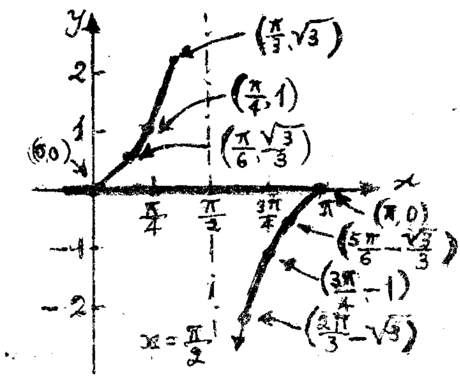
oo u taagan $\tan \frac{\pi}{2}$. Marka x u dhowaato $\frac{\pi}{2}$, $|\tan x|$ xad

la'aan bay u korodha. Markaa $x = \frac{\pi}{2}$ waa taabta $\tan x$.

Immika, haddii aan baraha tusaha kor ku magacaaban dhigno, waxan heleynaa baraha shaxanka 31.



Hadda, haddii aan u qaadanno in tan x iska haysto meel allaale meeshii u ka qeexan yahayba, waxan isugu xiri karnaa baraha sida shaxanka 32 ku muujisan.

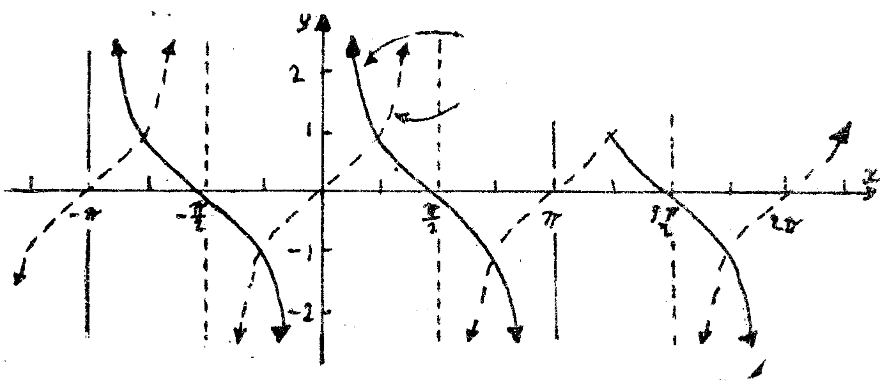


U fiirso tan x wuu kordhaa marka x ay ka korodho

min ‹ ilaa $\frac{\pi}{2}$ iyo min $\frac{\pi}{2}$ ilaa π.

Waxan ‹ ay in tan (x + π) = tan x. Markaa gaalis kasta o ererkiisu yahay π garaafkiisu wuxu

Waxan ognahay in cot $(x + \pi) = \cot x$; markaa gaalis kasta oo dhererkiisu yahay π, garaafkiisa wuxu noqonayaa ka Sh. 35aad oo kale. Guud ahaan garaafka $y = \cot x$ markaa ay x tahay tiro kasta oo maangal ah waa ka ku muujisan shaxanka 36.

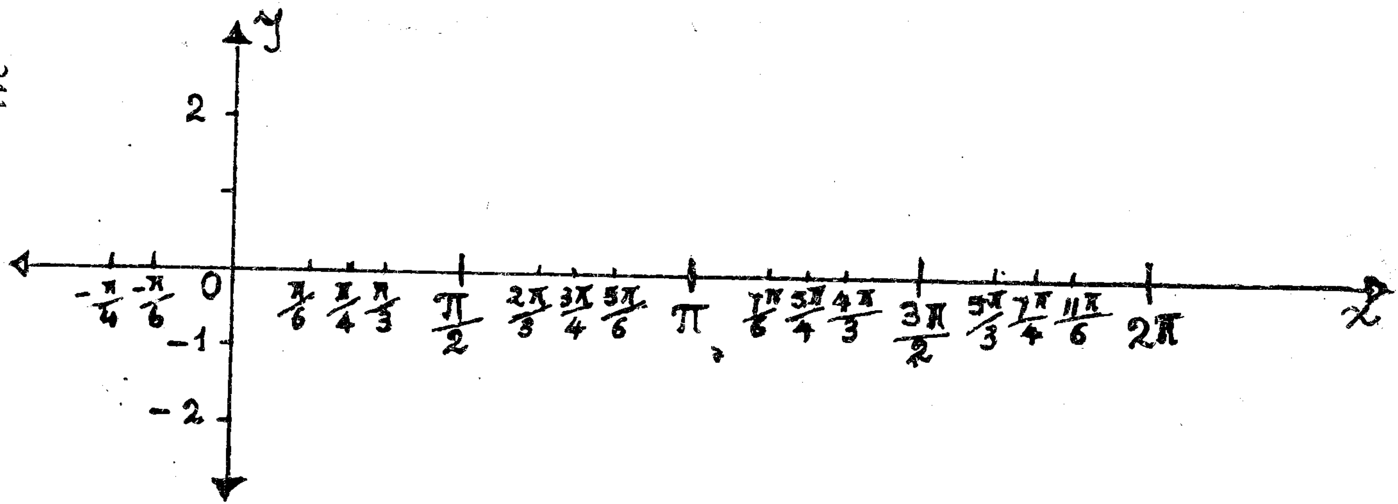


Madeyaasha garaafka waa garaafyada $x = k\pi + \frac{\pi}{2}$ oo ay k tahay abyoone. Baraha u garaafku ka gooyo dhidibka $-x$ waa $(k\pi, \cot k\pi)$, oo ay k tahay abyoone. Dandeedka cotaanjanku waa dhamman tirooyinka maangalka ah.

Garaafka $y = \csc x$ waa la heli karaa haddii tusaha hoose lala kaashado.

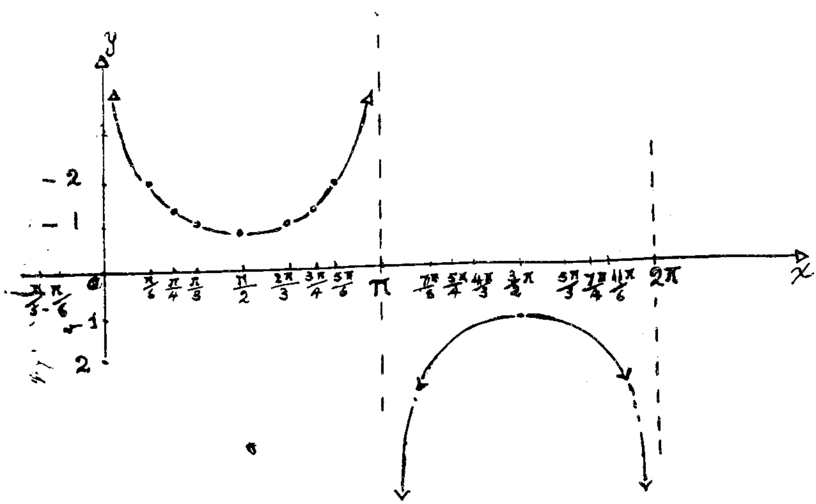
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{2}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
csc x		2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2		-2	$-\sqrt{2}$	$\frac{-2\sqrt{3}}{3}$	1	$\frac{-2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	

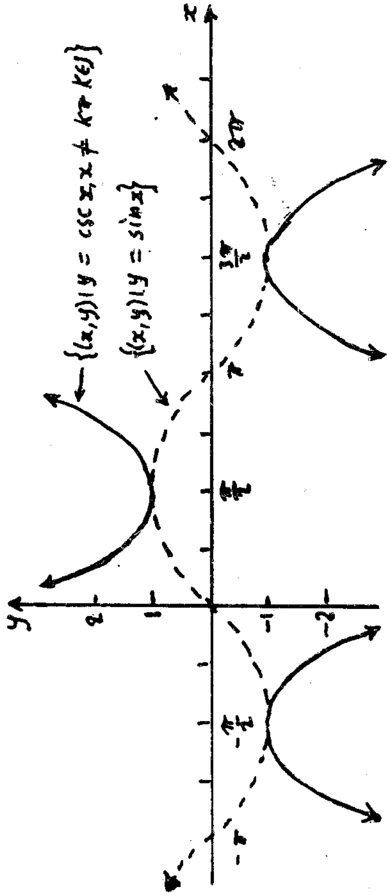
Marka barahaa la dhigo waxa heli baraha shaxanka 37.



Waxan ognahay csc x uuna qeexnayn marka x tahay 0, π, ama 2π waayo $\sin 0 = 0$ $\sin \pi = 0$, $\sin 2\pi = 0$ isla markaa $\csc x = \dfrac{1}{\sin x}$. Waliba, marka x u dhawaato eber, π, ama 2π, $|\csc x|$ aad iyo aad bay u weynaataa. Markaa garaafyada $x = 0$, $x = \pi$ iyo $x = 2\pi$ waa madeyaasha garaafka kosiinkanka.

Hadda, haddii baraha shaxanka 37 aan isku xirno waxaannu heli xoodka shaxanka 38aad.




{(x,y) | y = csc x, x ≠ kπ, k∈I}
{(x,y) | y = sin x}
x
y
2π
3π/2
π
π/2
-π/2
-π
2
1
-1
-2

Mar haddii csc $(x + 2\pi) = \csc x$, markaa garaafka gaalis kasta oo dhererkiisu yahay 2π wuxu noqonayaa ka shanxanka 38aad oo kale. Shaxanka 39aad waa garaafka $y = \sin x$ oo xarriiq googo'an ah iyo garaafka $y = \csc x$ marka ay x tahay tiro kasta oo maangal ah.

U fiirso in danbeedka $y = \csc x$, u yahay ururka dhammaan tirooyinka maangalka ah ee qiimahooda sugan le'eg yahay ama ka weyn yahay 1, t.a.,

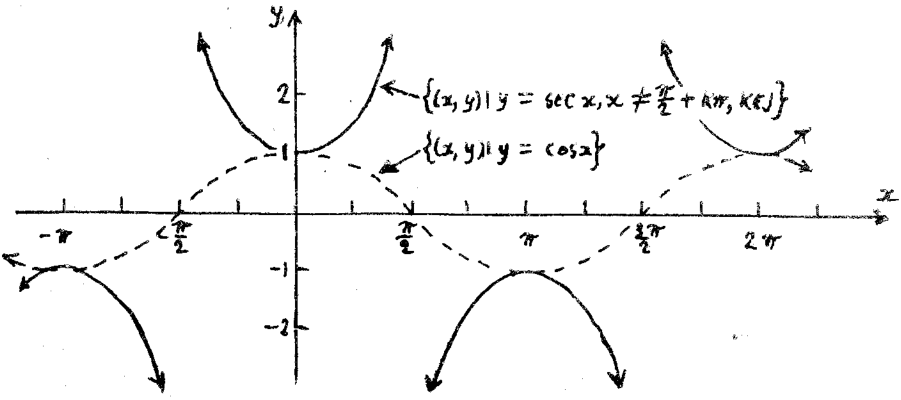
D (csc) $= \{y \mid y$ tahay tiro maangal ah, isla markaa $|y| \geq 1\}$. Madeyaasha kosiikanku waa $x = k\pi$ marka k tahay abyoone.

Sidoo kale, garaafka siikanka waan heli karnaa haddii tusahan la dhammaystiro, dabadeedna baraha la dhigo.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sec x																

Ogow :

Sec $(x + 2\pi) = \sec x$. Markaa haddii aad hesho garaafka gaalis dhererkiisu yahay 2π, waad heli kartaa garaaf guud ee $y = \sec x$ marka ay x tahay tiro kasta oo maangal ah. Haddaba, marka aad dhammaystirto tusaha kore, ee aad baraha dhigto, isku xir baraha. Danbadeedna adoo la kaashana kalgalidda siikanka, dhammaystir garaafka y = sec x marka ay x ∈ R. Ma heshay garaafka shaxanka 40aad oo kale.



Madeyaasha y = sec x waa xarriiqaha $x = \frac{\pi}{2} + k\pi$

ee k tahay abyoone. Danbeedka siikanku waa ururka dhammaan tirooyinka maangalka ah ee qiimahooda sugani le'eg yahay ama ka weyn yahay 1, t.a.,

$$D\ (\text{siikan}) = \{y \mid y \in R,\ \ |y| \geq 1\}$$

Layli :

Samee garaafka

1) – tan x
2) – cot x
3) – sec x
4) – csc x
5) Tan (–x)
6) Cot (–x)
7) Sec (–x)
8) Csc (–x)

Isla dhidbo ku samee garaafyada

b) $y = \sin x$ iyo $y = \csc x$

t) $y = \cos x$ iyo $y = \sec x$

j) $y = \tan x$ iyo $y = \cot x$

WEYDAARRADA FANSAARRADA GOOBO IYO GARAAFYADOODA

Fansaar kasta oo goobo waxay leedahay xiriir weydaar, laakiin weydaarradaa midna fansaar maaha. Cutubkii xiriir iyo fansaar waxan ku dhiganay in weydaarka xiriir lagu helo haddii xubnaha lammaaneyaasha la isku bedello, t.a., haddii xubinta hore ee lammaane kasta oo horsan laga dhigo xubinta dambe, ta dambena laga dhigo xubinta hore.

Waxa kale oon ognahay in weydaarka xiriir yahay fansaar haddii iyo haddii oo qura oo fansaarku isu beegnaan mid-mid ah yahay.

Hadda, bal tixgeli fansaar saynka

$$\{(x, y) \mid y = \sin x\}$$

iyo weydaarka fansaarka oo ah

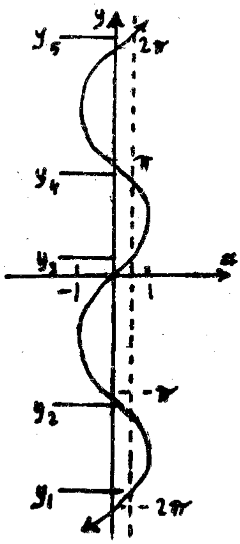
$$\{(x, y) \mid x = \sin y\}$$

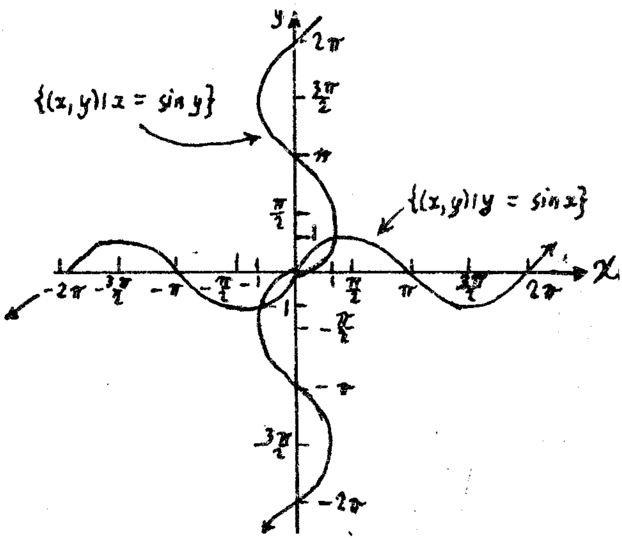
Garaafka labadaaba waxay ku muujisan yihiin shaxanka 41. U fiirso, isleegta $x = \sin y$ fansaar ma qeexdo waayo, kutirsane kasta oo horaadka waxa ku lammaan tirobeel kutirsane oo dambeedka (eeg sh. 42).

Xiriirka weydaarka fansaarka sayn waxa la yiraa: **xiriirka aarkosayn.**

$$\text{Aarkosayn} = \{(x, y) \mid x = \sin y\}$$

Horaadka aarkosayn waa $\{x \mid x \in R, \ -1 \leq x \leq 1\}$ dambeedkiisuna waa ururka dhammaan tirooyinka maangal ah.

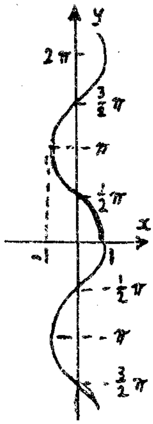




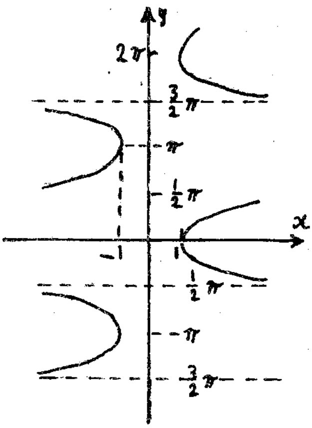
Sidoo kale, weydaarka fansaar kasta oo goobo waa xiriir. Markaa:

aarkosayn	=	$\{(x, y) \mid x = \cos y\}$
aartaanjant	=	$\{(x, y) \mid x = \tan y\}$
aarkotaanjant	=	$\{(x, y) \mid x = \cot y\}$
aarkosiikant	=	$\{(x, y) \mid x = \csc y\}$
aarsiikant	=	$\{(x, y) \mid x = \sec y\}$

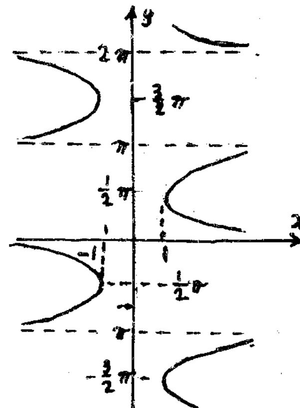
Garaafyada xiriiryadaas oo dhani waxay ku muujisan yihiin shaxanka 43.



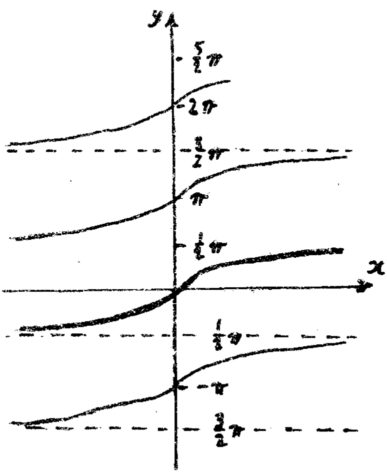
$\{(x, y) \mid x = \cos y\}$



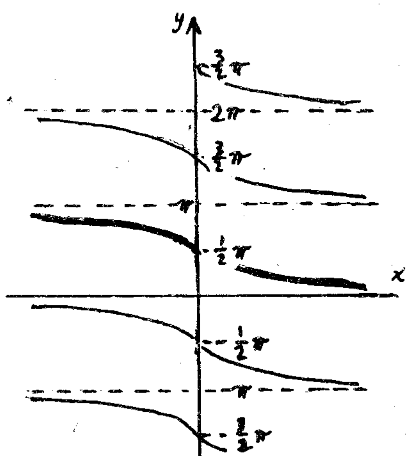
$\{(x, y) \mid x = \sec y\}$



$\{(x, y) \mid x = \csc y\}$



$\{(x, y) \mid x = \tan y\}$



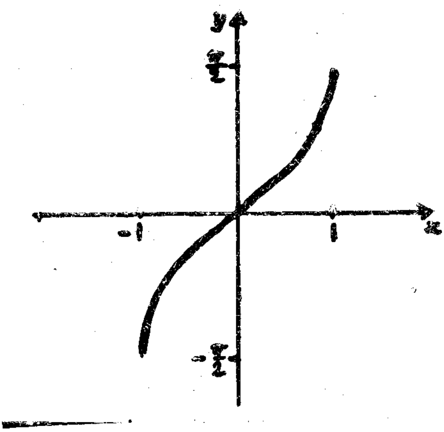
$\{(x, y) \mid x = \cot y\}$

Qiimayaalka Doorka ah ee Weydaarrada

Haddii aan horaaddada fansaarrada goobo aan si habboon u xannibno t.a., haddii si habboon aan u xannibno dambeeddada weydaarradooda, waxan u heli karraa fansaar kasta oo goobo Fansaar-weydaar. Fansaarradaa waxa la yiraa: **fansaar-weydaarka qiime doorka leh.** Si aan xiriir weydaarka looga sooco, waxa lagu magacaaba xarafyo waaweyn. Hadda, fansaar-weydaarka fansaarka sayn waxa la oran, Aarkosayn.

$$\text{Aarkosayn} = \left\{(x, y) \mid y = \sin y, \ -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right\}.$$

Shaxanka 44 wuxu muujinayaa garaafka Aarkosayn.



Mar haddii kutirsane madi ah y, oo dambeedka $\left\{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right\}$ uu ku beegan yahay kutirsane kasta oo horaadka $\{x \mid -1 \leq x \leq 1\}$, Aarsayn waa fansaar. Markaa waxan ku adeegsan karnaa qormadii fansaarka:

$$y = \text{Aarkosayn } x$$

oo la micno ah $x = \sin y$, isla markaa $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Waxa kale oo aan qori karraa.

$$y = \sin^{-1} y$$

FANSAARRO WEYDAARRADA IYO FANSAARRADA GOOBO

Qeex :

Aarsayn $= \left\{ (x, y) \mid y = \sin^{-1} x, \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \right\}$

Aarkosayn $= \{ (x, y) \mid y = \cos^{-1} x, \quad 0 \leq y \leq \pi \}$

Aartaanjant $= \left\{ (x, y) \mid y = \tan^{-1} x, \quad -\frac{\pi}{2} < y < \frac{\pi}{2} \right\}$

Aarkotaanjant $= \{ (x, y) \mid y = \cot^{-1} x, \quad 0 < y < \pi \}$

Aarkosiikant $= \left\{ (x, y) \mid y = \csc^{-1} x, \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, \quad y \neq 0 \right\}$

Aarsiikant $= \left\{ (x, y) \mid \sec^{-1} x, \quad 0 \leq y \leq \pi, \quad y \neq \frac{\pi}{2} \right\}$

In kasta oo danbeeddada la qaatay ay yihiin kuwa badanaaba la qaato, haddana micno aad u weyn kuma fadhiyaan. Matalan, Aarkosaynka danbeed doorkiisu waa $0 \leq y \leq \pi$.

Haddii aan qaadanno danbeedka $-\pi \leq y \leq 0$, kosayn-weydaarku waa fansaar. Sidoo kale, haddii aan qaadanno danbeedka $-2\pi \leq y \leq -\pi$, kosayn-weydaarku waa fansaar.

Tusaale :

Raadi Aarkosayn $\frac{1}{2}$

Furfuris :

Aarkosayn $\frac{1}{2}$ wuxu u la mid yahay, qaansada kosaynkeedu $\frac{1}{2}$ yahay. Waxan ognahay in qaansada kosaynkeedu 1 yahay ay tahay $\frac{\pi}{3} \pm 2n\pi$ ama $\frac{5\pi}{6} + 2n\pi$ laakiin, danbeedka Aarkosayn waa $\left\{ y \mid 0 \leq y \leq \pi \right.$ markaa, $\left. \cos^{-1} \frac{1}{2} = \frac{\pi}{3} \right\}$.

Layli :

Raadi qiimaha mid kasta oo hoos ku taal.

1) $\text{Sin}^{-1} \frac{\sqrt{3}}{2}$
2) $\text{Cos}^{-1}\ 0{,}5$
3) $\text{Tan}^{-1} \frac{\sqrt{3}}{3}$
4) $\text{Cot}^{-1}\ 1$
5) $\text{Sin}^{-1}\ -1$
6) $\text{Cot}^{-1}\ 0$
7) $\text{Sec}^{-1}\ 2$
8) $\text{Tan}^{-1}\ \sqrt{3}$
11) $\text{Tan}^{-1}\ 0.1003$
12) $\text{Sin}^{-1}\ 0.3802$
13) Aarkosayn -0.8624
14) Aarkotan 3.467
15) $\text{Cos}^{-1}\ 0.6675$
16) $\text{Csc}^{-1}\ 1.422$
17) $\text{Cos}^{-1}\ 0$
18) $\text{Tan}^{-1}\ 1$

9) $\operatorname{Tan}^{-1}\left\{-\frac{1}{\sqrt{3}}\right\}$

10) $\operatorname{Cos}^{-1} 2$

19) $\operatorname{Csc}^{-1} \frac{2\sqrt{3}}{3}$

20) $\operatorname{Sec}^{-1} \sqrt{2}$

Tusaale :

Raadi $\cos^{-1}(\tan \pi)$.

Mar haddii $\tan \pi = 0$, markaa

$$\cos^{-1}(\tan \pi) = \cos^{-1}(0) = \frac{\pi}{2}.$$

Raadi qiimaha mid kasta oo hoos ku taal.

1) $\operatorname{Sin}^{-1}\left\{\cos \frac{\pi}{4}\right\}$

2) $\operatorname{Tan}^{-1}\left\{\tan \frac{\pi}{3}\right\}$

3) $\operatorname{Cos}^{-1}\left\{\sin \frac{\pi}{2}\right\}$

4) $\operatorname{Sin}^{-1}\left\{\sin 3 \frac{\pi}{2}\right\}$

5) $\operatorname{Sin}\left\{\cos^{-1} \frac{1}{2}\right\}$

6) $\operatorname{Tan}\left\{\tan^{-1}\left(\frac{3}{2}\right)\right\}$

7) $\operatorname{Cos}\{\cot^{-1}(-\sqrt{3})\}$

8) $\operatorname{Sin}\{\tan^{-1}(-1)\}$

9) $\sin\left[2\sin^{-1}\frac{1}{2}\right]$

10) $\sin\left[2\cos^{-1}\frac{3}{5}\right]$

11) $\tan\frac{1}{2}\left[\sin\frac{12}{13}\right]$

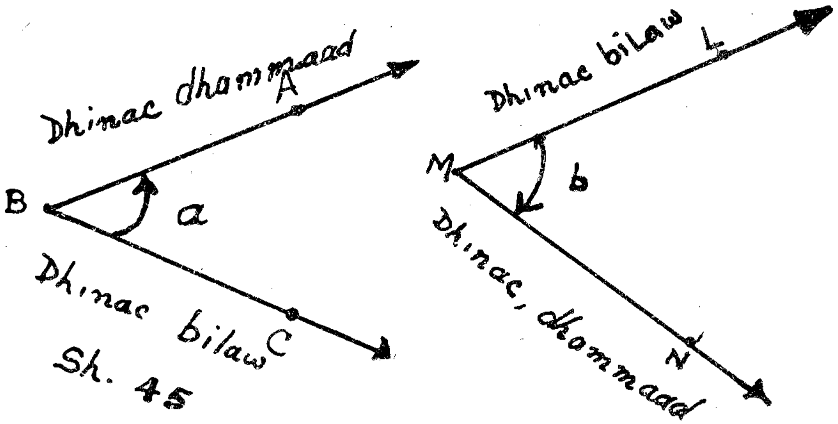
12) $\cos\frac{1}{2}(\tan^{-1} 0)$

13) $\cos\{\sin[\tan^{-1}(-1)]\}$

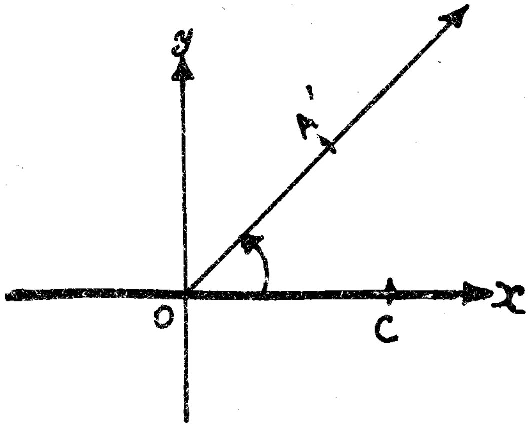
14) $\sin[\cos^{-1}(\tan 0)]$

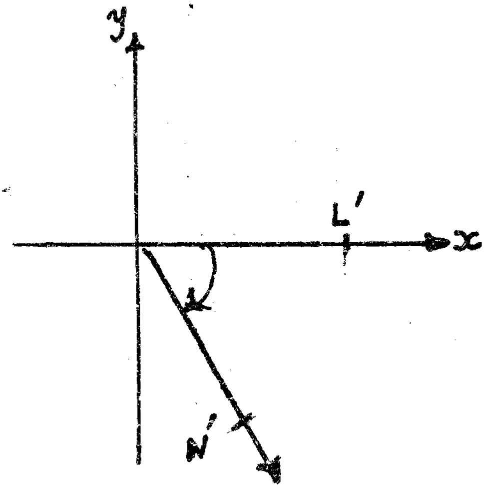
XAGLAHA IYO CABBIRRADOODA

Xagali waa isutagga laba fallaarood oo isla bar dhammaad ah (Sh. 45) iyo waniinka mid u dira ka kale. Bar dhammaadka ay wadaagaan waxa la yiraa **Geeska xagasha**, fallaarahana dhinacyaha xagasha.



Xagal kasta oo sallax waxay ku sargo'an tahay (≡) xagal kale oo dhinac bilaw ku leh dhidibka – x togan, isla markaa geeskeedu, ku yaal unugga (Sh. 46). Xaglaha noocaas oo kale ah waxa la yiraa **Xagal Rug Door.**

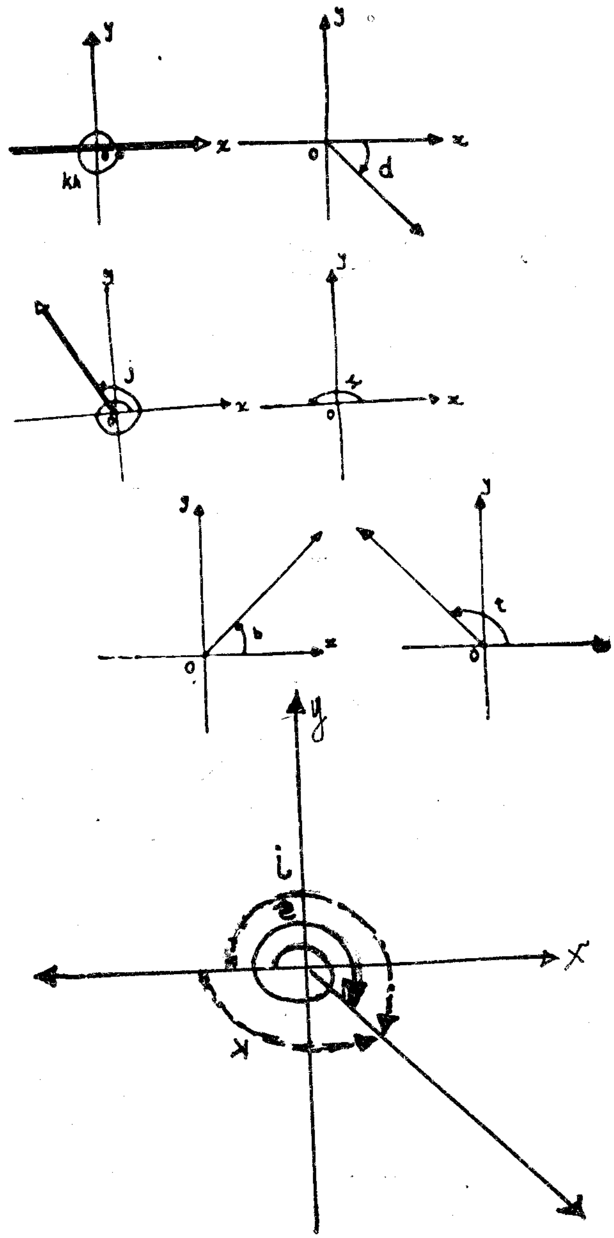




Xagal Rug Door

Qeex :

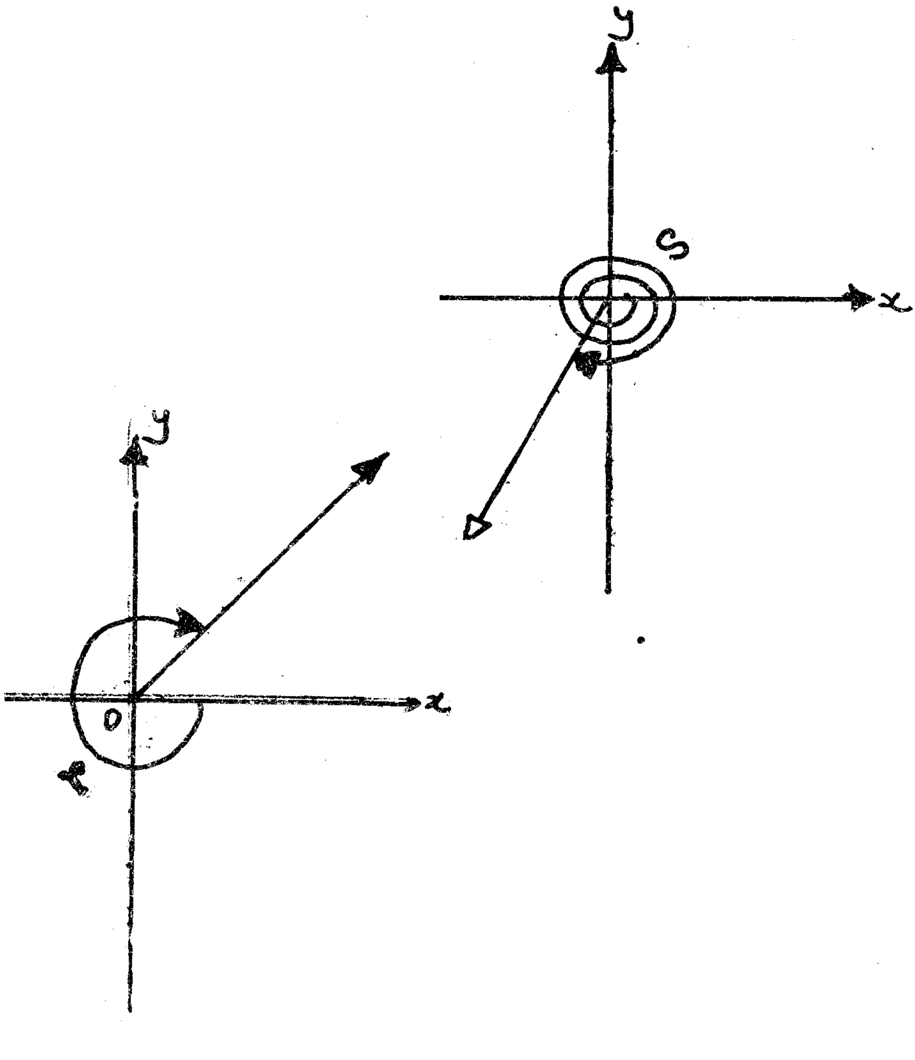
Xagasha dhinac bilawgeedu yahay dhidibka $-x$ togan waxa la yiraa xagal rug door.



Sh. 48

Xaglaha shaxanka 47 waa xaglo rug door, b, t, j, x iyo kh waa kuwa togan. Xaglaha d, r iyo s waa xaglo taban.

Xaglaha isla dhinac bilaw iyo dhinac dhammaad ah waxa la yiraa **xaglo isku dhammaad ah.**



Shaxanka 48, xaglaha k, s iyo j waa xaglo isku dhammaad ah.

Marka xaglo la cabbirayo, labada halbeeg ee badanaaba lagu shaqaystaa waa digrii iyo gacansiin.

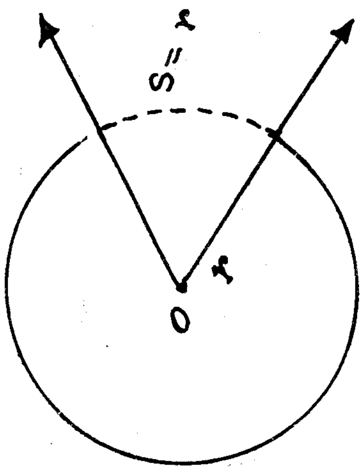
Qeex : DIGRII

Haddii meeriska goobo kasta loo qaybiyo 360 qaanso oo isle'eg, markaa qaanso kasta oo dhererkeedu yahay $\frac{1}{360}$ meeriska, xagasha ay xuddunta ku sameyso waxa la yiraa 1 **digrii** waxana loo qoraa 1°.

Qeexda waxa cad in xagasha u meerisku ku sameyo xuddunta ay le'eg tahay $360 \times 1^\circ = 360^\circ$.

Qeex : GACANSIN

Xagasha qaanso dhererkeedu le'eg yahay gacanka goobo ay ka sameyso xuddunta goobada waxa la yiraa **Gacansni** waxana loo qoraa 1^R.



Hadda, xagal kasta cabbirkeedu waa inta halbeeg (digrii ama gacansin) ee xagasha ku jirta.

Tusaale 1:

Waa imisa digrii xagasha ay sameyso qaanso S, oo dhererkeedu yahay $\frac{1}{8}$ meeriska?

Furfuris :

Ka soo qaad in x tahay cabbirka xagasha ay S ku sameyso xuddunta. Haddaba, waxan ku barnay joomatariga in dhererka qaansooyinka goobo iyo cabbirka xaglaha ay ku sameeyaan xuddunta ay saamigal yihiin, markaa:

$$\text{Meeris} : S = 360^\circ : x$$

$$\frac{S}{\text{Meeris}} = \frac{x}{360}$$

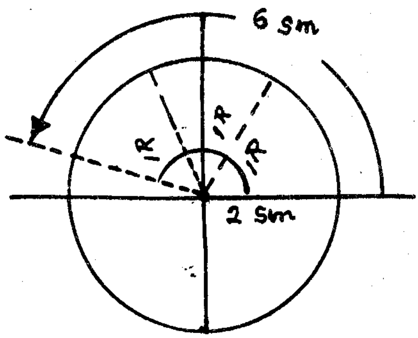
$$\text{Laakiin } S = \frac{1}{8} \text{ meeriska}$$

$$\frac{\frac{1}{8}}{1} \text{ meeris} = \frac{x}{360^\circ}$$

$$\therefore \quad x = \frac{1}{8} \times 360^\circ = 45^\circ$$

Tusaale 2:

Waa imisa gacansin xagasha ay ku sameyso xuddunta qaanso goobo dhererkeedu yahay 6 sm. haddii gacanka goobadu yahay 2 sm.



Furfuris :

Ka soo qaad in cabbirka xagashu yahay y.

$$\text{Markaa } y = \frac{6 \text{ sm.}}{2 \text{ sm.}} = 3^R$$

$$\text{ama} \quad 6 \text{ sm.} : 2 \text{ sm.} = y : 1^R$$

$$\frac{6 \text{ sm.}}{2 \text{ sm.}} = \frac{y}{1^R}$$

$$y = \frac{6}{2} \times 1^R$$

$$= 3 \times 1^R = 3^R$$

Tusaale :

Waa imisa gacansin xagasha u meeriska goobo ku sameeyo xuddunta, haddii gacanka goobadu yahay r halbeeg.

Furfuris :

Ka soo qaad in x tahay cabbirka xagashaasi. Dhererka meeriska goobada gacankeedu yahay r waa $2\pi r$.

Markaa, haddii S tahay qaanso le'eg gacanka, t.a., S = r, waxannu heli.

$$S : 2\pi r = 1^R : x$$

$$\therefore \quad \frac{2\pi r}{S} = \frac{x}{1^R} \Longrightarrow x = \frac{2\pi r}{S} \times 1^R$$

Laakiin S = r

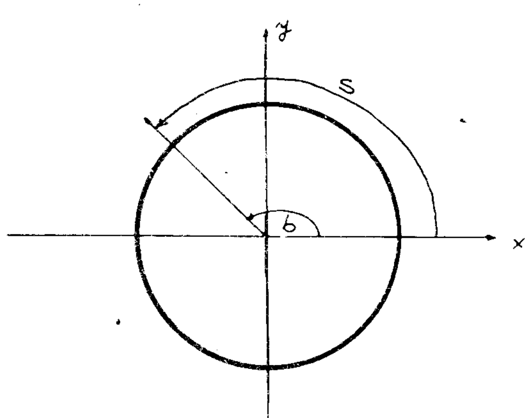
$$\therefore \quad x = \frac{2\pi r}{r} \times 1 = 2\pi$$

Tusaale 4:

Waa imisa gacansin xagasha ay qaansada dhererkeedu yahay S ka sameyso xuddunta haddii gacanka goobadu yahay r.

Furfuris :

Xagasha cabbirkeeda la rabaa waa xagasha b ee shaxanka 51.



Sh. 51

ka soo qaad in cabbirka b yahay Θ gacansin. Markaa, su'aasha aan isweydiinaynaa waa: Imisa gacan, r ayaa ku jira qaansada S? Hubaal S waxa ku jira $\frac{S}{r}$ gacan. Markaa $\Theta = \frac{S^R}{r}$.

OGOW: Haddii dhererka qaanso goobo iyo gacanka goobada lagu siiyo, oo lagu waydiiyo cabbirka xagasha ay ka sameyso xuddunta, waxad oran

$$\text{xagashu} = \frac{(\text{Qaansadu})^R}{\text{Gacanka}}.$$

Hadda, haddii gacanka r iyo xagasha Θ lagu siiyo, ma soo saari karta dhererka qaansada S? Ma oran karnaa $S = r\Theta$? Bal ka waran haddii xagasha iyo qaansada sameysay lagu siiyo. Ma soo saari kartaa dhererka gacanka r? Ma oran kartaa $r = \frac{S}{\Theta}$?

Tusaale 5:

b) Qaanso goobo ayaa dhererkeedu yahay 12 sm., xagasha ay xuddunta ku sameysaana waa 3^R. Waa imisa gacanka goobadu?

t) Qaanso goobo gacankeedu yahay 4 m. ayaa xuddunta ku sameysa xagal 3^R. Waa imisa dhererka qaansadu?

b) Qaansada S = 12 sm.
Xagasha $\Theta = 3^R$.
Gacanka r = ?

Waxan naqaan in $\Theta = \frac{S}{r}$

$$\therefore \quad r = \frac{S}{\Theta}$$

$$\therefore \quad r = \frac{12}{3} \text{ sm.} = 4 \text{ sm.}$$

Gacanku waa 4 sm.

t) Gacanka $r = 4$ m.
Xagasha $\Theta = 3$.
Qaansada $S = ?$

$$\therefore \quad S = r\Theta$$
$$= 4 \text{ m.} \times 3 = 12 \text{ m.}$$

XIRIIRKA KA DHEXEEYA DIGRII IYO GACANSIN

Waxan qeexdii digrii ka baranay in xagasha u meerisku ka sameeyo xuddunta goobo ay tahay 360°. Tusaale 3aad waxan ka hellay in xagasha u meerisku ku sameeyo xuddunta goobo ay tahay $2\pi^R$. Markaa waxa cad in $2\pi^R = 360°$.

$$\therefore \quad \pi = 180°, \quad 1^R = \frac{360°}{2\pi} = \frac{180°}{\pi}.$$

$$1° = \frac{2\pi}{180°} = \frac{\pi^R}{180°}$$

T u s a a l e :

30° u beddel gacansin.

F u r f u r i s :

$$30 = 30 \times 1°$$
$$= 30 \times \frac{\pi^R}{180°} = \frac{\pi^R}{6}$$

Tusaale 2:

$\frac{\pi^R}{12}$ u beddel digrii

Furfuris :

$$\frac{\pi^R}{12} = \frac{\pi}{12} \times 1^R = \frac{\pi}{12} \times \frac{180^\circ}{\pi} = \frac{180^\circ}{12} = 15^\circ$$

Layli :

1. U beddel gacansin.

   b) 270° t) 35° j) 45° x) 135°
   kh) 1050° d) 22.5° r) 75° s) 112.5°
   sh) 105° dh) 265°

2. U beddel digrii

b) $\frac{3\pi^R}{2}$ t) $\frac{1\pi^R}{2}$, r) $\frac{19\pi^R}{6}$ s) $4\pi^R$

j) $\frac{7\pi^R}{4}$ x) $\frac{5\pi^R}{4}$ sh) $\frac{2\pi^R}{5}$ dh) $6\pi^R$

kh) $\frac{11\pi^R}{6}$. d) $3\pi^R$

3. Waxa lagu siiyey dhererka qaansada S, iyo gacanka goobada oo ah r. Markaa raadi cabbirka xagasha θ.

b) S = 14 sm.
r = 8 sm

t) S = 4 m.
r = 8 dm.

j) $S = 28$ m.
$r = 3.5$ m.

kh) $S = \pi$ m.
$r = 0.5$ m.

x) $S = 2\pi$ sm.
$r = 1$ sm.

d) $S = \frac{44}{7}$ sm.
$r = 3.5$ sm.

4. Raadi dhererka qaansada S, haddii lagu siiyay cabbirka xagasha ay ku sameyso xuddunta oo ah Θ iyo gacanka goobada, r.

b) $r = 7$ mm.
$\Theta = 2^R$

t) $r = 12$ km.
$\Theta = 6$

j) $r = 14$ sm.
$\Theta = 0.25^R$

x) $r = 14$ mm.
$\Theta = \pi^R$

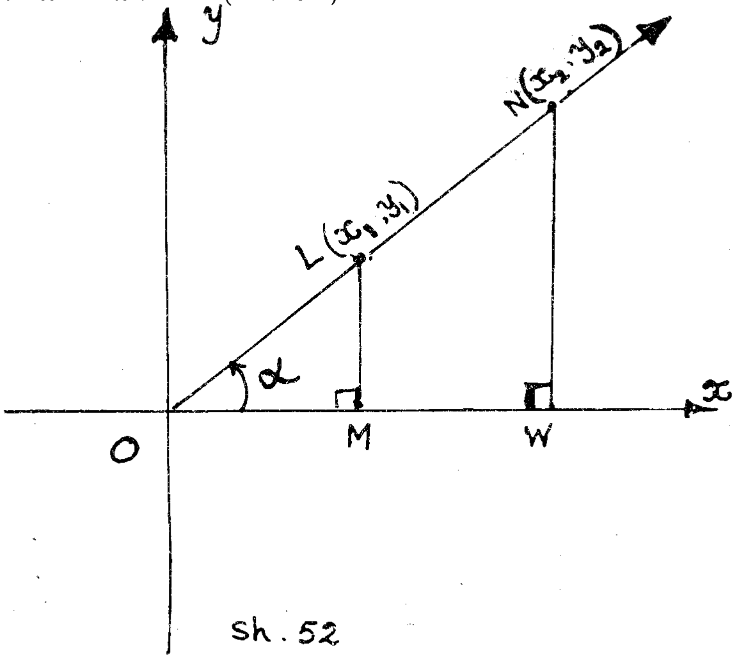
kh) $r = 14$ sm.
$\Theta = 3^R$

d) $r = 8$ sm.
$\Theta = 4^R$

FANSAARRO TIRIGNOOMETERI

Taariikh ahaan, barashada fansaarrada goobo waxay bilaabantay markii xaglaha iyo saddexagallada la bartay. Markii aan qeexnay fansaarrada goobo waxaan qaadanay dhererka qaanso goobo halbeeg dhererka qaansada oo laga bilaabay barta (0, 1), kuna dhammaatay bar ku taal goobo halbeegga), oo ah tiro maangal ah, waxana aan ku lammaana yahay tiro kale oo maangal ah, kulanka hore ama ka dambe ee bartaa. Markaa, horaadka iyo dambeedka fansaarku waxay noqdeen tirooyinka maangal ah.

Hadda, hal tixgeli xagasha α oo ah xagal rug door. Ka soo qaad in $(x_1\ y_1)$ iyo $(x_2\ y_2)$ ay yihiin laba barood oo kala geddisan oo aan ahayn unugga oo ku yaal dhinac dhammaadka (sh. 52).



Sh. 52

Markaa waxan caddayn karraa in

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} \quad (y_1,\ y_2 \neq 0), \qquad \frac{x_1}{\sqrt{x_1^2 + y_1^2}} = \frac{x_2}{\sqrt{x_2^2 + y_2^2}}$$

isla markaa

$$\frac{y_1}{\sqrt{x_1^2 + y_1^2}} = \frac{y_2}{\sqrt{x_2^2 + y_2^2}}$$

Caddayn

Ka soo qaad in L tahay barta (x_1, y_1), N tahay barta (x_2, y_2). Ka soo qaad in M tahay isgoyska dhidibka $-x$ iyo qotomaha dhidibka $-x$ ee mare L, W-na tahay isgoyska dhidibka $-x$ iyo qotomaha dhidibka $-x$ ee mare N. LM iyo NW waa barbarro waayo waxay ku wada qotomaan isla xarriiq.

Tixgeli : $\triangle$ QLM iyo $\triangle$ ONW

$\triangle$ OLM wuxu u eg yahay $\triangle$ ONW. (Waayo)

$$\frac{OM}{OW} = \frac{OL}{ON} = \frac{LM}{NW} \ldots . 1. \quad \text{(Waayo)}$$

Laakiin OM $= x_1$, OW $= x_2$

LM $= y_1$, NW $= y_2$

$$OL = \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2} = \sqrt{x_1^2 + y_1^2}$$

$$ON = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2} = \sqrt{x_2^2 + y_2^2}$$

Waayo?

$$\therefore \frac{OM}{OW} = \frac{LM}{NW} \Longrightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2}$$

$$\text{Laakiin } \frac{x_1}{x_2} = \frac{y_1}{y_2} \Longrightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

Inta hartay waxa looga tegay in uu ardaygu caddeeyo.

Inkasta oo shaxanka u tusayo marka dhinac dhammaadka α u ku yaallo waaxda Iaad, haddana saamiyaddaasi waxay isle'eg yihiin marka (x_1, y_1) iyo (x_2, y_2) ay yihiin baro ku yaal dhinac kasta oo waaxdii la doono ku yaal.

Hadda waxan qeexi karraa fansaarro cusub oo mid kastaba horaadkeedu yahay xaglaha rug door, danbeedkeeduna yahay urur tirooyin maangal ah.

Qeex :

Haddii α tahay xagal rug door, $(x, y) \neq (0, 0)$ ay tahay bar ku taal dhinac dhammaadka α, markaa

$$\text{Kotaanjant} = \left\{ (\alpha, \cot \alpha) \mid \cot \alpha = \frac{x}{y}, \quad y \neq 0 \right\}$$

$$\text{Sayn} = \left\{ (\alpha, \sin\alpha) \mid \sin\alpha = \frac{y}{\sqrt{x^2+y^2}} \right\}$$

$$\text{Kosayn} = \left\{ (\alpha, \cos\alpha) \mid \cos\alpha = \frac{x}{\sqrt{x^2+y^2}} \right\}$$

$$\text{Taanjant} = \left\{ (\alpha, \tan\alpha) \mid \tan\alpha = \frac{y}{x}, \quad x \neq 0 \right\}$$

$$\text{Siikan} = \left\{ (\alpha, \sec\alpha) \mid \sec\alpha = \frac{\sqrt{x^2+y^2}}{x}, \quad x \neq 0 \right\}$$

$$\text{Kosiikan} = \left\{ (\alpha, \csc\alpha) \mid \csc\alpha = \frac{\sqrt{x^2+y^2}}{y}, \quad y \neq 0 \right\}$$

Fansaarradaa waxa la yiraa **fansaarro tirignoometeri.** Ogow, $\sqrt{x^2 + y^2}$ waa xididka togan.

Tusaale 1:

Raadi kutirsanaha danbeedka fansaar kasta oo tirignoomatari (lixda fansaar) ee ku lammaan α haddii α u yahay kutirsane horaadk, isla markaa, ay barta $(-3, 5)$ ku jirto dhinac dhammaadka α.

Furfuris :

Qeex ahaan,

$$\text{Sin } \alpha = \frac{y}{\sqrt{x^2+y^2}}$$

$$= \frac{5}{\sqrt{(-3)^2 + 5^2}} = \frac{5}{\sqrt{9+25}} = \frac{5}{\sqrt{34}}$$

$$\text{Cos } \alpha = \frac{x}{\sqrt{x^2 + y^2}} = \frac{-3}{\sqrt{34}}$$

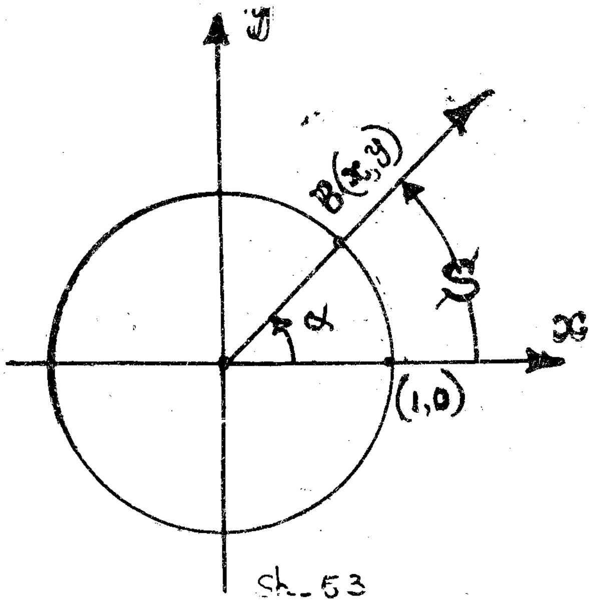
$$\text{Tan } \alpha = \frac{y}{x} = \frac{5}{-3} = \frac{-5}{3}$$

$$\text{Cot } \alpha = \frac{x}{y} = \frac{-3}{5} = \frac{-3}{5}$$

$$\text{Sec} = \frac{\sqrt{x^2 + y^2}}{x} = \frac{\sqrt{34}}{-3} = \frac{-\sqrt{34}}{3}$$

$$\text{Csc} = \frac{\sqrt{x^2 + y^2}}{y} = \frac{\sqrt{34}}{5} = \frac{\sqrt{34}}{5}$$

Waxan ognahay in xagal kasta oo sallax ku taal ku sargo'an tahay xagal rug door. Markaa, qeexdii fansaarrada tirignoometeri waa la fidin karaa oo waxa la oran xagal kasta oo α ku sargo'an waxay ku lammaan tahay isla tiradii α ku lammaanayd. Markaa inkastoo fansaarrada ku qeexnay xaglo rug door, waxa la arki karaa in ay ku run yihiin, urur kasta oo ka koobma xaglo ku yaal sallax. Waliba, waxan ognahay in xaglaha isku sargo'an ay isku cabbir yihiin, markaa xaglaha ku jira horaadka fansaar kasta waxan ku sheegi karraa cabbirkooda. Metalan, waxan qori karraa sin 30° iyo sin $\frac{\pi^R}{6}$. Sin 30° waxay u taagan tahay «saynka xagasha cabbirkeedu yahay 30°». Sidoo kale, sin $\frac{\pi^R}{6}$ waxay u taagan tahay «saynka xagasha cabbirkeedu yahay $\frac{\pi}{6}$ gacansin».



Sh. 53

Xiriirka ka dhexeeya Fansaarrada

Tirignoometeri iyo kuwa Goobo

Fansaarrada tirignoomatari ee xagasha α waxan ku qeexnay kulammada bartii la doono ee ku taal dhinac dhammaadka α oo aan unugga ahayn. Ka soo qaad in bartaasi tahay barta B(x, y) oo ku taal goobo halbeegga shaxanka 53. Marka $\cos \alpha = \frac{x}{1}$ ama $x = \cos \alpha$.

Sidoo kale, $\sin \alpha = \frac{y}{1}$ ama $y = \sin \alpha$. Waliba, waxan ognahay in $\cos S = x$ iyo in $\sin S = y$.

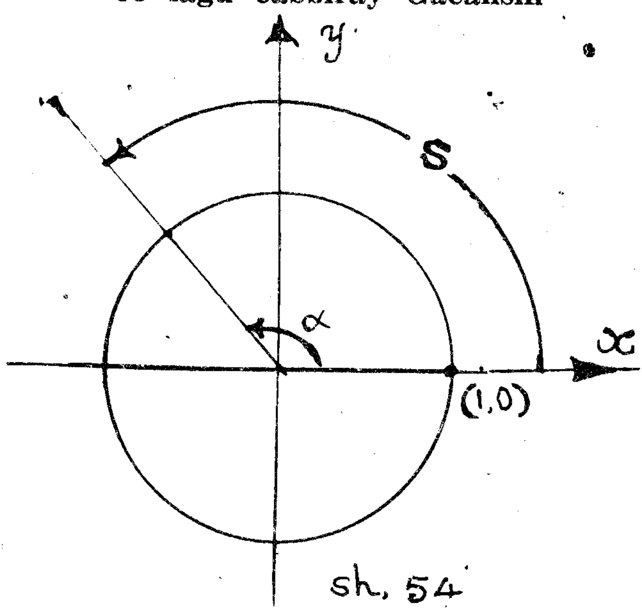
Markaa $\sin \alpha = y = \sin S$; $\cos \alpha = x = \cos S$.

Hadda, halkan waxa ka cad in kutirsaneyaasha danbeedka fansarraada tairignoometeri ay le'eg yihiin kutirsaneyaasha ku beegan ee danbeedka fansaarrada goobo ee la sifada ah. Taas oo ah, haddii S tahay qaanso goobo halbeeg xuddunta ku sameysa xagasha α, markaa $\sin S = \sin \alpha$, $\cos S = \cos \alpha$, $\tan S = \tan \alpha$, iwm.

Waxan niri xagal waxan ku magacaabi karnaa cabbirkeeda. Markaa haddii $\alpha = 30°$, $B = \frac{2\pi^R}{3}$, S_1 iyo S_2 ay yihiin qaansooyinka goobo halbeegga ee ku beegan siday u kala horreeyaan. Markaa, $\sin S_1 = \sin 30°$, $\cos S_1 = \cos 30°$, iwm. Sidoo kale

$$\sin S_2 = \sin \frac{2\pi^R}{3}, \quad \cos S_2 = \frac{2\pi^R}{3}, \text{ iwm.}$$

Xiriirka ka dhexeeya S iyo α oo lagu cabbiray Gacansin



sh. 54

Shaxanka 54, S waa dhererka qaanso goobo halbeeg. α^R waa cabbirka xagasha ay qaansadaasi ku sameyso xuddunta. Waxan naqaan in xagasha (ku cabbiran gacansin) $= \dfrac{\text{Qaansada}}{\text{Gacanka}}$.

$$\alpha = \frac{S}{1} \text{ gacanka goobo halbeeg waa } 1.$$

$$\therefore \quad \alpha = S$$

Guud ahaan, waxan arkeynaa in dhererka qaanso goobo halbeeg iyo xagasha ay ku sameyso xuddunta oo ku cabbiran gacansin ay astiro ahaan isle'eg yihiin. Markaa waxan gaari karraa in sin S = sin α, t.a.

$$\text{Sin } \frac{\pi}{3} = \sin \frac{\pi^R}{3}$$

$$\text{Sin } \frac{2\pi}{3} = \sin \frac{2\pi^R}{3}$$

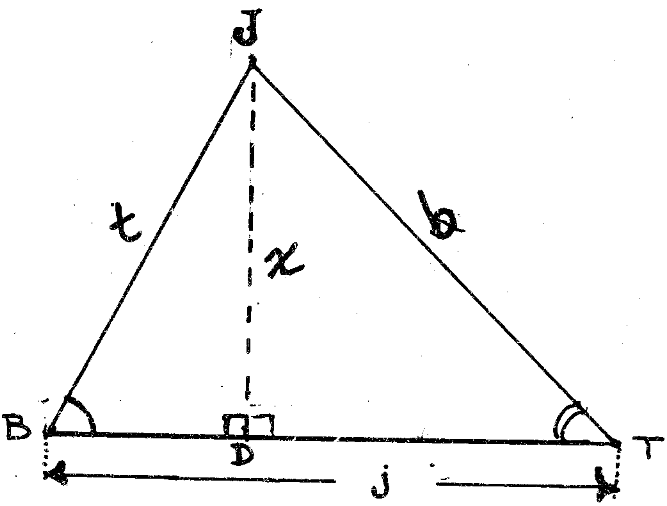
$$\text{Cos } \frac{\pi}{3} = \cos \frac{\pi^R}{3}, \text{ iwm.}$$

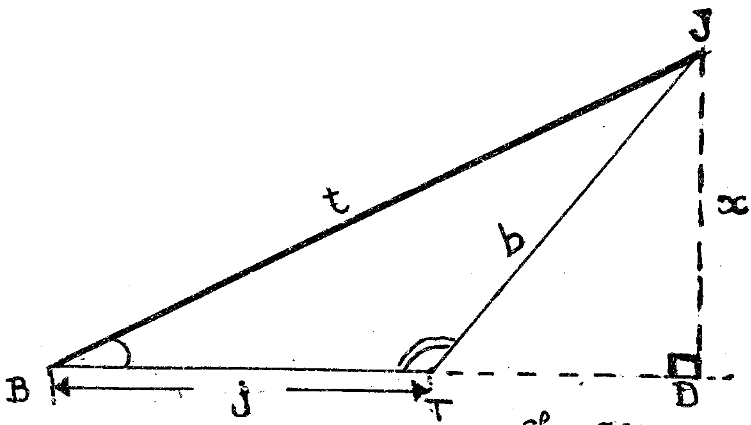
Hadda, waxan leenahay, xeerarkii aan hore u qaadnay ee fansaarrada goobo waa ku run fansaarrada tirignoometeri taa micnaheedu waxa weeye, S waxan u arki karraa cabbirka xagasha ay S xuddunta ku sameyso oo ku cabbiran gacansin.

XEERKA SAYNKA IYO KOSAYNKA

1. Xeerka saynka

Tirignoometeriga iyo ku adeeggeeda waxay badanaaba inna kar siiyaan in aan soo saarro dhinacyada iyo xaglaha saddexagal qaarkood markaa qaar la inna siiyo. Jidad dhowr ah ayaa jira oo taa ku lug leh. Bal ka u horreeya oo ah xeerka saynka aan soo diirro.





sh. 56

Bal tixgeli seddaxagallada shaxanka 55 iyo shaxanka 56 ee dhinacyadooda yihiin b, t, j xaglahooduna yihiin B, T, J,. Samee qotomaha mara geeska J ee dhinaca BT, ama BT oo la fidiyay. D u bixi meesha qotomahu ka gooyo dhinaca BT, dhererkiisana u bixi x.

$$\text{Markaa } \sin B = \frac{x}{t}$$

$$\text{Markaa } x = t \sin B \ldots\ldots (i)$$

Shaxanka 56, sin < B T J = sin < J T D, waayo

B T J = (π − J T D). Markaa $\sin T = \frac{x}{b}$.

$$\therefore \quad x = b \sin T \ldots\ldots (ii).$$

Haddii aan isle'eg kaysiinno tibaaxaha x ee (i) iyo (ii) waxannu heli T sin B = b sin T.

Marka aan labada dhinacba u qaybinno sin B iyo sin T, waxannu heli $\frac{b}{\sin B} = \frac{t}{\sin T}$ (ii)

Haddii qotomaha laga soo jeexi lahaa geeska T, waxan heli lahayn $\frac{b}{\sin B} = \frac{j}{\sin J}$ (iv)

Ugu dambeyn, isleegyada (iii) iyo (iv) waxannu ka gaari in $\frac{b}{\sin B} = \frac{t}{\sin T} = \frac{j}{\sin J}$ (i)

Kani waa xeerka saynka oo u qoran summad ahaan. Weedh ahaan wuxu noqonayaa, seddaxagal kasta, dhinac loo qaybiyay saynka xagasha ka soo horjeeda wuxu le'eg yahay dhinac kasta oo kale oo loo qaybiyay saynka xagasha ka soo horjeeda.

Xeerka saynka waxa lagu adeegsan karaa:

b) Haddii laba xaglood iyo dhinac la ogyahay .
t) Haddii laba dhinac iyo xagal aan u dhexayn la ogyahay.

Xaaladda labaad waxa la yiraa **xaaladda dahson** ee xeerka saynka, waxanan dhigan doonaa mar kale.

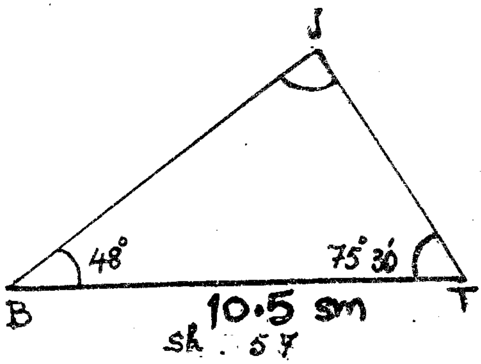
TUSAALOOYIN KU SAABSAN

XAALADDA (b)

1. Raadi qaybaha maqan ee saddexagalka B, T, J, haddii: $\measuredangle B = 48°$, $\measuredangle T = 75° 30'$, J = 10.5 sm.

Furfuris :

Dhismaha shaxanku waa ka hoos ku muujisan



$$\begin{aligned} J &= 180° - (<B + <T) \\ &= 180° - (48° + 75° 30') \\ &= 180° - 123° 30' \\ &= 56° 30' \end{aligned}$$

Marka aan isticmaalno xeerka saynka, waxaanu heli

$$\frac{b}{\sin B} = \frac{j}{\sin J}$$

1) $$\frac{b}{\sin 48^\circ} = \frac{10.5 \text{ sm.}}{\sin 56^\circ 30'}$$

$$b = \frac{\sin 48^\circ \times 10.5 \text{ sm.}}{\sin 56^\circ 30'}$$

Marka aan logardamka isticmaallo, waxaanu heli

$\log b = \log 10.5 + \log \sin 48^\circ - \log \sin 56^\circ 30'$

Tiro	Logardam
10.5 sm.	1.0212
sin 48°	$\bar{1}$.8745
	0.8957
sin 56° 30′	$\bar{1}$.9211
	0.9746

lidlog (0.9746) = 9.43 sm.

$\therefore$ b = 9.43 sm.

U dambayn, si t loo helo, waxaan ognahay

$$\frac{t}{\sin T} = \frac{j}{\sin J}, \quad t = \frac{j \sin t}{\sin J}$$

$$t = \frac{10.5 \text{ sm.} \times \sin 75^\circ 30'}{\sin 56^\circ 30'}$$

$\text{Log } t = \log 10.5 \text{ sm.} + \log \sin 75^\circ 30' - \log 56^\circ 30'$

Tiro	Log
10.5 sm.	1.0212
sin 75° 30′	$\bar{1}$.9859
	1.0071
sin 56° 30′	$\bar{1}$.9211
	1.0860

liglog (1.0860) = 12.2 sm.

∴ t = 12.2 sm.

Layli :

Masalooyinka 1 ilaa 5, raadi qaybaha maqan ee saddexagalka BTJ.

1)	B = 73° 25′	T = 67° 20′	b = 115 sm.
2)	J = 26° 31′	T = 78° 02′	j = 1.16 sm.
3)	T = 105°	B = 21° 30′	t = 16.68 sm.
4)	B = 57° 30′	T = 61° 26.8′	t = 63.26 sm.
5)	T = 111° 43′	J = 26° 26′	b = 0.905 sm.

6) Xaglogooyaha barbarroole ayaa dhererkiisu yahay 289 sm. Raadi dhinacyadiisa haddii xaglaha u dhexeeya dhinacyadiisa iyo xaglagooyuhu ay yihiin 28° 40′ iyo 43° 10′.

7) Laba nin oo A iyo B kala jooga ayaa isku jiro 362 m. waxayna wada eegayaan barta C. Imisa ayey C u jirtaa nin kasta haddii xagasha CBA = 37° 20′, xagasha CAB = 68° 30′.

8) Tiir qotoma ayaa ku yaal degaandeg ugu janjeerta jiifka 8°. Harka tiirku ee degaandegga ku dhaca-

yaa waa 82 m. Waa immisa dhererka tiirku haddii xagasha kacsan ee cadeeddu tahay 28°?

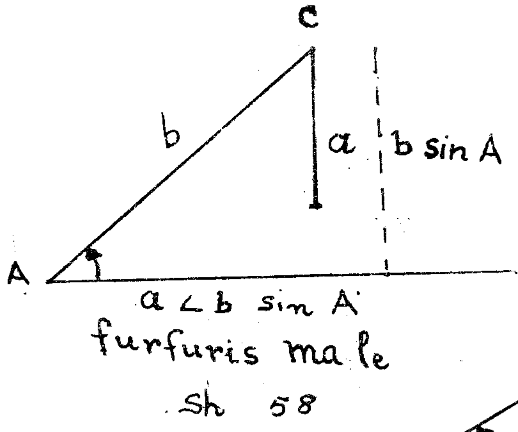
9) Xagal qardhaaseed ayaa ah 42° 38′. Haddii dhniacyadeedu ay yihiin 57.63 sm., raadi dhererka xaglogooyaha dheer.

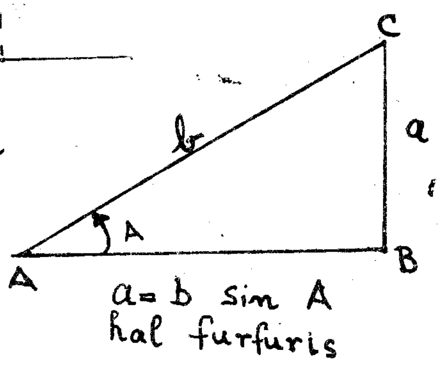
XAALADDA DAHSON

Hore waxan u sheegnay in xeerka saynka lagu shaqeeyo marka la inna siiyo laba dhinac iyo xagal aan u dhexayn. Xaaladdaa waxa la yiraa **xaaladda dahsoon** waayo waxa dhici kara inuu furfur yeelan ama in hal furfur ama laba furfur u yeesho. Bal aan eegno sida loo ogaado inta furfur iyo sida jawaabta loo helo.

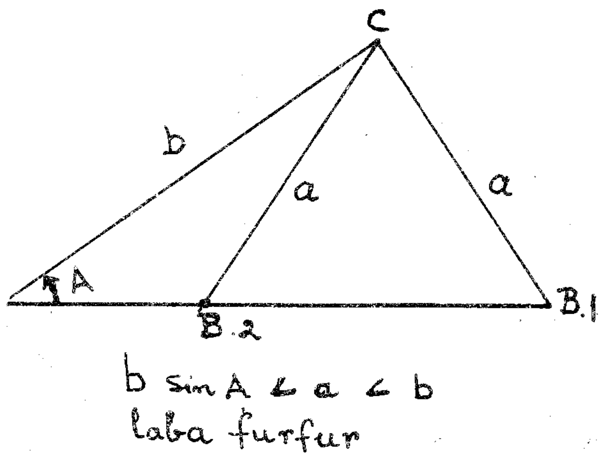
Ka soo qaad in la inna siiyay laba dhinac a iyo b iyo xagasha A oo fiiqan. Bal an tixgelinno marka $a<b$ oo keliya, maxaa yeelay haddii $b>a$ waxan heleynaa saddexagal keliya oo raalligeliya, dabadeedna waxa jiro hal furfur.

Marka $a<b$, $<A$-na fiiqan tahay, waxan heleynaa sida shaxanka hoose ku muujisan.



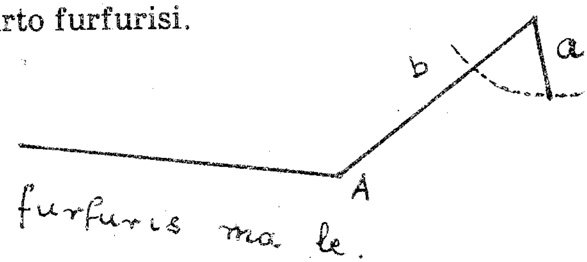


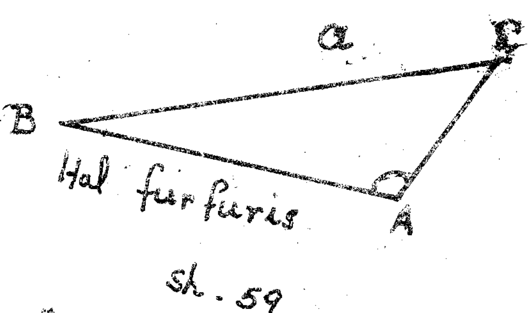
$b \sin A<a<b$. Laba furfur.



U faarso in aan hellay laba seddaxagal A B_1 C iyo A B_2 C. Waliba $\angle AB_2C = 180^\circ - \angle AB_1C$, AB_1C wuu fiiqan yahay.

Bal ka soo qaad in la inna siiyay a iyo b, iyo xagasha A oo furan. Haddii a > b waxa jira hal furfur. Haddii a < b, ma jirto furfurisi.





Hal furfur **Laba furfur**

Tusaale :

Saddexaaglka, ABC, haddii $a = 210$, $b = 317$, $\angle A = 62° 20'$, immisa furfur baa jira.

Furfuris :

$$b \sin A = 317 \sin 62° 20'$$
$$= 317 \; (0.8857)$$
$$= 280.7669$$

$\therefore$ $a < b \sin A$, waayo $a = 210$.

Furfur ma leh.

Tusaale :

Immisa furfur baa jira haddii $a = 341$, $b = 319$, $\angle A = 61° 30'$. Raadi mid kasta, hadday jiraan.

Furfuris :

$\angle A$ wuu fiiqan yahay.

Mar haddii $a > b$, waxan heleynaa hal furfur.

$$\therefore \quad \sin B = \frac{b \sin A}{a}$$

$$= \frac{319 \sin 61° 30'}{341}$$

$$= \frac{319 \; (0.8788)}{0.8221}$$

$\therefore$ $\angle B = 55° 20'$.

$B_1 = 180° - 55° 20' = 124° 40'$ iyo $B_2 = 55° 20'$ waa labada qiima ee B yeelan karto. Laakiin B ma noqon karto xagal saddexagalkaa waayo

$A + B = 61° 30' + 124° 40' = 186° 10' > 180.$

Markaa, waxa jira hal furfur oo keliya.

$\therefore \quad C = 180° - (61° 30' + 55° 20') = 63° 10'.$

Marka aan ku adeegsanno xeerka saynka, waxannu heli in

$$C = \frac{341 \sin 63° 10'}{\sin 61° 30'}$$

$$= 346$$

Tusaale :

Furfur saddexagalka ABC haddii $a = 303$, $b = 574$ $A = 29° 20'$. Taswiir shaxanka furfur kasta oo aad hesho.

Furfuris :

Marka aan ku adeegsanno xeerka saynka

$$\frac{a}{\sin A} = \frac{b}{\sin B}, \quad \text{markaa}$$

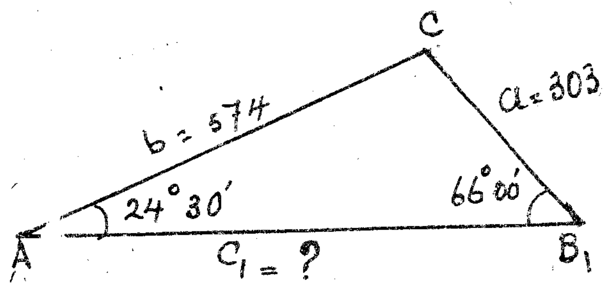
$$\sin B = \frac{574 \sin 29° 20'}{303}$$

$\therefore \quad \log \sin B = \log 574 + \log \sin 29° 20' - \log 303$

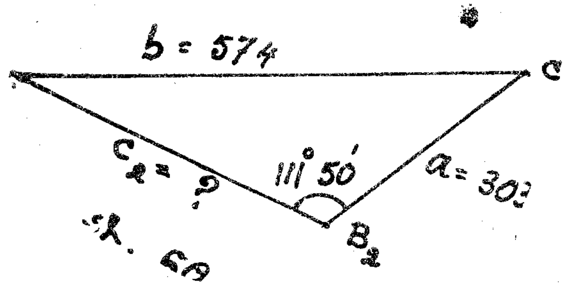
Tiro	Log
574	2.7589
sin 29° 20′	1.6901
	2.4490
303	2.4814
	1.9676

$\therefore \quad B_1 = 68° 10'$, markaa $B_2 = 180° - 68° 10' = 111° 50'$

Hadda waxa jira laba furfur, waayo B_1 iyo B_2 laba-duba waxay noqon karaan xaglo saddexagalkaa. Washirrada labada saddexagal waxay ku muujisan yihiin shaxanka 60. Ardayga ayaa looga tegay in C_1 iyo C_2 soo saaro.



sh. 60



sh. 60

Layli :

Laylisyada 1 — 5, raadi inta furfur ee mid kasta leeyahay.

1)	$\angle A = 31° 20'$	$b = 812$,	$a = 371$
2)	$\angle B = 37° 12'$	$c = 543$	$b = 6092$
3)	$\angle C = 61° 46'$	$a = 2267$	$c = 2574$
4)	$\angle A = 27° 27'$	$\angle B = 63°$	$c = 205$
5)	$\angle C = 97° 53'$	$\angle A = 36° 36'$	$b = 67$

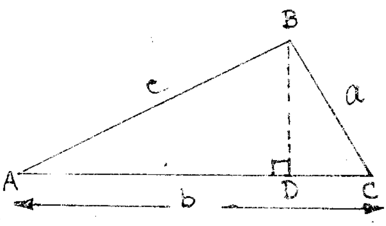
6) Duuliye ayaa A ka duulay oo B u socday, foolkiisuna wuxu ahaa 130°, dabadeedna B intuu ka tegay ayuu C u kacay, foolkiisuna wuxu ahaa 225°. Haddii A u jirto 538 mayl B, C-na 807 mayl, waa imisa fogaanta C iyo B? Sheeg foolka C marka A la joogo.

7) Salaan 32 m. ah ayaa marka gidaar lagu tiiriyo la sameeya xagal 61° jiifka. Waa imisa xagasha salaan 37 m. ahi u la sameynayo jiifka marka lagu tiiriyo isla gidaarkii ee uu gaaro isla meeshii kii hore ku tiirsanaa.

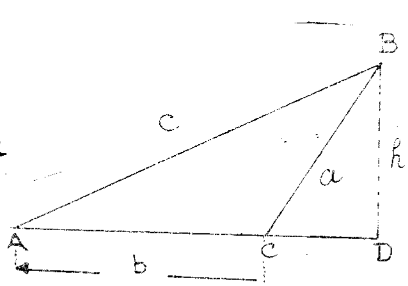
8) Bir-calan ayaa ku dul taagan daar. B waa meel 750 sm. u jirta bar ku taal salka daarta oo hoos ah bir-calanka. Haddii xaglaha kacsan ee gunta iyo baarka bir-calanku marka la jooga B ay yihiin 34° iyo 50° siday u kala horreeyaan, waa imisa dhererka bir-calanku?

XEERKA KOSAYNKA

Waxan soo diiri doonnaa jidka kale ee lagu furfuro saddexagallada, marka laba dhinac iyo xagasha u dhexay layna siiyo. Bal u fiirso shaxannada hoos ku yaal.



Sh. 61



Sh. 62

Haddii aan ku isticmaallo aragtiinka «Pythagoras», waxan saddexagal kasta ka heleynaa in

(1) $C^2 = h^2 + (AD^2$

Shaxanka 61, $AD = b - DC$

$$= b - a \cos C, \text{ waayo } \cos C = \frac{DC}{a}$$

Isla markaa, $h = a \sin C$, waayo $\sin C = \frac{h}{a}$

Shaxanka 62, $AD = b + DC$

$= b + a \cos DCB$

Laakiin, $\angle DCB = 180° - \angle C$

$h = a \sin BCB$

Haddaba, $AD = b + a \cos (180° - \angle C)$

$= b - a \cos C.$

$h = a \sin \angle BCB = a \sin (180° - \angle C) = a \sin C$

Labada shaxanba

(1) $C^2 = (a \sin C)^2 + (b - a \cos C)^2$

$= a^2 \sin^2 C + b^2 - 2ab \cos C + a^2 \cos^2 C$

$= a^2 (\sin^2 C + \cos^2 C) + b^2 - 2ab \cos C$

$= a^2 + b^2 - 2ab \cos C.$

OGOW: $\sin^2 C + \cos^2 C = 1.$

Markaa $C^2 = a^2 + b^2 - 2ab \cos C$

Sidoo kale $a^2 = b^2 + c^2 - 2bc.\cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$

Aragtiinka Kosaynka

Labajibbaarka dhinac kasta ee saddexagal wuxu le'eg yahay wadarta, labajibbaarrada dhinacyada kale iyo labanlaabka, taranka dhinacyadaa iyo Kosaynka xagasha u dhexaysa.

Isleegta (1) waxa loo yaqaan xeerka kosaynka si kale oo loo qori karaa waa

(2) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$, $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

Isleegta (1) iyo (2) waxa ka cad in xeerka kosaynka isticmaali karo.

1) Marka laba dhinac iyo xagasha u dhexaysa la ogyahay.

2) Marka saddex dhinac la ogyahay.

Tusaale 1:

Raadi dhinaca haray ee seddaxagalka ABC haddii: $c = 68$ sm. $b = 51$ sm. $\angle A = 37°$.

Furfuris :

$a^2 = b^2 + c^2 - 2bc \cos A$
$= 51^2 + 68^2 - 2(51 \times 68) \cos 37°$
$= 2601 + 4624 - 6936\ (0.7986)$
$= 1685 \cdot 9104$

Markaa $a \simeq \sqrt{1685 \cdot 9104} \simeq 41.$

Tusaale 2:

Raadi xagasha ugu yar ee seddaxagalka ABC haddii: $a = 234$, $b = 185$, $c = 297$.

Furfuris :

Xagasha la rabaa waa B, markaa b waa dhinaca ugu yar; markaa waxan ku adeegsan karraa isleegta (2).

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{(234)^2 + (297)^2 - (185)^2}{2\,(234 \times 297)}$$

$\cos B = 0.7823$

$\therefore \quad \angle B = 38^\circ\ 30'$ ku seeban $10'$ ee ugu dhow.

Layli :

Raadi dhinaca saddexaad ee saddexagalka ABC.

1)	$\angle A = 41^\circ$	$b = 19$	$c = 23$
2)	$\angle B = 73^\circ$	$a = 48$	$c = 69$
3)	$\angle C = 105^\circ$	$a = 24$	$b = 27$
4)	$\angle A = 50^\circ$	$b = 25$	$c = 30$
5)	$\angle C = 60^\circ$	$a = 7$	$b = 9$

Raadi xagasha ugu weyn ee saddexagalka ABC.

6)	$a = 9$	$b = 23$	$c = 27$
7)	$a = 48$	$b = 37$	$c = 52$
8)	$a = 3$	$b = 5$	$c = 7$
9)	$a = 13$	$b = 12$	$c = 20$
10)	$a = 3$	$b = 4$	$c = 5$

Furfur saddexagal kasta.

11)	$\angle A = 49°$	$c = 29$	$b = 39$
12)	$\angle B = 92°$	$a = 17$	$c = 23$
13)	$\angle C = 31°$	$b = 36$	$a = 42$
14)	$a = 71$	$b = 45$	$c = 51$
15)	$a = 35$	$b = 39$	$c = 44$

16) Orod-hawada dayuuradeed waa 400 km./saacad, foolkeeduna waa 135°. Haddii dabayshu ka dhacayso galbeed orodkeeduna yahay 50 km./saacad, waa immisa orod-dhulka dayuuraddu.

17) Dhul-cabbire C jooga ayaa ayaa eegay laba barood A iyo B oo ku kala yaal laba daannood oo webi. Haddii C u jirto B 500 mitir, Ana 7500 mitir, xagasha ACB-na ay tahay 30°, waa immisa ballaca webigu.

18) Markab ayaa 20 km. u socday jiho ah 35°, dabadeedna 30 km. ayuu u socday jiho ah 100°. Immisa ayuu u jiraa bar bilawgiisii?

19) Laba dayuuradood oo mid orodkeedu yahay 300 km. saacadiiba, midnaa 450 km. saacadiiba ayaa gego ka duulay isla mar. Saddex saacadood ka dib, haddii ay isku jiraan 1200 km. waa immisa xagasha u dhexaysaa waddooyinkooda?

20) Waa imisa xagasha u dhexaysa labada itaal oo kala ah 20 kg. iyo 15 kg. haddii wadarteerkoodu yahay 26 kg.?

DHERERKA QAANSO

Waxan hore u dhignay in haddii S tahay dhererka qaanso, Θ-na tahay xagasha S ay ku sameyso xudduntu oo ku cabbiran gacansin, r-na yahay gacanka goobada, in $S = r\Theta$. Isleegtaa waxan ka heli karraa dhererka qaanso, gacanka goobo ama xagasha ay qaansadu ku sameyso xuddunta. Marka laba ka mid ah saddexdaa aan naqaan, ka haray si dhib yar ayaa loo soo saari karraa.

Tusaale 1:

Raadi dhererka qaansada goobo gacankeedu yahay 6 sm. ee xuddunta ku sameysa xagal ah $\frac{\pi}{8}$, (u qaado $\pi \simeq 3.142$).

Furfuris :

$$S = ?$$

$$\Theta = \frac{\pi^R}{8}$$

$$r = 6 \text{ sm.}$$

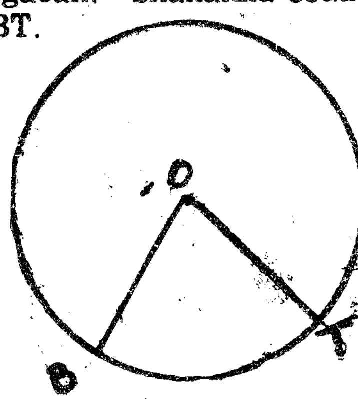
$$\therefore \quad S = r\Theta = \frac{\pi}{8} \times 6 \text{ sm.} = \frac{3\pi}{4} \text{ sm.}$$

$$= \frac{3}{4} \times 3.142 \text{ sm.} = \frac{9.426}{4}$$

$$= 2.3565 \text{ sm.}$$

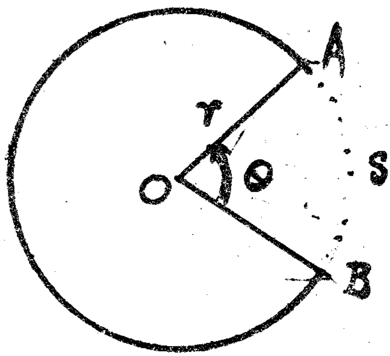
Bedka Fatuug

Fatuuq w aa qayb goobo ka mid ah oo ay soo oodan qaanso iyo laba gacan. Shaxanka 53aad wuxu muujinaa fatuuqa OBT.



Sh 63

U fiirso shaxanka 64. OAB waa fatuuq ku dhex oodan laba gacan OA iyo OB iyo qaansada S, Θ waa xagasha ay S ku sameyso xuddunta oo ku cabbiran gacansin.



Hadda, waxan ognahay in $2\pi^R$ ay tahay xagasha u meerisku ku sameeyo xuddunta. Markaa, waxa cad in

bedka fatuuqa OAB le'eg yahay $\frac{\Theta}{2\pi}$ bedka goobada.

$$\text{Bedka fatuuqa OAB} = \frac{\Theta}{2\pi} \times \pi r^2$$

$$= \frac{1}{2} r^2 \Theta$$

Soo gaabin

$\pi^R = 180°$

Dhererka qaanso $= r\Theta$

Bedka fatuuq $= \frac{1}{2} r^2 \Theta$

Tusaale 2:

Fatuuq goobo ayey soo oodaan laba gacan oo mid-kiiba dhererkiisu yahay 6 sm. iyo qaanso dhererkeedu yahay 5 sm. Raadi xagasha fatuuqa iyo bedka fatuuqa.

Furfuris :

Ka soo qaad in xagasha fatuuqu tahay Θ.

$\therefore \quad S = r\Theta$

$5 \text{ sm.} = 6 \times \Theta$

$$\therefore \quad \Theta = \frac{5^R}{6} = 0.8333$$

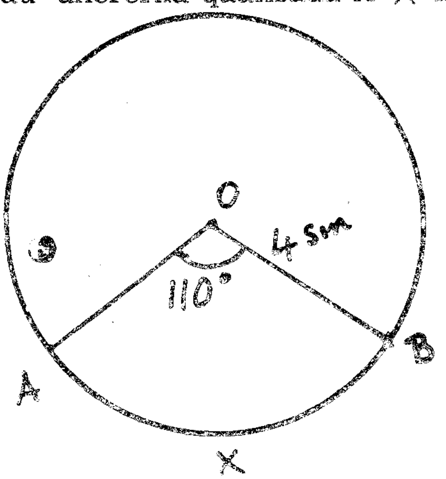
$$\text{Bedka fatuuqu} = \frac{1}{2} r^2\Theta = \frac{1}{2} \times 36 \times \frac{5}{6} \text{ sm}^2$$

$$= 15 \text{ sm}^2.$$

Tusaale 3:

AB wuxu u yahay boqon xuddunteedu tahay 0, gacan-keeduna yahay 4 sm. $\angle AOB = 110°$.

(i) Raadi bedka fatuuqa AOB eeg shaxanka 65.
(ii) Raadi dhererka qaansada A × B.



Furfuris :

110° u beddel gacansin

$$\therefore \quad 110^\circ = \frac{\pi^R}{180} \times 110 = \frac{11 \times \pi^R}{18}$$

I) Bedka fatuuqa AOBX $= \frac{1}{2} r^2\Theta = \frac{1}{2} \times 4 \times 4 \frac{11\pi}{18}$

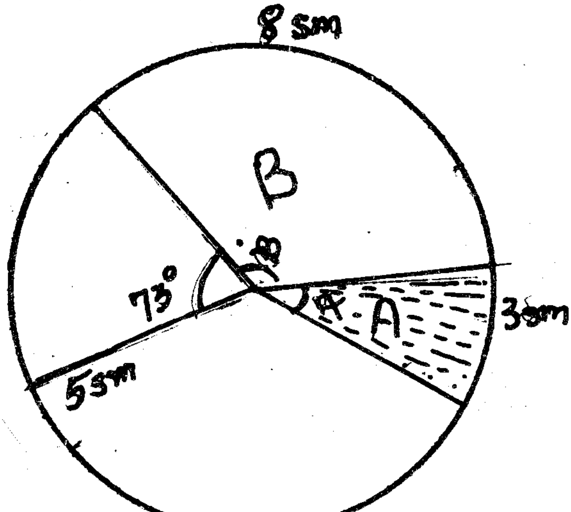
$$= \frac{44\pi}{9}\ \text{sm}^2$$

II) Dherer qaansada A × B $= r\Theta = 4 \times \frac{11\pi}{18}$ sm.

$$= \frac{22\pi}{9}\ \text{sm.}$$

Layli :

1) Shaxanka 67 (i) ku raadi digrii xaglaha α iyo β.
(ii) Raadi dhererka qaansooyinka X iyo Y.
(iii) Raadi bedadka fatuuqyada A iyo B.



2) XY waa qaanso dhererkeedu yahay 8 sm. oo ku taal goobo gacankeedu yahay 6 sm. Raadi bedka fatuuqa ku dhex oodan labo gacan iyo XY?

3) Raadi bedka goobo haddii dhexroorka goobadu 14 sm. yahay, qaansada fatuuquna yahay 10 sm.

4) Bedka fatuuq goobo ayaa 3 sm^2. ah, gacanka goobaduna waa 4 sm. Waa imisa dhererka qaansada fatuuqu.

5) AB waa boqon goobo oo dhererkiisu yahay 9 sm. gacanka goobaduna waa 5 sm. Raadi dhererka qaansada yar AB iyo bedka fatuuqa qaansada yar.

JIDADKA IYO MIDAALLADA TIRIGNOOMETERIGA

Fansaarradii tirignoometeri ee aan soo aragnay siyaabo badan ayay isugu xiran yihiin. Bal siyaabahaa qaar ka mid ah aan eegno.

Midaallo ku saabsan Xagal Keliya.

Isleeg leh u yaraan hal doorsame oo horaadkiisu yahay urur xagallo doorada ayaa la yiraa **isleeg tirignoometeri.** Isleeg tirignoometeri, sida

$$(2 \sin \Theta + 1)\ (2 \sin \Theta - 1) = 4 \sin^2 \Theta - 1,$$

oo ku run ah kutirsane kasta oo horaadka waxa la yiraa

midaal tirignoometeri.

Midaallada tirignoometeri waxay ku xiran yihiin qeexdii fansaarrada tirignometeri iyo aljebraha tiroo-yinka maangalka ah. Ma sheegi kartaa waxa hawraaraha soo socdaa ay ugu run yihiin xagal kasta Θ oo fansaarku ku qeexan yahay?

1. $\text{Tan } \Theta = \dfrac{\sin \Theta}{\cos \Theta}$

2. $\text{Cot } \Theta = \dfrac{\cos \Theta}{\sin \Theta}$

3. $\text{Sec } \Theta = \dfrac{1}{\cos \Theta}$

4. $\text{Csc } \Theta = \dfrac{1}{\sin \Theta}$

5. $\text{Cot } \Theta = \dfrac{1}{\tan \Theta}$

Midaallada 1 — 4 waxay ka yimaadeen qeexdii fansaarrada tirignoometeri, midaalka 5 wuxu ka yimid midaallada 1 iyo 2. U fiirso $\sin \Theta \neq 0$, $\cos \Theta \neq 0$.

Markaa

$$\therefore \quad \frac{1}{\tan \Theta} = \frac{1}{\dfrac{\sin \Theta}{\cos \Theta}} \text{ (mid. 1)}$$

$$= \frac{\cos \Theta}{\sin \Theta} \text{ (astaanta tirooyinka maangalkaal)}$$

$$= \cot \Theta \text{ (mid. 2)}.$$

$$\therefore \quad \cot \Theta = \frac{1}{\tan \Theta}$$

6. $\sin^2 \Theta + \cos^2 \Theta = 1$

Midaal 6 horaan ugu dhignay fansaarrada goobo.

Haddii dhinac kasta oo mdiaal 6 aan u qaybinno $\cos^2\Theta$, waxan soo diiri karnaa midaal kale.

$$\frac{\sin^2\Theta}{\cos^2\Theta} + 1 = \frac{1}{\cos^2\Theta}, \text{ ama}$$

$$1 + \left(\frac{\sin\Theta}{\cos\Theta}\right)^2 = \left(\frac{1}{\cos\Theta}\right)^2, \cos\Theta \neq 0.$$

Haddii aan la kashanno midaallada 1 iyo 3, waxannu heli

7. $\mathbf{1 + \cot^2\Theta = \csc^2\Theta}$

Ma sheegi kartaa sida loo soo diiro midaalka soo socdaa.

8. $\mathbf{1 + \cot^2\Theta = \csc^2\Theta}$

Midaallada 1 — 8 waxa la yiraa **midaallada tirignoometeriga ee doorka ah.** Iyaga ayaa naga caawin kara in aan soo saarro midaallo kale oo tirignoometeri.

Tusaalooyin :

1) Raadi tibaax kale oo u dhiganta oo ah tibaax $\cos\alpha.\ (1 + \sin\alpha)\ (\sec\alpha - \tan\alpha)$.

Furfuris :

Tibaaxda waxay u taagan tahay tiro maangal ah haddii $\cos\alpha \neq 0$. Markaa

$$(1 + \sin\alpha)\ (\sec\alpha - \tan\alpha) =$$

$$= (1 + \sin\alpha)\left(\frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}\right)$$

$$= (1 + \sin\alpha)\left(\frac{1 - \sin\alpha}{\cos\alpha}\right) = \frac{1 - \sin^2\alpha}{\cos\alpha}$$

$$= \frac{\cos^2 \alpha}{\cos \alpha} \text{ midaal } 6$$

$\therefore \quad (1 + \sin \alpha)(\sec \alpha - \tan \alpha) = \cos \alpha$, haddii $\cos \alpha \neq 0$

Layli:

U tibaax mid kasta oo soo socota tibaax fansaar keliya oo tirignoometeri.

1) $\dfrac{\sin \Theta}{\cos \Theta}$

2) $\dfrac{\cos^2 u}{\sin^2 u}$

3) $1 + \tan^2 B$

4) $1 - \cos^2 \phi$

5) $1 - \sin^2 \Theta$

6) $1 - \csc^2 \Theta$

7) $\tan \Theta \sec \Theta \cos \Theta$

8) $\csc \Theta \sin \Theta \cot \Theta$

9) $\sin^2 \Theta + \cos^2 \Theta + \tan^2 \Theta$

10) $\cos^2 \alpha + \sin^2 \alpha + \cot^2 \alpha$

11) $\csc^2 \phi - \cot^2 \phi + \tan^2 \phi$

12) $\tan a \cot a - \cos^2 a$

13) $\dfrac{(\sin^2 B + \cos^2 B)(\sec^2 B - \tan^2 B)}{\tan B}$

14) $\dfrac{\sin \Theta (\csc^2 \Theta - \cot^2 \Theta)}{\cos \Theta \sec \Theta}$

15) $\dfrac{\sqrt{\sec^2 \Theta - 1}}{\sqrt{\csc^2 \Theta - 1}}$

16) $\dfrac{\sqrt{1 - \sin^2 \Theta}}{\sqrt{1 + \tan^2 \Theta}}$

Laylisyada 17 — 20, u tibaax fansaarrada Sayn ama Kosayn oo keliya, dabadeedna fududee.

17) $\left\{\dfrac{\cos \alpha - \sec \alpha}{\sec \alpha} + \cos^2 \alpha \tan^2 \alpha\right\} \left\{\dfrac{\tan \alpha - \sin \alpha}{\tan \alpha}\right\}$

18) $(\tan \phi + \sin \phi)\ (1 - \cos \phi) + \dfrac{\cos \phi}{\csc \phi}$

19) $\left\{\dfrac{\sqrt{\cot^2 B + 1}}{\csc B}\right\} \left\{\dfrac{\cot^2 B \sec^2 B - 1}{\csc B \cot^2 B \sin B}\right\}$

20) $\sin \gamma \sec \gamma \left\{\cos \gamma + \dfrac{\csc \gamma}{\sec^2 \gamma}\right\} + (\csc \gamma + \sec \gamma)$

CADDEYNTA MIDAALLADA

Mararka qaarkood, waxan caddeyn karnaa in isleeg tirignoometeri ay tahay midaal tirignoometeri, innagoo la kaashanayna astaamaha tirooyinka maangalka ah iyo midaallo doorrada.

Tusaale 1:

Caddee midaalkan:

$$2 \csc^2 \Theta = \frac{1}{1 + \cos \Theta} + \frac{1}{1 - \cos \Theta}$$

C a d d e y n :

U fiirso in isleegta layna siiyey ay micno leedahay haddii iyo haddii oo qudha oo $1 \pm \cos \Theta \neq 0$, isla markaa $\sin \Theta \neq 0$ (waayo?).

1. Qaado dhinaca midig ee isleegta, t.a.,

$$\frac{1}{1 + \cos \Theta} + \frac{1}{1 - \cos \Theta}$$

$$\therefore \quad \frac{1}{1 + \cos \Theta} + \frac{1}{1 - \cos \Theta} =$$

$$= \frac{(1 - \cos \Theta) + (1 + \cos \Theta)}{(1 + \cos \Theta)\ (1 - \cos \Theta)}$$

$$= \frac{2}{1 - \cos^2 \Theta}$$

Laakiin $1 - \cos^2 \Theta = \sin^2 \Theta$ Midaal 6.

$$\therefore \quad \frac{1}{1 + \cos \Theta} + \frac{1}{1 - \cos \Theta} = \frac{2}{\sin^2 \Theta}$$

$$= \frac{2}{\left(\dfrac{1}{\csc \Theta}\right)^2}$$

waayo $\sin \Theta = \dfrac{1}{\csc \Theta}$

$$= \frac{2 \csc^2 \Theta}{1}$$

$$= 2 \csc^2 \Theta$$

Mar haddii tallaabooyinka dhinaca midig lagu saan-qaaday ayna keenin xannibaad cusub, midaalka waxa la caddeeyay inuu sax yahay.

Tusaale 2:

Caddee in $\frac{\sin \Theta}{1 - \cos \Theta} = \frac{1 + \cos \Theta}{\sin \Theta}$

Caddeyn:

1. Dhinacaad doonto qaado, ka dhig ka bidixdaba oo ah $\frac{\sin \Theta}{1 - \cos \Theta}$. Sarreeyaha iyo hooseeyahaba waxad ku dhufataa $(1 + \cos \Theta)$ oo ah sarreeya dhinaca midig. $(\cos \Theta \neq -1)$

$$\therefore \frac{\sin \Theta}{1 - \cos \Theta} = \frac{\sin \Theta}{(1 - \cos \Theta)} \frac{(1 + \cos \Theta)}{(1 + \cos \Theta)}$$

$$= \frac{\sin \Theta (1 + \cos \Theta)}{1 - \cos^2 \Theta}$$

$$= \frac{\sin \Theta (1 + \cos \Theta)}{\sin^2 \Theta}$$

waayo $1 - \cos^2 \Theta = \sin^2 \Theta$

$$= \frac{1 + \cos \Theta}{\sin \Theta} \quad (\sin \Theta \neq 0)$$

Tallaabooyinku ma keeneen xannibaad cusub? Bal aan eegno $\sin \Theta \neq 0$. Haddii $\sin \Theta = 0$, markaa $\Theta = 0$

ama 180°, laakiin $\cos \Theta = 1$ ama -1. Markaa waxa maaqata sansaanqaadku uuna xannibaad keenin waayo

$\dfrac{\sin \Theta}{1 - \cos \Theta}$ waxay malagelinaysaa in $\cos \Theta$ uuna noqon

karayn 1 (waayo?). Markaa, mar haddii sansaanqaad uuna xannibaad cusub keenin, midaalka waa la caddeeyay.

$$\therefore \quad \frac{\sin \Theta}{1 - \cos \Theta} = \frac{1 + \cos \Theta}{\sin \Theta}$$

Mararka qaarkood, waxa dhib yar in la sansaanqaado dhinac kasta ilaa la gaaro tibaaxo isle'eg.

Tusaale 3:

Caddee in $\tan B + \cot B = \sec B \csc B$

Caddeyn :

Dhinaca bidix

$$\text{Tan}\, B + \cot B = \frac{\sin B}{\cos B} + \frac{\cos B}{\sin B}$$

$$= \frac{\sin^2 B + \cos^2 B}{\cos B \sin B}$$

$$= \frac{1}{\cos B \sin B}.$$

Dhinaca midig

$$\sec B \csc B = \frac{1}{\cos B} \cdot \frac{1}{\sin B}$$

$$= \frac{1}{\cos B \ \sin B}$$

$\therefore \quad \tan B + \cot B = \sec B \ \csc B$

Layli :

1) $\sin \Theta \ \cot \Theta = \cos \Theta$

2) $\cos A \ \tan A = \sin A$

3) $\dfrac{\sin^2 \Theta + \cos^2 \Theta}{\cos \Theta} = \sec \Theta$

4) $\dfrac{\sin B - 1}{\cos B} = \tan B - \sec B$

5) $1 - \sin \Theta \ \cos \Theta \ \tan \Theta = \cos^2 \Theta$

6) $\dfrac{1 + \sin \Theta}{\sin \Theta} = 1 + \csc \Theta$

7) $\sin A + \cos A \ \cot A = \csc A$

8) $1 - 2 \sin^2 x = 2 \cos^2 x - 1$

9) $\cos A \ (\csc A - \sec A) = \cot A - 1$

10) $\csc \Theta \ (\csc \Theta + \cot \Theta) = \dfrac{1}{1 - \cos \Theta}$

11) $\sin^4 B - \cos^2 B = 2 \sin^2 B - 1$

12) $\tan^4 A - \sec^4 A = 1 - 2 \sec^2 A$

13) $\dfrac{\sin B + \tan B}{1 + \cos \Theta} = \tan B$

14) $\sec A + \tan A = \dfrac{\cos A}{1 - \sin A}$

15) $(1 + \csc A)(1 - \sin A) = \cot A \cos A$

16) $(1 + \tan \Theta + \sec \Theta)^2 = 2(1 + \sec \Theta)(\tan \Theta + \sec \Theta)$

17) $(1 + \sec B)(\sec B - 1) = \dfrac{\sin B \sec B}{\cos B \csc B}$

18) $(\csc B - 1)(1 + \csc B) = \dfrac{\csc B \cos B}{\sec B \sin B}$

19) $\dfrac{\sin A \cos A}{1 + \cos A} - \dfrac{\sin A}{1 - \cos A} = -(\cot A \cos A + \csc A)$

20) $\dfrac{\sin A + \cos A}{\sec A + \tan A} + \dfrac{\cos A - \sin A}{\sec A - \tan A} = 2 - 2 \sin^2 A \sec A$

21) $\dfrac{\sec B}{1 - \cos B} = \dfrac{\sec B + 1}{\sin^2 B}$

22) $\dfrac{\tan A}{\tan A + \sin A} = \dfrac{1 - \cos A}{\sin^2 A}$

23) $\dfrac{1 + \sec A}{\sec A - 1} + \dfrac{1 + \cos A}{\cos A - 1} = 0$

24) $\dfrac{\sec^2 \Theta (1 + \csc \Theta) - \tan \Theta (\sec \Theta + \tan \Theta) - 1}{\csc \Theta (1 + \sin \Theta)} = 0$

25) $$\frac{\tan A - \sin A}{\tan A \sin A} = \frac{\tan A \sin A}{\tan A + \sin A}$$

26) $$\frac{\csc A}{1 + \sec A} = \frac{\cot A}{1 + \cos A}$$

27) $$\frac{\csc B + \cot B}{\csc B - \cot B} = \csc^2 B \, (1 + 2 \cos B + \cos^2 B)$$

28) $$\frac{\sin B + \cos B - 1}{\sin B - \cos B + 1} = \frac{\cos B}{\sin B + 1}$$

29) $$\frac{\sin^3 T + \cos^3 T}{\sin^2 T + 2 \sin T \cos T + \cos^2 T} =$$

$$= \frac{1}{\sin T + \cot T} - \frac{\cos T}{1 + \cot T}$$

30) $$\frac{\cos B - \sin B}{\cos^3 B - \sin^3 B} = \frac{1}{\tan B \cos^2 B + 1}$$

CUTUB IV

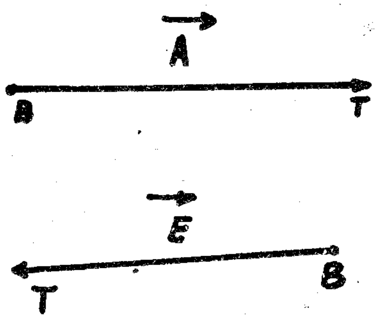
L E E B A B

ARAAR :

Xaddiyada fisikiska waxaynu u qaybin karnaa laba jaad, kuwo leh laxaad keliya iyo kuwa leh laxaad iyo jiho.

Xaddiga lagu asteeyo laxaad keliya, ama laxaad iyo summad, Aljebra waxaa lagu magacaabaa **Foolwaa.** Haddaba cuf, ammin, cufnaan waa foolwayo. Markaa halbeegyada cabbirrada la cugto ama la doorto, tiro maangal ahiba waxay u joogi ama u taagnaan, foolwaa, oo middiidin u noqota ama u hoggaansanta xeerarka Aljeebrada hoose oo dhan.

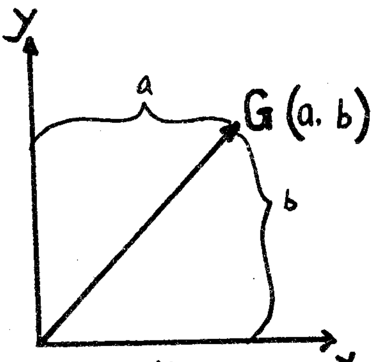
Xaddiga jiho iyo laxaad labadaba leh waxa lagu magacaabaa **Leeb:** xoog, kaynaan, karaar ayaa tusaale ahaan loo qaadan karaa. Xariijin jihan (Jiho leh), waxaynu uga gol leenahay ama u jeednaaba xarriijin jiho loo doortay. Jihada waxa lagu asteeyaa ama lagu tilmaamaa, Fiiqa madaxa leebka (eeg shaxanka 1aad).



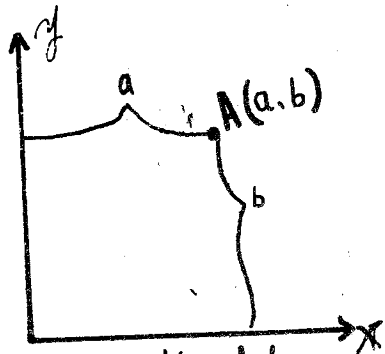
Shaxankan, B-da waxa la yiraa **Bar** bilowga T-dana yiraa **Bar dhammaadka xarriijinta jihan.**

Waxaynu xusuusanahay in ay $R \times R$ tahay Ururka «Kaartis» oo guud ahaanna ka kooban lammaanayaal horsan oo xubnahooda hore iyo kuwooda dambe yihiin tiro maangal ah. Waxaynu hore u garwaaqsanahay in lammaane kasta oo horsan oo tirada maangalka ahi uu yahay kulan bar ku jirta sallax. Waxaynu niri lammaane horsan oo tirada maangalka ahiba waa leeb, laba addimoole ah.

Hadday a iyo b tircoyin yihiin, waxa caado ah in loo muujiyo ama loo taago barta (a, b), bar ahaan, laguna magacaabo xarfaha waaweyn. (Eeg shaxanka 1.1).



shaxanka: 1. 2



Shaxanka: 1. 1

Turjumad Joometeri ah ayaa loo sameyn karaa Leebabkii Aljebra ee lammaanayaasha horsan ahaa, waayo lammaane horsan (a, b) oo kasta waxa loo maddeeyaa ama lagu soo soocaa xarriijin jihan ama leeb Joometeri ah oo ka unkanta (bar bilowga ku leh) unugga, ku dhammaatana (bar dhammaadka ku leh) bar sallaxa ku taal oo ku beegan lammaanaha horsan ee (a, b). (Eeg shaxanka 1.2). Waxaynu ugu yeeri (0, 0) leeb eber, oo loo qoro 0.

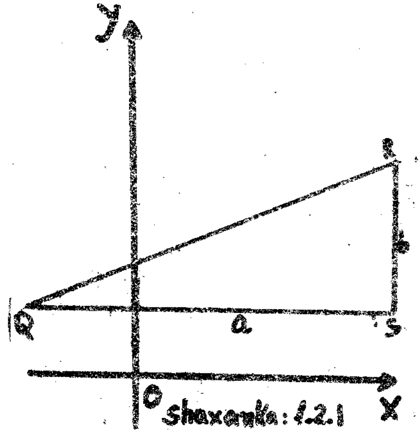
Layli 1.1:

U jooji ama u taag leebabka soo socda bar iyo leeb joometari ah.

b) $A = (3,4)$
t) $B = (5,1)$
j) $C = (3,-2)$
x) $D = (6,-6)$
kh) $E = (0,8)$
d) $F = (7,0)$
r) $G = 0,-3)$
s) $H = (-5,0)$

1.2 XUBNAHA LEEBKA

Haddii aynu haysanno leeb joometeri QR sida ku muujisan shaxanka 1.2.1, waxaynu sawiri karnaa saddexgal quman QRS, oo ku beegan oo QS-du jiifto RS-duna ku qotonto.



Shaxanka: 1.2.1

Dhererka QS waa «xubinta x» ee QR; a way togan tahay haddii QR ay u fiiqan tahay midigta, wayna taban tahay, hadday u fiiqan tahay bidixda. Sidaas oo kale SR waa «xubinta —y, b» ee QR; b way togan tahay haddii QR ay u fiiqan tahay sare (kor) wayna taban tahay hadday u fiiqan tahay hoos. Waxa caddaan ah in xubnaha la yaqaan ama la ogyahay haddii leeb la yaqaan ama la ogyahay; iyo roggeeda oo ah laba xubnood (lammaanayaal xubno ah) waxay sugaan leeb. Hore waxaynu u gorfaynay in leebabku bar bilowga ku leeyihiin unugga, markaa kulammada bar dhammaadku waxay le'eg yihiin xubnaha leebka. Haddaba leeb kasta oo sallax ku jira waxa lagu sugaa tiro lammaane horsan (a, b). Sidaas oo kale leebab dulalaati yaallaa waxay leeyihiin saddex xubnood; waxaana lagu sugaa saddexan horsan (a, b, c), ama leeb saddex addimoole ah.

Cutubkan waxaynu ku shaqayn Leebabka laba addimoodka ah, haddii kalese waa lagu sheegi.

Tusaale:

Sug xubnaha leebabka soo socda:
$D = (a, b)$. Xubnuhu waa a iyo b.
$R = (8, -3)$. Xubnuhu waa 8 iyo -3.
$S = (a, b, c)$. Xubnuhu waa a, b iyo c.

QEEXO

Eegga aynu isku dayno inaynu qeexno leebabka addimo kasta ha lahaadee.

Qeex 1:

Leeb waa teed kasta oo tiro ah, lehna hal dhinac u tax ama hal joog u tax.

Qormo Leeb

Waxaynu u qori doonnaa leebabkeenna sida leeb dhinac u tax (a, b), (a, b, c) ama sida «leeb-joog-tax» $\begin{pmatrix} a \\ b \end{pmatrix}$, $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$. Wax weyn oo aynu ku kala soocnaa ma jirto leeb-dhinac u tax, iyo leeb-joog u tax, hase ahaatee waxa inoo fudud ama habboonba inaynu labada qormaba adeegsanno ama gargaarsanno meelaha qaarkood.

Leeb Eber

Qeex 2:

Leebkii dhererkiisu eber yahay waxa la yiraa **Leeb eber** waxaana loo qoraa 0. Waxay u dhigantaa xarriijin jihan oo ka timid bar una socota, ama u jeedda bartaa (bartaa ayaa bar bilow iyo bar dhammaadba u ah).

Mar hadduu leeb eber ku beegan yahay bar wuxuu u jeeraaran yahay jiho walba.

Leeb Halbeeg ah

Qeex 3:

Leebka laxaadkiisu (dhererkiisu yahay hal (kow) waxa la yiraa **Leeb-halbeeg ah.**

Isle'egkaanshaha Leebab

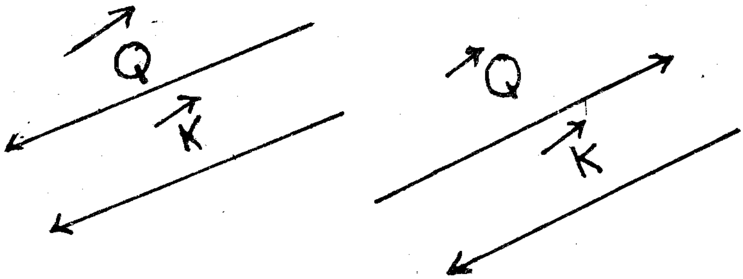
Qeex 4:

Laba leeb waxay isle'eg yihiin hadday isku laxaad (dheerar) iyo isku jiho yihiin. Waxa kaloo dhihi karnaa. haddii ay xubnaha isku beegani isle'eg yihiin, labada leebna way isle'eg yihiin.

Leebab Barbarro ah

Qeex 5:

Laba leeb Q iyo K waa barbarro, haddii ay isku ama kala jiho yihiin. **Ogow:** 0 waa la barbarro leeb kasta.



sh 1.2.2

Layli 1.2:

U taag leebabkan soo socda bar ama leeb joometeri ah markaana sug xubnahooda.

1) B = (4, 3)
2) T = (2, −1)
3) J = (−3, 2)
4) X = (−5, −4)
5) KH = (0, 1)
6) D = (1, 0)
7) R = (0, −1)
8) S = (−1, 0)

1.3 ISUGEYNTA IYO ISKUDHUFASHA LEEBABKA

1.3.1 Isugeynta Leebabka.

Mar haddii leebab aanay ahayn tirooyin, wadarta laba leeb waa fikrad cusub, una baahan qeex.

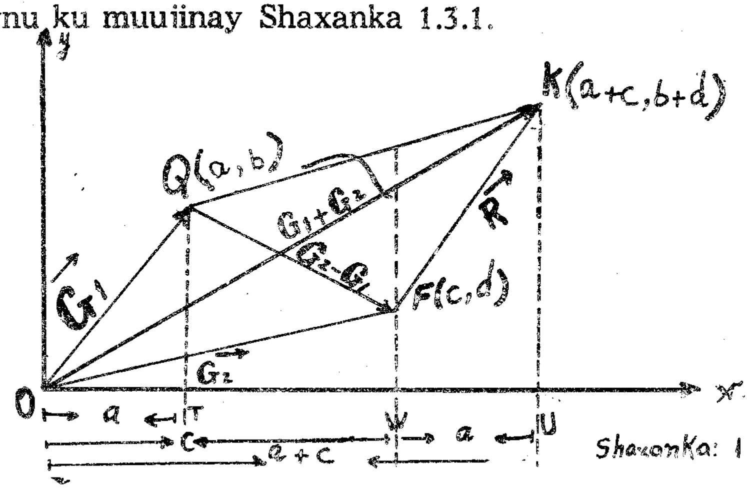
Qeex :

Wadarta laba leeb waxa lagu qeexaa jid xisaabeedkan.

$(a, b) + (r, s) = (a + r, b + s)$

$(a, e, u) + (d, r, s) = (a + d, e + r, u + s)$ Saddex aaddimo.

Haddaba laba leeb oo isku aaddimo ah, isugeyntood, waxaynu isugeynaa xubnahooda isku beegan; tani waxay leedahay turjumad joometeri ah oo lama huraan ah sida aynu ku muujinay Shaxanka 1.3.1.



Shaxanka: 1.3

Qaado leebka $\vec{R}$ oo le'eg leebka $\vec{G}$; saar bar bilowgeeda, bar dhammaadka leebka G_2, ku xir bar-dhammaadka cusub ee R unugga kulammada dhidibyada.

Dhererka OW waa «c — xubinta x» ee G_2.
Dhererka WU waa «a — xubinta x» ee G_1.
Haddaba, dhererka OU waa «OW + WU xubinta x» ee leeb OK.

$$OU = OW + WU = c + a = a + c$$

Sidaas oo kale, $KU = b + d$.

Haddaba $OK = OF + OQ = G_2 + G_1$.

$OK = (a + c,\ b + d)$ isla sidaas $QF = G_2 - G_1$ markaas $[a + (-c),\ b + (-d)] = (a - c,\ b - d)$.

Waxaynu ku soo gabagabayn karnaa wadarta leebab laba aaddimoole waa leeb labo aaddimoole ah. Tiro kasta (a) oo maangal ahi waxay leedahay ama u jirta madiga (—a) oo tiro maangal ah, taasoo ah $a + (-a) = 0$. Haddaba, bal aynu u bixinno leebka $(-a, -b)$ leeb $-\vec{A}$. Waxaynu caddeyn doonnaa in $\vec{A} + (-\vec{A})$ ay la mid tahay (le'eg tahay) leeb eber.

Caddeyn :

Xulo leeb $\vec{A} = (a.b)$ iyo leeb $-\vec{A} = (-a, -b)$

$\vec{A} + (-\vec{A}) = [a + (-a),\ b + (-b)]$ Qeexda isugeyn-wadarta tiro maangal ah iyo weydaarka isugeynta waa 0 (eber).

$= (0, 0)$

$= 0$ Qeexda leeb eber.

Sidaas oo kale, waxaynu caddeyn karaa in $-\vec{A} + \vec{A} = 0$. Haddii lagu siiyo (a, b) waxaan madmadow kaaga jirin markaad fiiriso astaamaha tirada maangalka ah, in (—a, —b) ay madi tahay. Haddaba leeb kasta oo A; —A waa madi. Waxaynu ugu yeeri doonnaa weydaarka isugeynta ee $\vec{A}$.

Kala goynta Leebabka.

$\vec{A} - \vec{B}$ waxay la mid tahay $\vec{A} + (-\vec{B})$.

Leebab Isle'eg.

Leeb $\vec{A} = (a, b)$ iyo $\vec{B} = (c, d)$ way isle'eg yihiin haddii iyo haddii qura oo $a = c$, $b = d$.

1.3.2 XEERARKA ISUGEYNTA LEEBABKA

1. Ururka leebabka laba aaddimoole, wuu oodmaa isugeynta. Hadday A iyo B yihiin leebab labo aaddimoole A + B waa leeb laba aaddimoole ah.
2. Isugeynta leebabku way kala hormartaa $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.
3. Isugeynta leebabku way hormogashaa $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$.
4. Waxaa jira leeb eber 0, kaasoo leebkii kasta $\vec{A}$, ay $\vec{A} + 0 = 0 + \vec{A} = \vec{A}$.
5. Leeb kasta oo A, waxa uu leeyahay ama u jira leeb $(-\vec{A})$ kaasoo $\vec{A} + (-\vec{A}) = (-\vec{A}) + \vec{A} = 0$
6. Leebabka $\vec{A}$, $\vec{C}$, $\vec{D}$, haddii $\vec{C} = \vec{D}$ markaa $\vec{C} + \vec{A} = \vec{D} + \vec{A}$: isla markaa haddii $\vec{C} + \vec{A} = \vec{D} + \vec{A}$ markaa $\vec{C} = \vec{D}$.

Layli 1.3:

1) Haddii lagu siiyo leebabka $\vec{A} = (-3, 1)$ $\vec{B} = (-4, -2)$, $\vec{C} = (5, 7)$ iyo $\vec{D} = (0, -8)$; Raadi leebabka soo socda:
b) $\vec{A} + \vec{B}$ t) $\vec{A} + \vec{C}$ j) $\vec{C} + \vec{A}$ x) $\vec{A} + \vec{D}$
kh) $\vec{B} + \vec{C}$ d) $\vec{B} + \vec{D}$ r) $\vec{C} + \vec{D}$ s) $\vec{D} + \vec{C}$.
Jaantuus ku muujin (b) iyo (s).

2) Qor weydaarka isugeynta ee leebabka A, B, C iyo D ee masalada koowaad.

3) Goob x iyo y si ay labada leeg isu le'egkaadaan.

Tusaale :

$(-5, 3)$; $(x + 2, y - x)$.

Furfuris :

Laba leeb waxay isle'eg yihiin haddii xubnahoodu isle'eg yihiin.

Haddaba $x + 2 = -5$
$y - x = 3$

Markaa $x + 2 = -5$
$x = -7$

Dabadeed $y - x = 3$
$y - (-7) = 3$
$= -4$

Sidaa awgeed $x = -7$, $y = -4$.

b) $(11, 0)$; $(2x - 1, y + 5)$

t) $(2, -7)$; $(x - y, x + 2y)$

j) $(4, -9)$; $(x - 2y, 3x + 4)$

x) $(2x, x + 3y)$; $(-1, 1/4)$

kh) $(0, y - x)$; $(3x + 2y), -5)$

4) b. $(2x - 3, x + 5)$ iyo $(7, 2)$ ma isle'egkaan karaan?

t. $(3x - 1, 4x)$ iyo $(2, 3)$ ma isu noqon karaan weydaarka isugeynta.

5) Raadi x-da mid kasta oo kuwan soo socda ah, si ay A, B, C iyo D u noqdoon weydaarrada isugeynta, sida ay u kala horreeyaan, ee A, B, C iyo D ee weydiinta koowaad.

1.3.3 TARANTA LEEBABKA

1.3.3.1 **Ku Dhufasho Foolwaa.**

Marka aynu leebab ka hadlayno waxaynu u aqoonsanaanaa tirada maangalka ee caadiga ah foolwaa. Hadda aynu qeexno taranta ka soo baxa marka foolwaa lagu dhufto leeb.

Qeex: Haddii (a, b) leeb yahay, K-na foolwaa yahay, waxaynu u qeexi taranta K (a, b) in ay noqoto leebka (Ka, Kb).

Tusaale :

b) $2(1, -4) = (2, -8)$
t) $-1(2, 3) = (-2, -3)$
j) $c(2, 3) = (2c, 3c)$
x) $0(c, d) = (0, 0) = 0$

Marka joometeri ahaan loo muujiyo arrintani waxay sheegtaa, haddii m tahay tiro togan, jihada leebku isma beddelo, hase yeeshee dhererka ayuunbaa lagu dhuftaa m. Waxaynu ku fekeri karnaa in ku dhufasho foolwaa ay kala jiiddo ama isku rooriso leeb. Hadday m tahay tiro taban, waxay leebka ujeeddiisaa jihada tiisa ku lid ah (ka horjeedda).

Tusaalooyin ku dhufasho foolwaa ayaa ku muujisan shaxannada 1.3.3, 1.3.3.1 iyo 1.3.3.2.

Si tifaftiran haddii aynu isugu duno intii aynu kor ku soo sheegnay, waxaynu ka gaari doonnaa astaamaha soo socda, hadday m foolwaa tahay, A tahay leeb:

1) $m = 0$, mA waxay leebka A u roorisaa 0. (Leebkii way liqday).

2) $0 < m < 1$, mA jihada madooriso, leebkase way roorisaa.

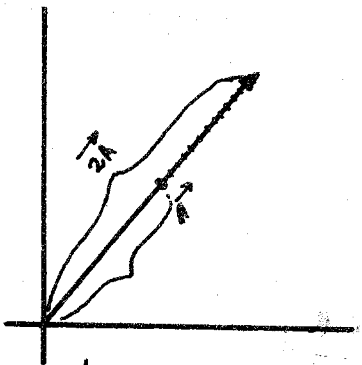
3) $m = 1$ mA jihaadka iyo laxaadka leebka madooriso.

4) $-1 < m < 0$, mA jihada way doorisaa leebkana way roorisaa.

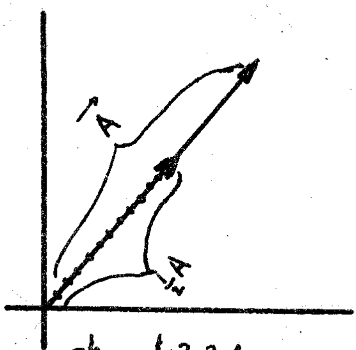
5) $m = -1$, jihada way doorisaa, laxaadka leebkase ma dooriso.

6) $m > 1$ leebka way kala jiidaa, jihadase ma dooriso.

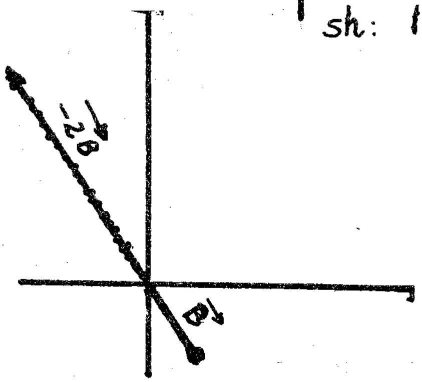
7) $m < -1$ leebka way kala jiidaa, jihadana way doorisaa.



sh: I.3.3



sh: I.3.3.1



Sh: I.3.3.2

Astaamaha Aljebrada hoose ee ku dhufashada foolwaa waxay ku jiraan aragtiinyada soo socda. Aragtiinyada, c iyo d waa tirooyin maangal ah, A iyo B waa leebabka.

1) $1\vec{A} = \vec{A}$

2) $c(d\vec{A}) = (cd)\vec{A}$

3) $c(\vec{A} + \vec{B}) = c\vec{A} + c\vec{B}$

4) $(c + d)\vec{A} = c\vec{A} + d\vec{A}$

5) $0\vec{A} = 0$

6) $(-c)\vec{A} = -c\vec{A}$

Waxaynu caddayn doonnaa Qaybta 3aad.

$C(A + B) = C[(a_1, a_2) + (b_1, b_2)]$ Midaal gaar loojik

$= C(a_1 + b_1,\ a_2 + b_2)$ Qeexda isugeynta leebabka

$= [C(a_1 + b_1,\ c(a_2 + b_2)]$ Qeexda ku dhufashada leebabka.

$= (ca_1 + cb_1,\ ca_2 + cb_2)$ Astaanta kala dhigga.

$= (ca_1, ca_2) + (cb_1, cb_2)$ Qeexda isugeynta laababka.

$= C(a_1, a_2) + (b_1, b_2)$ Qeexda ku dhufashada foolwaa.

$= CA + CB$ Midaal loojika.

Qaybaha kale layli ahaan baa ardayga loogu dhaafay.

Tusaale :

U qor leebabka soo socda saansaanka (a_1, a_2) oo ay a_1 iyo a_2 tirooyinka maangal ah yihiin.

b) $5(0, 1) + (-2)(6, -3)$

t) $2(-1, -2) + 6(-3, 0) + 0(7, 1)$

Furfuris b:

$$= (0, 5) + (-12, +6)$$
$$= [0 + (-12), 5 + 6]$$
$$= (-12, 11)$$

Furfuris t:

$$= (-2, -4) + (-18, 0) + (0, 0)$$
$$= (-2 + (-18) + 0, \quad -4 + 0 + 0$$
$$= (-20, -4)$$

1.3.3.2 Taran Dhexaad.

Hore waxaynu labadii leebba uga soo saaray mid saddexaad oo aynu niri waa wadartooda. Haddana waxaynutixgelin doonnaa xisaab falka, ku aaddiya foolwaa, lammaanihii leebab ahba. Foolwaaga waxa la yiraa **taran dhexaadkii leebabka.** Taranka ka soo baxa laba leeb waa fikrad kale oo in la qeexo u baahan. Run ahaantiina waxa jira saddex jaad, oo taran ah oo joogto ahaan loo adeegto, hase ahaatee, halkan waxaynu ku falanqeyn taran dhexaadka oo keliya.

Qeex :

Taran dhexaadka laba leeb A: (a_1, b_1) iyo B: (a_2, b_2) waxa lagu qeexaa in uu yahay foolwaaga $a_1 a_2 + b_1 b_2$.

Tarantan waxa lagu asteeyaa bar, markaas

$$A \cdot B = (a_1, b_1) \cdot (a_2, b_2) = a_1 a_2 + b_1 b_2$$

Tusaale :

b $(3, -2) \cdot (1, 4) = (3)(-1) + (-2)(4) = -5$

t. $(5, 2) \cdot (1, 1) = (5)(1) + (2)(1) = 7$

j. $(-4, 1) \cdot (0, 0) = (-4)(0) + (1)(0) = 0$

x. $(1, 0) \cdot (0, 1) = (1)(0) + (0)(1) = 0$

Taran dhexaadku wuxuu u hoggaansamaa xeerar-

1) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ Sharciga kale hormarinta.

2) $\vec{A} \cdot \vec{A} = 0$ Haddii iyo haddii qura oo ay $A = 0$

3) $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$ Sharciga kala dhigga.

4) $(K\vec{A}) \cdot \vec{B} = \vec{B} \cdot (K\vec{A}) = K(\vec{A} \cdot \vec{B})$

Waxaynu caddayn doonaa qaybta 4aad.

Caddeyn :

$(K\vec{A}) \cdot \vec{B} = K(a_1, a_2) \cdot (b_1, b_2)$ midaal loojik.

$= (Ka_1, Ka_2) \cdot (b_1, b_2)$ Qeexda ku dhufashada foolwaa.

$= Ka_1 b_1 + Ka_2 b_2)$ Qeexda taran dhexaadka.

$= K(a_1 b_1 + a_2 b_2)$ Astaanta kala dhigga, isugeynta ee isku dhufashada tirada maangalka ah.

$= K(a_1, a_2) \cdot (b_1, b_2)$ Qeexda taran dhexaadka.

$= K(\vec{A} \cdot \vec{B})$ Midaal loojik.

Isla markaana $(K\vec{A}) \cdot \vec{B} = \vec{B} \cdot (K\vec{A})$. Waayo mar hadday KA leeb ku noqotay qeexda ku dhufasho foolwaa, ee aynu ku xusnay cutubkan xubintiisa 1.3.3.1, marka la cuskado sharciga kala hormarnita ee xeerka Taran dhexaadka, $(K\vec{A}) \cdot \vec{B} = \vec{B} \cdot (K\vec{A})$. Tusaale ahaan hadday:

$\vec{A} = (3, 1)$, $\vec{B} = (2, -1)$, $\vec{K} = 2$, marka

$$K\vec{A} \cdot \vec{B} = 2\,(3, 1) \cdot (2, -1) = (6, 2) \cdot (2, -1)$$
$$= 12 + (-2) = 10.$$

Isla markaa $\vec{B} \cdot (K\vec{A}) = (2, -1) \cdot 2\,(3, 1)$

$$= (2, -1) \cdot (6, 2)$$
$$= 12 + (-2) = 10$$

Sidaa awgeed, $(KA) \cdot B = B(KA) = 10$. Qaybaha kale ardayga ayaa layli ahaan loogu dhaafayaa.

Layli 1.3.1:

1. U qor mid kasta oo leebabka soo socda ka mid ah saansaanka (a_1, a_2) oo ay a_1 iyo a_2 yihiin tirooyin maangal ah.

b) $6(1, 0) + 4(-2, 5)$

t) $1(-2, 1) + 0(6, -4)$

j) $8(1, -1) + 6(4, -3)$

x) $-2(7, 11) + (5(-3, 6)$

kh) $4(-3, -1) + -5(6, 0) + 7(8, -3)$

2. Fududee mid kasta oo kuwan soo socda ka mid ah.

a) $3(a, -b) + 6(2a, b)$
e) $-2(a, 0) - 5(0, b)$
i) $-2(x + y, -4) - 4(-2x, x - y)$
o) $-10(0, 0) + 2(x + y, x - y)$

3. Waa maxay qiimaha x iyo y si ay:

b) $x(2, -3) + y(-1, 0 \overset{?}{=} (0, -3)$

t) $x(-4, -8) + y(3, 3) \overset{?}{=} 1, 5)$

4. Soo saar taran dhexaadka

i) $(2, 1) \cdot (1, -2)$

ii) $(6, -2) \cdot (-2, 0)$

iii) $(1, 3) \cdot (2, -4)$

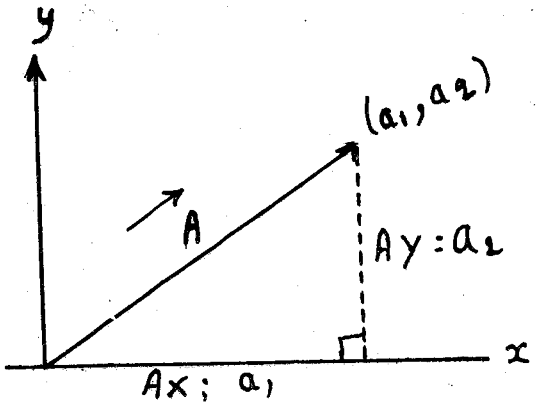
iv) $(4, -1) \cdot (-2, -1)$

5. Caddee in $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

1.4 LAXAADKA LEEBABKA

Markii aynu baranaynay sida Leeb loo muujiyo Joometeri ahaan, waxaynu u taagnay leebka xarriijin jihan. Fiiqda madax leebku wuxuu sheegayay jihada leebka; dhererka xarriijintuna waxay u taagneyd laxaadka leebka.

Saxanka 1.4 waxaynu ka aragnaa in aynu gargaarsan karno, Aragtiinka «BITAAGORAS» si aynu u helno dhererka leebka. Dhererka leebka A: (a_1, a_2) waxa lagu asteeya $|A|$ waana $\sqrt{a_1^2 + a_2^2}$. Haddii aynu u qeexno $\vec{A}x$ — xubinta x ee $\vec{A}$, $\vec{A}y$ — xubinta y ee A, waxaynu arkaynaa in $|a|^2 = \vec{A}x^2 + \vec{A}y^2$.



sh. 1.4

Waxaynu qeexi karnaa dhererka leebka innagoo cuskanayna taran dhexaadka leebabka.

Qeex :

Dhererka leebka (a, b) waa xididka laba jibbaarka taran dhexaadka (a, b) · (a, b) ee togan. Taasina waa dhererka $(a, b) = \sqrt{(a, b) \cdot (a, b)} = \sqrt{a^2 + b^2}$.

Tusaale :

b) Dhererka $(3, 4) = \sqrt{9 + 16} = \sqrt{25} = 5$

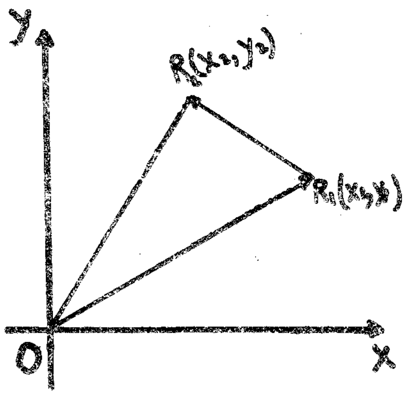
t) Dhererka $(1, 0) = \sqrt{1 + 0} = \sqrt{1} = 1$

j) Dhererka $(0, 0) = \sqrt{0 + 0} = \sqrt{0} = 0$

x) Dhererka $(3, -4) = \sqrt{9 + 16} = \sqrt{25} = 5$

kh) Dhererka $(-3, -4) = \sqrt{9 + 16} = \sqrt{25} = 5$

Laxaadka leebka aan bar bilowga ku lahayn unugga, waxa loo helaa sidan soo socota: ka dhig inay $R_1\,(x_1, y_1)$ iyo $R_2\,(x_2, y_2)$ yihiin laba barood (eeg shaxanka 1.4.1)



$$\begin{aligned} \text{Haddaba } R_1R_2 &= OR_2 - OR_2 \\ &= (x_2 - x_1) + (y_2 - y_1) \end{aligned}$$

Eegga, haddii aynu u qorno $A = R_1R_2$ oo aynu go'aankii horana waafajino, waxaynu heleynaa.

$$\begin{aligned} |A| = |R_1R_2|^2 &= Ax^2 + Ay^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \end{aligned}$$

ama

$$|R_1R_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\,.$$

Tani waa jidka fogaanta ee jometeriga «Saafan». Leebabka saddex-aaddimoole, taasi waxay noqotaa:

$$|R_1 R_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (w_1 - w_2)^2}$$

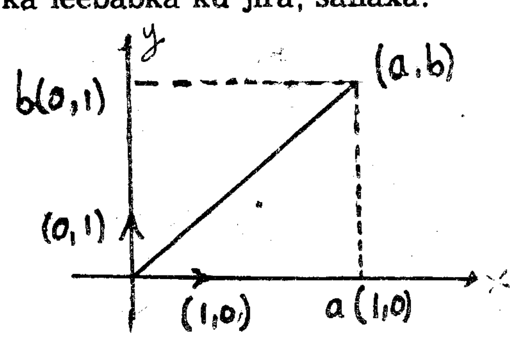
L a y l i 1.4:

1) $(-7, 0)$
2) $(4, -\sqrt{2})$
3) $(\sqrt{+2}, \sqrt{5})$
4) $(3, -2) + (-3, -2)$
5) $(1 - \sqrt{7}, 1 + \sqrt{7})$
6) $(4, \sqrt{2}) + (-3, -\sqrt{2})$
7) $(\sqrt{2}, \sqrt{5}) - 3(\sqrt{5}, -\sqrt{2})$
8) $(2 - \sqrt{6}, 3 + \sqrt{2}) + (-1 + \sqrt{6}, 4 - \sqrt{2})$
9) $(1, 2, 2)$
10) $(1, 3, -4)$

1.5 LEEBABKA BEEGALKA AH EE KU JIRA SALLAX

Leeb kasta oo ku jira sallax waxa loo dhigi karaa racayn toosan oo laba leeb. Taasi waxay tahay in leeb kasta oo (a, b) yahay wadarta taran dhexaad $(1, 0)$ iyo $(0, 1)$. $[(a, b)] = a(1, 0) + b(0, 1)$.

Ururka leebabka $\{(1, 0), (0, 1)\}$ waxa la yiraa: **gundhiga G** ee ururka leebabka ku jira, sallaxa.



Ururka $\{(1,0), (0,1)\}$ waxa la yiraa **gundiga beegalka** ee **G.** Leeb kasta oo ka mid ah kutirsanayaasha ururkaa waxa loo yaqaan Halbeeg Leeb, waayo laxaadkiisu waa 1, (kow). Waxaynu kan ku soo aragnay cutubkan xubintiisa 1.4. Qormada beegalka ah ee gundhigyada leebku waa $(1,0) = I$ iyo $(0,1) = J$. Markaa $(a,b) = ai + bj$. Haddaba leeb kasta oo lagu siiyo waxa lagu dhigi karaa gundhiyadaas leebabka. Tusaale ahaan $(4,2) = 4i + 2j$. Bal haddaba aynu gargaarsanaba gundhiyada beegalka ah.

Tusaale:

$$\vec{A} = (a,b) \cdot \vec{B} = (c,d)$$
$$\vec{A} + \vec{B} = (a+c,\ b+d)$$

Gargaarsiga gundhiyada waa:

$$\vec{A} = (a,b) = ai + bj$$
$$\vec{B} = (c,d) = ci + dj$$
$$\vec{A} + \vec{B} = (a+c,\ b+d) = (a+c)\,i + (b+d)\,j.$$

1.6 TARAN DHEXAADKA KU JIRA SALLAXA

Waxaynu cutubkan xubintiisa 1.4 ku soo qaadanay gundhigyada leebka ku jira sallaxa. Bal eegga aynu faaqidno taran dhexaadkooda.

$$\vec{i} \cdot \vec{i} = (1,0) \cdot (1,0) = 1 + 0 = 1$$
$$\vec{j} \cdot \vec{j} = (0,1) \cdot (0,1) = 0 + 1 = 1$$
$$\vec{i} \cdot \vec{j} = (1,0) \cdot (0,1) = 0 + 0 = 0$$

Haddaba waxaynu qeexi in

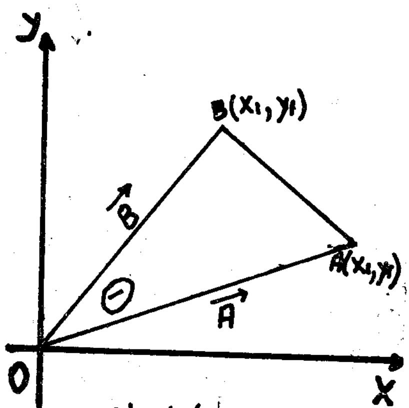
$$\vec{i} \cdot \vec{i} = 1;\ \vec{j} \cdot \vec{j} = 1 \text{ iyo } \vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{i} = 0.$$

Tusaale:

$$(ai + bj) \cdot (ci + dj) = ac + bd.$$

Haddaba waxaynu joognaa heer aynu si dhab ah u sharaxno turjumadda Joometeriga ah ee taran dhexaadda. Waxaynu naqaan haddii ay $A = (a_1, b_1) \cdot B = (a_2, b_2)$ in ay $A \cdot B = a_1 a_2 + b_1 b_2$.

Dhis (washir) saddexagalka OAB sida Shaxan 1.6 ku tusan.



Sh: 1.6

Haddaba cusko sharciga kosaynta ee

$$|AB|^2 = |OA|^2 + |OB|^2 - 2|OA|\,|OB| \operatorname{Cos} \Theta.$$

Adeegashada jid fooganta ee cutubkan xubintiisa 1.5 waxaynu tani u qori karnaa:

$$\begin{aligned}
|AB|^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\
&= x_2^2 - 2x_1 x_2 + x_1^2 + y_1^2 - 2y_1 y_2 + y_1^2 \\
&= (x_1^2 + y_1^2) + (x_2^2 + y_2^2) - 2\,(x_1 x_2 + y_1 y_2) \\
&= |OA|^2 + |OB|^2 - 2\,(OA \cdot OB)
\end{aligned}$$

Haddaynu isle'egkaysiino labada tibaaxood ee $|AB|^2$ waxaynu heli $-2\,(OA \cdot OB) = -2\,|OA|\,|OB| \operatorname{Cos} \Theta$, taasoo ah in $A \cdot B = |A|\,|B| \operatorname{Cos} \Theta$. Hawraar ahaan wax kaloo aynu oran karnaa in taran dhexaadka laba leeb uu yahay taranta dhererkooda oo lagu dhuftay kosaynka xagal dhexaadka. Jidkan cusubi wuxuu ina siiyaa hab fudud oo habboon laguna heli karo xagasha u dhexaysa laba leeb oo aan ahayn leeb eberro, markaa xubnaha leebabka la ogyahay, taasina waa

$$\vec{A} \cdot \vec{B} = |\vec{A}|\ |\vec{B}| \operatorname{Cos} \Theta$$

$$\frac{\vec{A} \cdot \vec{B}}{|A|\ |B|} = \operatorname{Cos} \Theta.$$

$$\therefore \quad \operatorname{Cos} \Theta = \frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2}\ \sqrt{x_2^2 + y_2^2}}$$

Dheeho taranta $|A|\ |B| \operatorname{Cos} \Theta$ waa taran saddex tiro oo maangal ah mar hadday $|A|\ |B| \operatorname{Cos} \Theta = 0$. Oraahyadan soo socda ugu yaraan mid uunbaa run ah $|A| = 0$; $|B| = 0$ ama $\operatorname{Cos} \Theta = 0$, kol hadday A iyo B ahayn leeb eber, $\operatorname{Cos} \Theta = 0$, oo $0 \leq \Theta \leq 180°$; dabadeeto $\Theta = 90°$. Hadday $|A| = 0$, A waa leeb eberka, weliba hadday $|B| = 0$, $\vec{B}$ waa leeb eberka. Haddii aynu ku heshiinno in leeb eberku ku qotomo leeb kasta, waxaynu heli karnaa qeexda soo socota.

Leebab isku qotoma

Qeexid :

Laba leeb oo ah $\vec{A}$ iyo $\vec{B}$ way isku qotomaan, haddii iyo haddii qura ah oo taran dhexaadka $\vec{A} \cdot \vec{B}$ ay tahay eber.

Tusaale :

Ku raadi digriiga ugu dhow xagasha u dhexaysa lammaankii leebab ahba.

b. (2, 1) iyo (3, 6)
t. (−1, 2) iyo (2, 1)

Furfuris 1:

b) $\vec{A} \cdot \vec{B} = (2, 1) \cdot (3, 6)$
$= 6 + 6 = 12$

$$\text{Cos } \Theta = \frac{12}{\sqrt{5}\ \sqrt{45}} = \frac{12}{15}$$
$$= 0.8000$$

$\therefore \quad \Theta = 36°$ waa digrii ugu dhow.

Furfuris 2:

t) $\vec{A} \cdot \vec{B} = (-1, 2) \cdot (2, 1)$
$= -2 + 2 = 0$

$$\text{Cos } \Theta = \frac{0}{\sqrt{5}\ \sqrt{5}} = 0$$

$\therefore \quad = 90°$

Laylis 1.6:

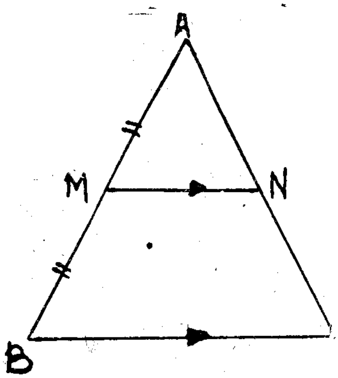
1) Raadi taran dhexaadka mid kasta oo lammaan-
   - b) $-2j$ iyo $-3i$
   - t) $5i - 5j$ iyo $3j$
   - j) $2i + 6j$ iyo $-5i + 5j$
   - x) $-3i - 4j$ iyo $3i + 4j$
2) Soo saar Kosaynka xagasha u dhexaysa lammaanka leebabka ah ee weydiinta koowaad.
3) Leebabka soo socda lammaankeebaa isku qorma:
   - b) $(3, 1)$ iyo $(-1, 3)$
   - t) $(4, 0)$ iyo $(0, 2)$
   - j) $(-5, -2)$ iyo $(4, 10)$
   - x) $(0, 0)$ iyo $(6, 3)$
4) Caddey haddii $|\vec{B} - \vec{A}|^2 = |\vec{A}|^2 + |\vec{B}|^2$ in $\vec{A} \perp \vec{B}$.

1.7 KU MIDIISIGA JOOMETERIGA

Mararka qaarkood fikradda leebabka waxay ina awood siiyaan caddeynta Aragtiino badan oo Joometeri ah, sida kuwan ku jira tusaalooyinkan.

Tusaale 1:

Xarriiqda marta bar dhexaadka hal dhinac oo saddexagal oo dhinaca labaadna barbarro laha, way kala badhaa dhinaca saddexaad.



sh 1.7

$\overrightarrow{MN} = \frac{1}{2}\,\overrightarrow{BC}$ Ka timid Saddexagallo isu'eg, Eegga

$\overrightarrow{MN} = \frac{1}{2}\,\overrightarrow{BA} + \overrightarrow{AN}$

$\frac{1}{2}\,\overrightarrow{BC} = \frac{1}{2}\,\overrightarrow{BA} + \overrightarrow{AN}$

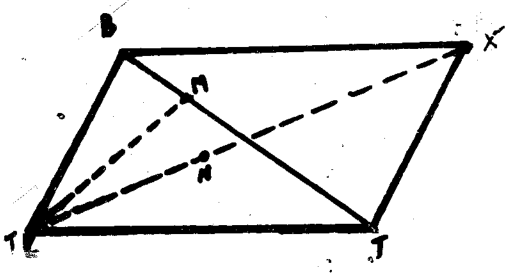
$\overrightarrow{AN} = \frac{1}{2}\,(\overrightarrow{BC} - \overrightarrow{BA})$

$= \frac{1}{2}\,(\overrightarrow{BA} + \overrightarrow{AC} - \overrightarrow{BA})$ Haddaba N waa bar bartanka AC.

$= \frac{1}{2}\,\overrightarrow{AC}.$

Tusaale 2:

Caddee: xaglogooyaasha barbarroole way is kala baraan.



Caddeyn :

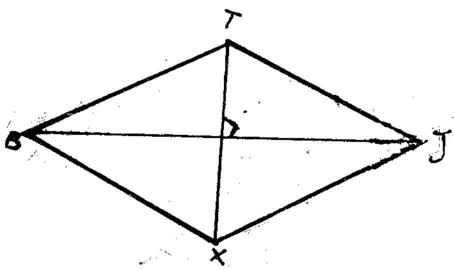
Ka dhig in M iyo N yihiin baro bartannada BJ iyo TX sida ay u kala horreeyaan, dabadeeto:

$$\begin{aligned} TM &= XJ + \tfrac{1}{2} JB = XJ + \tfrac{1}{2} (JX + XB) \\ &= (XJ - \tfrac{1}{2} XJ) + \tfrac{1}{2} XB \\ &= \tfrac{1}{2} (XJ + XB) = \tfrac{1}{2} (XJ + JT) \\ &= \tfrac{1}{2} XT. \end{aligned}$$

Haddaba XM = XN, marka M iyo N waa isdhuldhac.

Tusaale 3:

Caddee: Xaglogooyaasha Qardhaasto way isku qotomaan.



Caddeyn:

$$BJ = BX + XJ$$

$$TX = TJ + JX = BX - XJ$$

$$BJ \cdot TX = (BX + XJ) \cdot (BX - XJ)$$

$$= |BX|^2 - BX \cdot XJ + XJ \cdot BX - |XJ|^2$$

$$= |BX|^2 - |XJ|^2$$

$= 0$ Mahadhada Qardhaasta (dhibicyadu way isle'eg yihiin).

Sidaa awgeed $BJ = TX$.

Layli 1.7:

Caddee:

1) Haddii xaglogooyaasha laydi ay isku qotomaan, laydigu waa laba jibbaarrane.

2) Xarriiqda isku xirta baro bartanka laba dhinac oo saddexagal waa la barbarro dhinaca saddexaad, waana dhererkeeda barkeed.

3) Haddii xaglogooyaasha Afargeesle uu midba midka kale kala badho, afargeesluhu waa barbarroole.

4) Dhexfurka salka saddexagal labaale, wuxuu ku qotomaa salka.

5) Dhexfurrada Saddexagal waxay ku kulmaan bar taasoo dhexfur kasta Saddexgoysa.

CUTUB V

TAXANEYAAL

Qaybtan waxaan ka baranaynaa fikrad xisaab ah oo la yiraahdo **Taxaneyaal**, taasoo waxtar joogto ah u leh furfurista habdhiska isle'egyada toosan. Taxaneyaalka siyaale kale oo badanna waa loogu shaqaysan karaa:

Qeex :

Taxane waa teed laydi oo ka kooban m dhinactax iyo n joogtax oo tirooyin maangal ah. Taxane waxaa aalaaba lagu muujiyaa tibixda $m \times n$ (loo akhriyo «ma, na») m waxay u taagan tahay inta dhinactax ee taxanuhu leeyahay, n inta joogtax ee taxanuhu leeyahay. Haddii $m = n$, taxanaha waxa la yiraahdaa **Taxane Labajibbaarane ah.** Taxane waxa lagu dhex xiraa laba bilood ama laba sakal.

Tusaalooyinka soo socdaa waa taxaneyaal:

b) $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ 3×1 — b. Taxane 3-dhinactax 1-joogtax ah.

t) $\begin{bmatrix} 2 & 4 \\ 3 & 0 \end{bmatrix}$ 2×2 — t. Taxane 2-dhinactax 2-joogtax ah.

j) $(1\ 2\ 0\ 4)$ 1×4 — j. Taxane 1-dhinactax 4-joogtax ah.

x) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ — x. Taxane 2-dhinactax 4-joogtax ah.

Taxanihii hal dhinactax keliya leh waxa la yiraa **Taxane dhinactax.** Markuu hal joogtax keliya leeyahayna waxaa la yiraa **Taxane joogtax.** Kujirayaalka midmidka ah ee taxanuhu ka kooban yahay waxa la yiraa

Kutirsanayaal. Taxanaha waxaa lagu magacaaba xaraf weyn, sida B, T, J iwm. ama summadda B m x n, taasoo u taagnaan karta taxane kasta oo leh m-dhinactax iyo n-joogtax. Hoos dhiga (muujiyaha) m $\times$ n wuxu u taagan yahay aaddimaha ama heerka taxanaha.

Inta kutirsane ee taxanuhu leeyahay waxa lagu helaa m oo lagu dhuftay n. Haddaba, haddaan u noqonno tusaalaha kor ku qoran, waxaan aragnaa in: taxanaha (b) u leeyahay aaddimo ah 3×1; kan (t) 2×2; kan (j) 1×4; kan (x) uu leeyahay 2×4 sidaas oo kale taxanihii ah heerka m $\times$ n wuxuu ka kooban yahay taxaneyaal ah m-dhinactax iyo taxaneyaal ah n-joogtax.

T u s a a l e 1:

Qor taxenaha $B_{2\times3}$. Waa hubaal ni $B_{2\times3}$ uu leeyahay $2 \times 3 = 6$ kutirsaneyaal.

F u r f u r i s :

$B_{2\times3} = \begin{bmatrix} b_1 & b_2 & b_3 \\ t_1 & t_2 & t_3 \end{bmatrix}$ B waxay leedahay laba taxane dhinactax oo ah ($b_1\, b_2\, b_3$) iyo ($t_1\, t_2\, t_3$) iyo saddex taxane oo ah joogtax.

$\begin{bmatrix} b_1 \\ t_1 \end{bmatrix}$, $\begin{bmatrix} b_2 \\ t_2 \end{bmatrix}$ iyo $\begin{bmatrix} b_3 \\ t_3 \end{bmatrix}$

Si ballaaran taxaneyaalka waxa loogu isticmaalaa ugga jebayto qoridda sida tan soo socota oo kale:

Tusaale: Shirkad baabur sameysaa waxay soo saartaa basas, laandaroofarro iyo fatuurado oo casaan, madow iyo buluug isugu jira. Waxan laga yaabaa in wakaalad dalaal ahi ku muujiso baabuurtaa iibka ah tuse sida midka hoos ku qoran. Tusuhu waa taxane 3×3 ah. U fiirso in taxane kasta ee dhinactax ahi uu muujinaayo inta baabuur ee isku jaadka ah kalase midab ah.

	Basas	Laanda-roofar	Fatuu-rado
Cas	16	30	4
Buluug	20	25	15
Madow	8	5	12

Haddii qof doonayo in uu ogaado inta baabuur cas iib ah, waxaa ku filan in uu isugeeyo kutirsanayaasha dhinactaxa koowaad; kuwaasoo ah $16 + 30 + 4 = 50$.

Waxaan taxane guud kutirsaneyaalkiisa u joojinaa xaruuf yaryar; dhinactax walbana waxaan u qaadannaa xaraf gaar ah oo leh hoos dhig qura oo muujinaya joogtaxa gaar ee kutirsanahaas ku jiro. (Fiiri tusaalahan 2aad) hase ahaatee, kutirsaneyaalka joogtax waan u qaadan karnaa xaraf gaar ah oo hoos dhiggiisu muujinayo dhinactaxa laga helo. Labada dariiqoba waxay keenayaan dhibaato haddii kutirsaneyaalka taxanaha laga helayaa ay farabataan, maxaa yeelay xuruuftaa naga madhan. Haddaba dariiqo dhibaatadaa looga bixi karaa waa innagoo kutirsaneyaalka taxanaha oo idil u qaadanna xaraf qura oo laba hoos dhig leh, hoos dhigga hore oo muujinaaya dhinactaxa kutirsanahaasu ku jiro kan dambena joogtaxa uu ku jiro.

Tusaale 2:

Taxanaha guud oo heerka 2×3 waxa loo qori karaa

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Tusaale 3:

Taxanaha guud oo heerka $m \times n$ waxa loo qori karaa

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{mn} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix}$$

waxaana loo soo gaabiyaa (a_{ij}) marka $i = (0, 1, 2, ...m)$; $j = (1, 2, 3, ..., n)$ imika waxaan tixgelineynaa eraybixinta taxaneyaalka.

Taxane madhan.

Haddii taxane kutirsaneyaalkiisu dhammaan yihiin, eber, taxanaha waxaa la yiraa **taxane madhan** ama **taxane eber**, waxaana lagu asteeyaa $0\ m \times n$.

Melmel Taxane.

Melmelka taxane B, loona qoro B^m (u akhri «melmel B») waxa weeye taxane cusub oo ka yimid, B oo dhinactaxyadeeda iyo joogtaxyadeeda la isku beddelay.

Isle'ekaanshaha Taxaneyaal.

Laba taxane B iyo T oo isla aaddima ah waa isle'eg yihiin haddii iyo haddii qura oo kutirsaneyaashoodu isku beegan isle'eg yihiin.

Tusaale 4:

Haddii $B = T$; $B = \begin{pmatrix} b_1 & b_2 & b_3 \\ t_1 & t_2 & t_3 \end{pmatrix}$, $T = \begin{pmatrix} j_1 & j_2 & j_3 \\ d_1 & d_2 & d_3 \end{pmatrix}$

Markaa isle'ekaanshaha soo socdaa waa inuu jiraa:

$$b_1 = j_1 \qquad t_1 = d_1$$

$$b_2 = j_2 \qquad t_2 = d_2$$

$$b_3 = j_3 \qquad t_3 = d_3$$

OGOW: Sidey qeexda sare sheegayso, laba taxane ismale'eka, haddii aanay isku aaddimo ahayn,

Layli :

1) Sheeg aaddimaha taxaneyaalka soo socda.

b) $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 4 \\ 2 & 4 & -1 \end{bmatrix}$ t) $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

j) $\begin{bmatrix} 0 & -7 \\ 0 & -7 \\ 0 & -4 \end{bmatrix}$ x) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & -1 & 0 & 6 \end{bmatrix}$

kh) $\begin{bmatrix} 25 \\ 38 \end{bmatrix}$

2) Qor melmelka taxaneyaalka 1b - 1kh ee layliga 1aad.

3) Ka jawaab kuwa soo socda:

ka dhig B = $\begin{matrix} 5 & 6 & 1 & 2 \\ 2 & 3 & 4 & 0 \\ 10 & -1 & 8 & 7 \end{matrix}$

b) Sheeg kutirsaneyaalka dhinactaxa ugu dambeeya.

t) Sheeg aaddimmada B.

j) Sheeg kutirsaneyaalka dhinactaxa labaad iyo kuwa joogtaxa ugu dambeeya.

x) Qor melmelka B.

4) Qor taxane madhan oo la aaddimo ah B.

5) Soo saar x, y iyo w.

b) $\begin{bmatrix} x & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 3 & 4 \end{bmatrix}$

t) $\begin{bmatrix} 8 & y \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 4 & 3 \end{bmatrix}$

j) $\begin{bmatrix} y & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 3 \end{bmatrix}$

x) $\begin{pmatrix} 2 & 0 & 1 \\ w & x & y \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 4 & -1 \end{pmatrix}$

kh) $\begin{pmatrix} x & 4 & 1 \\ 2 & -6 & 0 \end{pmatrix} = \begin{pmatrix} y & 4 & 1 \\ 2 & y & 0 \end{pmatrix}$

d) $\begin{pmatrix} x & 6 \\ y & -8 \end{pmatrix} = \begin{pmatrix} 21 & y \\ 6 & -1 \end{pmatrix}$

ISUGEYN TAXANEYAAL

Wadarta laba taxane, B iyo T, oo isku aaddimo ah waxa loo joojiyaa taxane qura, $(B + T)m \times n$ kaasoo kujirihiisa dhinactax i-da iyo joogtaxa j-da yahay $b_{ij} + b_{ij}$ marka $i = (1, 2, 3, \ldots, m)$, $j = (1, 2, 3, \ldots, n)$. Taasu waxay tahay in kutirsaneyaalka B iyo T ee isku beegan la isugeyay. Haddaba taxanaha soo baxaa waa isugeynta B iyo T ee la doonayay.

Tusaale 5:

Ka soo qaad $B = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 4 & 3 \end{pmatrix}$; $T = \begin{pmatrix} 4 & 0 & 5 \\ 6 & 1 & 2 \end{pmatrix}$

$$(B + T) = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 4 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 5 \\ 6 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+4 & 3+0 & 2+5 \\ 0+6 & 4+1 & 3+2 \end{pmatrix} = \begin{pmatrix} 5 & 3 & 7 \\ 6 & 5 & 5 \end{pmatrix}$$

U fiirso inay B iyo T isku aaddimo yihiin.

Wadarta $B_{m \times n} + (- T_{m \times n})$ waxaa la yiraa **Faraqa** $B_{m \times n}$ iyo $T_{m \times n}$ waxaana loo qoraa $B_{m \times n} - T_{m \times n}$; markaan, waa in kujirayaalka $B_{m \times n}$ laga gooyaa kuwa $T_{m \times n}$ ee ku beegan.

ASTAAMAHA ISUGEYNTA TAXANEYAAL

Haddii B, T iyo J ay yihiin taxanyaal heerka $m \times n$, marka:

1. $(B + B)_{m \times n}$ waa taxane leh kutirsaneyaal maangal ah. Oodanta isugeynta.

2. $(B + T) + J = B + (T + J)$ Hormogelinta isugeynta.

3. Taxanaha $O_{m \times n}$ wuxuu astaan u leeyahay, haddii $O_{m \times n}$ loo geeyo taxane kasta $B_{m \times n}$ in:
$B_{m \times n} + O_{m \times n} = O_{m \times n} + B_{m \times n} = B_{m \times n}$ Asal madoorshaha isugeynta.

4. Taxane kasta $B_{m \times n}$ waxaa ku beegan taxanaha $-B_{m \times n}$ kaasoo leh astaan ah:
$B + (-B) = (-B) + B = 0$ Isweydaarka isugeynta.

Tabanaha taxanaha $B_{m \times n}$ waa taxanaha $-B_{m \times n}$ kaasoo kutirsaneyaalkiisu yihiin, tabanaha kutirsaneyaalka $B_{m \times n}$ ee ku beegan.

Tusaale 6:

1) Haddii $B = \begin{bmatrix} b & t \\ j & d \end{bmatrix}$, marka $-B = \begin{bmatrix} -b & -t \\ -j & -d \end{bmatrix}$

waayo

$$B + (-B) = \begin{bmatrix} b & t \\ j & d \end{bmatrix} + \begin{bmatrix} -b & -t \\ -j & -d \end{bmatrix}$$

$$= \begin{bmatrix} b-b & t-t \\ j-j & d-d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Guud ahaan, sida tirooyinka maangal marka taxanaha $B_{m \times n}$ laga gooynayo taxanaha $T_{m \times n}$, macnuhu waxa weeye adoo $-B_{m \times n}$ u geeya $T_{m \times n}$. Marka aynu ka hadlayno taxaneyaal, tira kasta oo maangal ah (sida r) waxaan niraahnaa **Foolwaa.** Taranka foolwaaga r iyo taxane waxay la mid tahay iyadoo kujire kasta oo taxanaha lagu dhufto foolwaaga r. Haddii $B_{m\ n}$ uu taxane yahay, markaa taranka $B_{m\ n}$ iyo r waa $r \cdot B_{m\ n}$.

Tusaale 7:

$$r \cdot \begin{pmatrix} b_1 & b_2 \\ t_1 & t_2 \end{pmatrix} = \begin{pmatrix} rb_1 & rb_2 \\ r\,t_1 & r\,t_2 \end{pmatrix}$$

$$3 \cdot \begin{pmatrix} 4 & 3 \\ -1 & 6 \end{pmatrix} = \begin{pmatrix} 3 \cdot 4 & 3 \cdot 3 \\ 3 \cdot (-1) & 3 \cdot 6 \end{pmatrix} = \begin{pmatrix} 12 & 9 \\ -3 & 18 \end{pmatrix}$$

Layli :

I. Hel taxane qura oo le'eg kuwa soo socda:

1) $\begin{pmatrix} 2 & 5 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix}$

2) $\begin{pmatrix} 8 & 9 \\ 4 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 1 \\ 2 & 6 \end{pmatrix}$

3) $\begin{pmatrix} 4 & 5 & 6 \\ 7 & 8 & 1 \\ 2 & 2 & 2 \end{pmatrix} - 2 \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 8 \\ 6 & 4 & 5 \end{pmatrix}$

4) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - 4 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

II. Isle'egyada soo socda u furfur taxane doorsoome.

Tusaale 1:

$\begin{pmatrix} 4b & 4t \\ 4r & 4d \end{pmatrix} = \begin{pmatrix} 12 & 16 \\ 8 & 20 \end{pmatrix}$ Taxanaha la doonaayaa waa $\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$

Furfuris :

$$4b = 12 \quad 4t = 16 \quad 4r = 8 \quad 4d = 20$$
$$b = 3 \quad t = 4 \quad r = 2 \quad d = 5$$

Marka laga shaqaynayo furfurista layliyadan oo kale, ugu horrayn isle'egkeysii kujirayaalka isku beegan ee labada taxane, dabadeed qabo wixii fal ah oo loo baahan yahay ilaa iyo inta taxanaha doorsoome uu le'egkaanayo taxane kale.

Tusaale 2:

$$4\begin{pmatrix} b & t \\ j & d \end{pmatrix} - 2\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} = 5\begin{pmatrix} 0 & 3 \\ 4 & 5 \end{pmatrix}$$

1. Foolwaaga ku dhufo taxane kasta:

$$\begin{pmatrix} 4b & 4t \\ 4j & 4d \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ -4 & -8 \end{pmatrix} = \begin{pmatrix} 0 & 15 \\ 20 & 25 \end{pmatrix}$$

2. U gee isweydaarka $\begin{pmatrix} -2 & 0 \\ -4 & -8 \end{pmatrix}$ Dhinac kasta ee isleegta.

$$\begin{pmatrix} 4b & 4t \\ 4j & 4d \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ -4 & -8 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 0 & 15 \\ 20 & 25 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 4 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 4b & 4t \\ 4j & 4d \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 15 \\ 24 & 31 \end{pmatrix}$$

3. U furfur b, t, j iyo d.
$4b = 2 \quad 4t = 15 \quad 4j = 24 \quad 4d = 31$

$$b = \frac{1}{2} \qquad t = \frac{15}{4} \qquad j = 6 \qquad d = \frac{31}{4}$$

Taxanaha la doonayey waa $\begin{pmatrix} \frac{1}{2} & \frac{15}{4} \\ 6 & \frac{31}{4} \end{pmatrix}$

1. $\begin{pmatrix} x & y \\ w & h \end{pmatrix} + 3\begin{pmatrix} 2 & 5 \\ -3 & 4 \end{pmatrix} = 4\begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}$

2. $\begin{pmatrix} x & y \\ w & h \end{pmatrix} - 2\begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix} = 3\begin{pmatrix} -5 & 6 \\ 8 & 10 \end{pmatrix}$

3. $\begin{pmatrix} b & t & j \\ d & r & s \end{pmatrix} + 3\begin{pmatrix} 0 & -1 & 6 \\ 4 & 3 & 2 \end{pmatrix} = -1\begin{pmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

III. Hel wadarta

1) $\begin{pmatrix} 2 & 1 & -2 \\ 4 & 0 & -4 \end{pmatrix} + 2\begin{pmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \end{pmatrix}$

2) $(2 \ \ -2 \ \ 4) + 5\,(3 \ \ 8 \ \ -7)$

3) $\begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \\ -5 \end{pmatrix}$

4) $3\,(1 \ \ 0 \ \ 4) + (9 \ \ 4 \ \ 0)$

5) $\begin{pmatrix} -1 & -3 & -4 \\ 2 & 4 & 5 \\ 3 & 6 & 7 \end{pmatrix} + \begin{pmatrix} -6 & 2 & 5 \\ -8 & 7 & 1 \\ 0 & 4 & -5 \end{pmatrix}$

IV. Ku caddee tusaale. Haddii $B_{3\,3} = T_{3\,3}$. Markaa, $B_{3\,3} + J_{3\,3} = T_{3\,3} + J_{3\,3}$.

OGOW: Astaamaha soo socdaa waa qaar ka mid ah astaamaha Aljebra ee iskudhufashada foolwaa iyo Taxane. Haddii B iyo T ay yihiin taxaneyaalka heerka $m \times n$ c iyo d ay tiro maangal yihiin.

Haddaba:

b) cB waa taxane $m \times n$ ah.
t) $c\,(dB) = (cd)\,B$
j) $(c + d)\,B = cB + dB$
x) $c\,(B + T) = cB + cT$
kh) $1B = B$
d) $0B = 0$

Astaamaha kor ku qoran caddeyntooda layli u qaado waa fudud yihiine.

ISKU DHUFASHADA TAXANEYAASHA

Ka soo in ay shirkadi leedahay Fatuurado, Basas iyo Xamuulqaadyo Siisowyo, kana soo qaad in midabkoodu yihiin:

	fatuurado	**basas**	**xamuul-qaadyo**
Buluug	15	25	5
Casaan	10	10	15
Madow	20	5	10

Ka dhig in fogaanta celceliska ee baabuurkiiba maalintii gooyo tahay: buluug 30 Km.; cas 60 Km.; madow 75 Km.

Wadarta fogaaneed ee fatuuraduhu maalintii gooyaan waa:

$$30 \times 15 + 60 \times 10 + 75 \times 20 = 2550 \text{ Km.}$$

tan basaskuna waa:

$$30 \times 25 + 60 \times 10 + 75 \times 5 = 1725 \text{ Km.}$$

iyo tan xamuulqaadayda oo ah

$$30 \times 5 + 60 \times 15 + 75 \times 10 = 1800 \text{ Km.}$$

Shaqadaas waxaa loo qaban karaa sidan:

$$(30 \ 60 \ 75) \begin{bmatrix} 15 & 25 & 5 \\ 10 & 10 & 15 \\ 20 & 5 & 10 \end{bmatrix} = (2550 \ 1725 \ 1800)$$

Habkaas tixraaciisa waxaan ka ogaaneynaa in:

1. Taxanaha bidixdu yahay 1×3 kan midigtuna yahay 3×3. Sida muuqata iskudhufashada taxaneyaal uma baahna aaddimo isle'eg.

2. Tirada joogtaxyada ee taxanaha bidix waxay le'eg tahay tirada dhinac u taxyada taxanaha midigta.

3. Taranku wuxuu le'eg yahay inta dhinactax ee taxanaha bidixdu leeyahay iyo inta joogtax ee kan midigtu leeyahay.

4. Sida la isugeynayo tarannada laga helay isku dhufashada kutirsanayaalka dhinactax taxane iyo kutirsaneyaalka ku beegan ee joogtaxa taxanaha kale, waxay ina siinaysaa macne buuxa oo aynu u eegno taranka laba taxane sida hoos ku muujisan.

$$\begin{pmatrix} b & t \\ j & d \end{pmatrix} \begin{pmatrix} x & s \\ w & y \end{pmatrix} = \begin{pmatrix} bx + tw & bs + ty \\ jx + dw & js + dy \end{pmatrix}$$

$$jx + dw$$

$$js + dy$$

U fiirso: In kutirsanaha dhinactaxa labaad joogtaxa koowaad ee taranka lagu helay isku dhufashada dhinactaxa labaad ee taxanaha bidixda iyo joogtaxa koowaad ee taxanaha midigta, dabadeedna la isugeyay. Ma aragta sida kutirsanaha dhinactaxa 2aad ee taxanaha taranka loo helay.

Tusaale 8:

$$(1 \quad 3 \quad -1) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 3 \cdot 2 + (-1) \cdot 3) = (4)$$

Deris (baro) labadan tusaale ee isku dhufashada taxanyaal.

Tusaale 1:

Ka dhig in $B_{2\ 2} = \begin{pmatrix} b_1 & b_2 \\ t_1 & t_2 \end{pmatrix}$, $T_{2\ 2} = \begin{pmatrix} j_1 & j_2 \\ d_1 & d_2 \end{pmatrix}$

$$(BT)_{2\ 2} = \begin{pmatrix} b_1 & b_2 \\ t_1 & t_2 \end{pmatrix} \begin{pmatrix} j_1 & j_2 \\ d_1 & d_2 \end{pmatrix} = \begin{pmatrix} b_1j_1 + b_2d_2 & b_1j_2 + b_2d_2 \\ t_1j_1 + t_2d_1 & t_1j_2 + t_2d_2 \end{pmatrix}$$

Tusaale 2:

Haddii $B = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$, $T = \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix}$, markaa

$$(BT) = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot (-3) + 3 \cdot 4 & 1 \cdot 2 + 3 \cdot 1 \\ (-2)(-3) + 1 \cdot 4 & -2 \cdot 2 + 1 \cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 5 \\ 10 & -3 \end{bmatrix}$$

Laakiinse $(TB) = \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -3 + (-4) & -9 + 2 \\ 4 + (-2) & 12 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -7 \\ 2 & 13 \end{bmatrix}$$

Sidaas daraadeed tusaalahani wuxuu caddeynayaa in isku dhufashada taxanyaal aanay, guud ahaan, kala hormarin. Bal aynu u diyaar noqonno qeexda isku dhufashada taxanayaal.

Qeexid :

Ka dhig B in ay tahay taxane $m \times p$ ah, T taxane $p \times n$ ah, markaa taranka BT wuxuu ku qeexan yahay in uu yahay taxanaha C oo ah $m \times n$. Kaasoo kutirsaneyaashiisa dhinactaxa i-aad iyo joogtaxa j-aad lagu helay isku dhufashada kutirsaneyaalka dhinactaxa i-aad ee B iyo joogtaxa j-aad ee T-deedna tarannadaas la isugeyay.

Summad ahaan haddii $B = (bij)_{m\ p}$, $T = (tij)_{p\ n}$, markaa $BT = (cij)_{m\ p}$, meesha $Cij = \Sigma\, b_{ik}\, t_{kj}$

OGOW: In labada taxane ee la isku dhufanayaa ay yihiin: tirada kutirsaneyaal dhinactax kasta ee taxanaha hore waxay le'eg tahay tirada kutirsaneyaal joogtaxa taxanaha labaad. Taasi waa haddii taxanaha bidix yahay $m \times n$, kan midig waa inuu noqdaa taxane $n \times p$, markaana tarankoodu waa taxane $m \times p$ ah.

Taxane labajibbaarrane oo xaglogooyihiisa doorka hi min bidix sare ilaa midig hoose marayo kutirsaneyaal wada kow ah, oo kutirsaneyaasha kale oo dhammi wada eber yihiin waxaa la yiraa **Taxane midaal.** Inta badanna waxaa loo joojiyaa 1. Waad caddeyn kartaa in taxaneyaalka midaal, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ iyo $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ yihiin asal madoorsheyaalka isku dhufashada ee ururka taxaneyaalka 2×2 iyo 3×3 sidey u kala horreeyaan.

Tusaale :

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} b & t \\ j & d \end{pmatrix} = \begin{pmatrix} b+0 & t+0 \\ 0+j & 0+d \end{pmatrix} = \begin{pmatrix} b & t \\ j & d \end{pmatrix}$$

Sidaas oo kale :

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b & t & j \\ d & k & s \\ c & g & f \end{pmatrix} = \begin{pmatrix} b & t & j \\ d & k & s \\ c & g & f \end{pmatrix}$$

Ururka tirada maangal, haddii $bt = 0$, markaa $b = 0$ ama $t = 0$, laakiin taranka

$$\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} -6 & 4 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Sidaa daraadeed, haddii B iyo T ay taxaneyaal yihiin markaa $BT = 0$ ma malagalinayso in $B = 0$ ama $T = 0$. Hase yeeshee sharciga hormogelinta $(BT)J = B(TJ)$ taxaneyaalku waa jiraa, sidoo kale sharciga kala dhigga taxaneyaalku waa jiraa

$$BT + BJ = B(T + J), \quad (B + J)J = BJ + TJ$$

Layli :

Iskudhufo:

1. $$(4\ \ 3\ \ 1)\begin{pmatrix}5\\6\\0\end{pmatrix}$$

2. $$\begin{pmatrix}8 & 9 & 6\\2 & 3 & 7\end{pmatrix}\begin{pmatrix}1 & 5 & 3\\2 & 0 & 1\\-2 & 2 & 0\end{pmatrix}$$

3. $$\begin{pmatrix}-1 & 4\\2 & -3\end{pmatrix}\begin{pmatrix}0 & 6\\-5 & -7\end{pmatrix}$$

4. $$\begin{pmatrix}5 & 0 & 2\\-1 & 4 & -3\\-2 & -3 & 6\end{pmatrix}\begin{pmatrix}0 & 5 & 0\\0 & -2 & 5\\3 & 6 & -3\end{pmatrix}$$

5. $$(1\ \ 3\ \ 2\ \ 0)\begin{pmatrix}6 & 2\\7 & -3\\8 & -4\\9 & -5\end{pmatrix}$$

6. $$\begin{pmatrix}1 & 0\\-2 & -3\\4 & 7\end{pmatrix}\begin{pmatrix}6 & 7 & 1\\-3 & 5 & 3\end{pmatrix}$$

7. $$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix}$$

8. $$\begin{pmatrix}1 & 0 & 1\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}b & t & j\\d & k & s\\c & g & f\end{pmatrix}$$

9. Haddii $B = \begin{pmatrix}0 & 2\\3 & 4\end{pmatrix}$ raadi B^2, B^m iyo $(-A)^3$

10. Haddii $T = \begin{pmatrix}0 & 3 & 4\\-2 & 1 & 0\\5 & 0 & 1\end{pmatrix}$, raadi T^2 iyo T^3

11. Haddii $B = \begin{pmatrix}1 & -2\\-3 & 0\end{pmatrix}$, $T = \begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$

$$J = \begin{pmatrix}0 & 2\\3 & 1\end{pmatrix}$$

Tus in $BT + BJ = B(T + J)$. Sidoo kale tus in $B(T + J) \neq (T + J)B$. Maxay xaaladda hore u jirtaa tan dambena ayna u jirin.

12. Haddii $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$, $T = \begin{bmatrix} 2 & -1 \\ 7 & 4 \end{bmatrix}$,
tus in $BT = TB = I$.

13. Haddii $B = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$, $T = \begin{bmatrix} 4 & 0 \\ 3 & 1 \end{bmatrix}$,
tus in $(B - T)^2 \neq B^2 - 2BT + T^2$.

SUGAHA FANSAAR

Isle'egta taxane $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{matrix} x = \\ y = \end{matrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$ waxay u dhigantaa habdhiska toosan

$$b_{11}x + b_{12}y = t_1$$
$$b_{21}x + b_{21}y = t_2$$

Haddaynu u furfurno isle'egta x iyo y waxaynnu heleynaa

$$x = \frac{b_{22}t_1 - b_{12}t_2}{b_{11}b_{22} - b_{12}b_{21}}, \quad y = \frac{b_{11}t_2 - b_{21}t_1}{b_{11}b_{22} - b_{12}b_{21}}$$

Sidaa daraadeed, waxaynnu isla baahayn karnaa tirada maangal ee ah $b_{11}b_{22} - b_{12}b_{21}$ iyo Taxanaha $\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$.

Qeex :

Sugaha taxanaha $B_{2\ 2} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ oo loo qoro $|B|$ waa tiro maangal ah oo lagu helo:

$$\begin{bmatrix} b_{11} & b_{12} \\ & \\ b_{21} & b_{22} \end{bmatrix} = b_{11}b_{22} - b_{21\ 12}.$$ Tirada $|B|$ waxaa la yiraa **Suge.**

Sidaynnu dib u arki doono, tiradaasi waxay sugtaa in taxane weydaar leeyahay iyo in kale.

Tusaale :

Sheeg sugaha taxanaha $B = \begin{bmatrix} 31 & 1 \\ 4 & 6 \end{bmatrix}$

Furfuris :

Waxaad isticmaali kartaa summadda δ (B) oo u taagan sugaha taxanaha B waxaa kalood isticmaali kartaa $|B|$.

$$(B) = \begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix} = 3 \cdot 6 - 4 \cdot 1 = 18 - 4 = 14.$$

Taxane kastoo leh aaddimo labajibbaar ah wuxuu leeyahay Suge. Kujirayaalka sugaha waxaa la yiraa **kutirsanayaal,** inta kutirsane ee ku jirta dhinactaxa ama joogtaxa waxaa la yiraa **Heerka sugaha.**

Tusaale :

Aynnu tixgelinno taxane labajibbaar oo heerkiisu yahay 3. Ka dhig $B = \begin{bmatrix} b_1 & t_1 & j_1 \\ b_2 & t_2 & j_2 \\ b_3 & t_3 & j_3 \end{bmatrix}$ Hel sugaha taxanaha.

Furfuris :

$$\delta(B) = |B| = \begin{vmatrix} b_1 & t_1 & j_1 \\ b_2 & t_2 & j_2 \\ b_3 & t_3 & j_3 \end{vmatrix} = b_1t_2j_3 + b_2t_3j_1 + b_3t_1j_2 - b_1t_1j_3 - b_1t_3j_2 - b_3t_2j_1.$$

Waxan aragnaa in wadarta isugeynta tarannada kor ku yaal ay inna siinayaan ratibaad kastoo suuragal ah oo muujiyaasha (hoos qorrada) b, t iyo j ay isu raaci karaan. Habkani wuxuu u baahan yahay aqoon racayn oo aan dhib yarayn. Hase yeeshee waxa jira hab ka fudud oo uu soo saaray xisaab yahanka la yiraa **Sarrus.** Habkaa oo tifaftiranina waa kan hoos ku qoran.

1. Guuri taxanaha lagu siiyay, joogtaxa ugu dambeeya midigtiisa mar labaad, ku qor labada joogtaxa ee ugu horreeya, taxanaha, say isugu xigaan.

$$\begin{matrix} b_1 & t_1 & j_1 & b_1 & t_1 \\ b_2 & t_2 & j_2 & b_2 & t_2 \\ b_3 & t_3 & j_3 & b_3 & t_3 \end{matrix}$$

2. Imika isku dhufo kujirayaalka saddexda ah ee xagalgooye kasta oo bidix sare ka socdaa marayo. Markan, tarannada la helay waa saddexda ugu horreeya sugaha ee dhammaan togan.

$$\begin{matrix} b_1 & t_1 & j_1 & b_1 & t_1 \\ b_2 & t_2 & j_2 & b_2 & t_2 \\ b_3 & t_3 & j_3 & b_3 & t_3 \end{matrix}$$

3. Sidoo kale, isku dhufo saddexda kujire ee xagalgooye kastoo midig sare ka socdaa marayo, taran kastana ka dhig tabane. Saddexdaa tibxood waa kuwa ugu dambeeya sugaha.

$$\begin{matrix} b_1 & t_1 & j_1 & b_1 & t_1 \\ b_2 & t_2 & j_2 & b_2 & t_2 \\ b_3 & t_3 & j_3 & b_3 & t_3 \end{matrix}$$

Tusaale :

Hel sugaha taxanaha $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 4 \\ 3 & 0 & -2 \end{bmatrix}$ adoo isticmaalaya habka «Sarrus».

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 4 \\ 3 & 0 & -2 \end{vmatrix} \quad \delta(B) = \begin{matrix} 1 & 2 & 3 & 1 & 2 \\ -2 & 1 & 4 & -2 & 1 \\ 3 & 0 & -2 & 3 & 0 \end{matrix}$$

$$\begin{aligned} \delta(B) &= 1 \cdot 1 \cdot (-2) + 2 \cdot 4 \cdot 3 + 3 \cdot (-2) \cdot 0 \\ &\quad - 2 \cdot (-2) \cdot (-2) - 1 \cdot 4 \cdot 0 - 3 \cdot 1 \cdot 3 \\ &= -2 + 24 + 0 (-8) + 0 (-9) = 5. \end{aligned}$$

Layliyo :

Hel δ(B). Haddii B tahay taxanaha layli kasta.

1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ 2. $\begin{bmatrix} -2 & 4 \\ -3 & 6 \end{bmatrix}$ 3. $\begin{bmatrix} 6 & -2 \\ -1 & 1 \end{bmatrix}$

6. $\begin{bmatrix} -5 & 3 \\ 6 & 4 \end{bmatrix}$ 7. $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 6 & -3 \\ 0 & 5 & 8 \end{bmatrix}$

8. $\begin{bmatrix} 5 & 0 & -6 \\ 0 & 8 & -2 \\ 5 & 1 & 0 \end{bmatrix}$ 9. $\begin{bmatrix} 8 & -2 & -5 \\ 3 & -3 & -6 \\ 1 & -4 & 8 \end{bmatrix}$

10. $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Haddii B iyo T ay yihiin taxanayaal 2×2 ah, «a» ay tahay foolwaa:

11. Tus in $\delta(aB) = a^2 \times (B)$
12. Tus in $\delta(B^m) = (B)$
13. Tus in $\delta(BT) \neq \delta(B) \times \delta(T)$
14. Tus in $\delta(B - B) \neq \delta(B^m - B)$

OGOW: Haddii sugaha taxane labajibbaar ahi yahay eber, taxanaha waxaa la yiraa **Kaaliyaale**, markaasna taxanuhu ma laha weydaar.

WEYDAARKA TAXANE

Markaan u noqonno ururka tirooyinka maangal, waxaynnu ognahay in haddii taranka laba tiro oo maangal ah yahay asal madoorshe 1, markaa labadaa tiro ee maangalka ahi waa weydaarro isku dhufasho, taas oo ah haddii $bt = 1$ markaa $b = t^{-1} = \frac{1}{t}$. Run ahaan, haddii taranka laba taxane B iyo T yahay 1 (t.a. $B \cdot T = 1$) markaa B iyo T waa weydaarro, sida caadiga ahna B waxa loo qoraa T^{-1}. Su'aasha aan laga fursaneyni waxa weeye sidee baynu u heli karnaa T^{-1}? Dhanka taxaneyaalka 2×2 ah, jawaabta su'aashani aad bay u sahlan tahay.

Ka soo qaad:

$$B = \begin{pmatrix} b & t \\ j & d \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} u & w \\ x & y \end{pmatrix} \text{ markaa } B \cdot B^{-1} = 1.$$

Taasi waa

$$\begin{pmatrix} b & t \\ j & d \end{pmatrix} \begin{pmatrix} u & w \\ x & y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} bu + tx & bw + ty \\ ju + dx & jw + dy \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Isle'egtan ugu dambeysya waa run haddii iyo haddii qura:

$$bu + tx = 1 \qquad bw + ty = 0$$

$$ju + dx = 0 \qquad jw + dy = 1$$

bishardi haddii $bd - jt \neq 0$. Waa maxay sababtu?

Waynu u furfuri karnaa isle'egyadan wada jira u, x iyo w, y siday isugu xigaan. Waxaynnu heleeynaa

$$U = \frac{d}{bd - jt} \qquad W = \frac{-t}{bd - jt}$$

$$X = \frac{-j}{bd - jt} \qquad Y = \frac{b}{bd - jt}$$

Mar haddii hooseeye kastaa yahay $|B|$, $|B| \neq 0$

$B^{-1} = \frac{1}{|B|} \begin{pmatrix} d & -t \\ -j & b \end{pmatrix}$. Markaa, waxaynu gaari karnaa in taxane kastoo 2×2 ahi wuxu leeyahay weeydaar haddaar haddii aan sugahiisu eber ahayn. Isle'egtu waxay inoo sheegysaa sida loo doono B^{-1} oo ah: in la isku bedbelayo b iyo d, iyo in j iyo t tabanno laga dhigo, markaana taxanaha soo baxa lagu dhufanayo $\frac{1}{|B|}$.

Tusaale :

Haddii $B = \begin{bmatrix} 3 & 2 \\ 4 & 6 \end{bmatrix}$, hel B^{-1}.

Furfuris :

$$|B| = \begin{vmatrix} 3 & 2 \\ 4 & 6 \end{vmatrix} = 18 - 2 \Rightarrow 10$$

$$B^{-1} = 1/|B| \begin{bmatrix} d & -t \\ -j & b \end{bmatrix}$$

$$B^{-1} = 1/10 \begin{bmatrix} 6 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 6/10 & -2/10 \\ -4/10 & 3/10 \end{bmatrix}$$

Markaad hesho weydaarka taxane kasta oo 2×2 ah, hubso in haddii weydaarradaa la isku dhufto ay ku siinayaan taxane-midaal, I, ama taxane asal madoorshe isku dhufasho. Taxane-labajibbaar kasta ee heerka $n > 2$ ahi waa leeyahay weydaar haddaan suguhiisu eber ahayn, laakiin habka loo helayaa isweydaarka uma dhib yara sida ka taxanaha 2×2 ah. Hase yeeshee hab loo helaa waa jiraa. Taxanihii isweydaar leh waxaa la yiraa **Weydaarle.**

Tusaale :

Haddii B iyo T ay yihiin taxaneyaal weydaarley ah, caddee in $(BT)^{-1} = T^{-1} B^{-1}$.

Caddeyn :

Mar haddii B iyo T yihiin weeydaarley $BB^{-1} = I$, $TT^{-1} = I$.

Taranka BT $(T^{-1}B^{-1}) = B(TT^{-1}B^{-1})$ Hormogelinta isku dhufashada

$= B(IB^{-1})$ Astaanta weeydaarka.
$= BB^{-1}$ Astaanta midaal.
$= I$ Astaanta weeydaarka.

Mar haddii BT $(T^{-1}B^{-1}) = I$, BT waa weeydaarka $T^{-1}B^{-1}$ ama $(BT)^{-1} = T^{-1}B^{-1}$.

Layli :

Soo saar weydaarka taxane kasta. Haddii aan taxanuhu weydaarle ahayn, sheeg sababta.

1. $\begin{pmatrix} 2 & 4 \\ 6 & 1 \end{pmatrix}$ 2. $\begin{pmatrix} 1 & 9 \\ -4 & 2 \end{pmatrix}$ 3. $\begin{pmatrix} 9 & 2 \\ -1 & -3 \end{pmatrix}$

4. $\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$ 5. $\begin{pmatrix} 8 & -3 \\ 4 & -1 \end{pmatrix}$ 6. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

7. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 8. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ 9. $\begin{pmatrix} 4 & 6 \\ 5 & 7 \end{pmatrix}$

10. $\begin{pmatrix} -1 & -2 \\ -4 & 6 \end{pmatrix}$ 11. $\begin{pmatrix} 3 & 8 \\ 9 & 1 \end{pmatrix}$ 12. $\begin{pmatrix} 6 & 0 \\ -3 & 0 \end{pmatrix}$

U furfur isle'egyada lagu siiyay B.

Tusaale :

$$\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix} B = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$$

Furfuris :

1) Hel weeydaarka $\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$. Sugeheedu waa -10.

$$\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}^{-1} = 1/-10 \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2/10 & 3/10 \\ 4/10 & -1/10 \end{pmatrix}$$

2) Bidixda kaga dhufo weeydaarka, dhinac kastaa isle'egta:

$$\begin{pmatrix} -2/10 & 3/10 \\ 4/10 & -1/10 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix} B = \begin{pmatrix} -2/10 & 3/10 \\ 4/10 & -1/10 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} B = \begin{pmatrix} 5/10 & 2/10 \\ 5/10 & 16/10 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/5 \\ 1/2 & 8/5 \end{pmatrix}$$

$$\therefore B = \begin{pmatrix} 1/2 & 1/5 \\ 1/2 & 8/5 \end{pmatrix}$$

13. $\begin{pmatrix} 1 & 4 \\ 3 & 6 \end{pmatrix} B = \begin{pmatrix} 4 & 6 \\ 5 & 7 \end{pmatrix}$

14. $\begin{pmatrix} -2 & 3 \\ 1 & 5 \end{pmatrix} B = \begin{pmatrix} 8 & 2 \\ 1 & -2 \end{pmatrix}$

15. $\begin{pmatrix} 0 & 4 \\ 5 & 2 \end{pmatrix} B = \begin{pmatrix} 3 & 2 \\ 5 & 6 \end{pmatrix}$

16. $\begin{pmatrix} 4 & -3 \\ 4 & 2 \end{pmatrix} B = \begin{pmatrix} -1/2 & 2 \\ 0 & 1 \end{pmatrix}$

17. $\begin{pmatrix} 7 & 3 \\ 1 & 6 \end{pmatrix} B = \begin{pmatrix} 1 & -5 \\ 2 & 8 \end{pmatrix}$

18. $\begin{pmatrix} 1 & -1 \\ 1 & 5 \end{pmatrix} B = \begin{pmatrix} -5 & 4 \\ 1 & 3 \end{pmatrix}$

19. $\begin{pmatrix} 5 & 3 \\ 2 & 6 \end{pmatrix} B + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 3 & 2 \end{pmatrix}$

20. $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} B + \begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 0 & 6 \end{pmatrix}$

21. Haddii $BX + T = J$, X u tibaax B, T iyo J.

FURFURISTA HABDHISYADA ISLE'EGTA TOOSAN

Tixgeli habdhiska
$$b_1x + t_1y = j_1$$
$$b_2x + t_2y = j_2$$

Haddii aynn u ka dhigno $B = \begin{bmatrix} b_1 & t_1 \\ b_2 & t_2 \end{bmatrix}$, $T = \begin{bmatrix} x \\ y \end{bmatrix}$,

$$J = \begin{bmatrix} j_1 \\ j_2 \end{bmatrix}$$

Markaa habdhiska sare wuxuu u dhigma taxanaha

$B \cdot T = J$ t.a $\begin{bmatrix} b_1 & t_1 \\ b_2 & t_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} j_1 \\ j_2 \end{bmatrix}$

furfuristuna tahay

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 & t_1 \\ b_2 & t_2 \end{bmatrix}^{-1} \begin{bmatrix} j_1 \\ j_2 \end{bmatrix} = 1/|B| \begin{bmatrix} t_2 & -t_1 \\ -b_2 & b_1 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \end{bmatrix}.$$

Taxanaha $\begin{bmatrix} b_1 & t_1 \\ b_2 & t_2 \end{bmatrix}$ waxaa la yiraa **Taxane weheliyeyaal.**

Tusaale :

Furfur
$$2x + 5y = 6$$
$$3x - 2y = -10$$

Furfuris :

Saansaanka taxane ee habdhisku waa

$\begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \end{bmatrix}$ Taas daraadeed

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix}^{1} \begin{bmatrix} 6 \\ -10 \end{bmatrix}$$

ugu dambeyn $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ t.a., $x = -2$, $y = 2$.

Layli:

Raadi ururka furfurista ee habdhisyada lagu siiyay adoo isticmaalaya taxanayaal. Haddii aanu habdhisku lahayn, furfuris, sheeg sababta.

1. $3x + 2y = 4$
   $5x + 3y = 0$
2. $x + y = 4$
   $2x - 2y = 3$
3. $4x - y = 0$
   $2x + 3y = 6$
4. $6x - 3y = 1$
   $x - 2y = 2$
5. $5x + 3y = 3$
   $2x - y = 1$
6. $6x - 2y = 4$
   $3x - y = 1$
7. $x - y = 4$
   $2x - 4y = -1$
8. $3x + 3y = 1$
   $4x - y = 2$
9. $10x + y = 5$
   $x - y = 4$
10. $4x + 4y = 4$
    $x + y = -4$

YARYAAL U KALA BIXINTA SUGAYAAL

Habkii aynnu ku isticmaaleynay kala bixinta sugayaal waa ku qalafsan tahay sugeyaalka heer sare ah, hase yeeshee, waxaa jirta hab kale oo la yiraa: **Yaryaal u kala bixinta.** Kaasoo lagu isticmaali karo suge kasta oo heer kasta ah. Yaraha kutirsane waa sugaha soo baxaya marka la reebo dhinactaxa iyo joogtaxa kutirsanahaasu kaga jiro sugaha lagu siiyay. Haddaba yaraha kutirsanaha 2 ee ku jira $\begin{vmatrix} 2 & 3 & 4 \\ 1 & -1 & -2 \\ 0 & 5 & -3 \end{vmatrix}$ waa $\begin{vmatrix} -1 & -2 \\ 5 & -3 \end{vmatrix}$, -3 yarihiisuna waa $\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$. Sheeg yaraha 1?

Qiimaha suge heerka saddexaad ah, sidii hore loogu qeexay waxaa loo sii qori karaa sidan:

$$\begin{vmatrix} b_1 & t_1 & j_1 \\ b_2 & t_2 & j_2 \\ b_3 & t_3 & j_3 \end{vmatrix} = b_1t_2j_3 - b_1t_3j_2 + t_1b_2j_3 - t_1b_3j_2 + j_1b_2t_3 - j_1b_3t_2.$$

Haddii aynnu isir wadaag u raadinno tirooyinka sare waxaynnu heli

$$b_1 (t_2j_3 - t_3j_2) + t_1 (b_2j_3 - b_3j_2) + j_1 (b_2t_3 - b_3t_2)$$

Haddaba tibaaxaha bilaha ku jiraa waa yarayaalka b_1, t_1 iyo j_1 sidey u kala horreeyaan. Haddii yarayaalkaa aynnu u joojinno, B_1, T_1 iyo J, waxaynnu heleynaa in

$$\begin{vmatrix} b_1 & t_1 & j_1 \\ b_2 & t_2 & j_2 \\ b_3 & t_3 & j_3 \end{vmatrix} = b_1B_1 + t_1T_1 + j_1T_1$$

Sidaa daraadeed tibaaxda midigta taalli waa yarayaal u kala bixinta sugaha ee loo eegay dhinactaxa 1aad. Guud ahaan, suge yarayaal waan u kala bixin karnaa haddii aynnu qaadanno dhinactaxa kasta ama joogtaxa kasta; habka loo shaqeynayaan waxaỹ ku kooban tahay Xeerka soo socda:

> Isku dhufasho kutirsane kasta ee dhinactax ama joogtax aad dooratay iyo yarahiisa. Ku dhufo taran kasta 1 ama -1 adoo u eegaya siday wadarta tirada dhinactax iyo joogtax ee kutirsanuhu u kala yahay dhaban ama Kisi. Ugu dambeyn isugee tarannada.

Tusaale :

U kala bixi $\begin{vmatrix} 2 & 3 & -1 \\ 4 & 2 & -3 \\ 5 & 0 & 2 \end{vmatrix}$ yarayaalka joogtaxa Koowaad.

Furfuris :

Haddii aynnu raacno xeerka kor ku qoran waxaan heleynaa:

$$\begin{vmatrix} 2 & 3 & -1 \\ 4 & 2 & 3 \\ 5 & 0 & 2 \end{vmatrix} = (+1)\, 2 \begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} + (-1)\, 4 \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix}$$

$$+ (+1)\,5\begin{vmatrix}3 & -1\\ 2 & 3\end{vmatrix}$$

$$= 2(4) - 4(6) + 5(11)$$
$$= 8 - 24 + 55 = 39$$

Guud ahaan, habka loo kala bixinayo sugayaal, waa sida soo socota:

Tixgeli Taxane 4×4 sida

$$B = \begin{bmatrix} b_1 & t_1 & j_1 & d_1\\ b_2 & t_2 & j_2 & d_2\\ b_3 & t_3 & j_3 & d_3\\ b_4 & t_4 & j_4 & d_4\end{bmatrix}$$

Haddaba yarayaalka $|B|$ ee loo eegay dhinactaxa 1aad waa

$$B_1 = \begin{vmatrix} t_2 & j_2 & d_2\\ t_3 & j_3 & d_3\\ t_4 & j_4 & d_4\end{vmatrix} \qquad J_1 = \begin{vmatrix} b_2 & t_2 & d_2\\ b_3 & t_3 & d_3\\ b_4 & t_4 & d_4\end{vmatrix}$$

$$T_1 = \begin{vmatrix} b_2 & j_2 & d_2\\ b_3 & j_3 & d_3\\ b_4 & j_4 & d_4\end{vmatrix} \qquad D_1 = \begin{vmatrix} b_2 & t_2 & d_2\\ b_3 & t_3 & d_3\\ b_4 & t_4 & d_4\end{vmatrix}$$

Markaa waxaan u qeexna in

$$|B| = b_1B_1 + t_1T_1 + j_1J_1 + d_1D_1.$$

OGOW in B_1, T_1, J_1 iyo D_1 ay isu egyihiin waxaynnu yarayaal ugu qeexnay sugayaalka heer saddexaad. Sidaa daraadeed waxaan aragnaa in ay yihiin yarayaalku sugaha heerka afraad oo loo eegay dhinactaxa 1aad. Haddii sugaha lagu kala bixin lahaa dhinactax kale ama joogtax kale, qiimaha sugahu ma beddelmo.

Yare kasta ee sugahu waa suge heerka 3aad ah, laakiin haddii naftiisa la yareyaal kala bixiyo waa loo gaabin karaa suge heerka 2aad ah. Haddaba suge heerka 4aad ahna waa loo gaabin karaa suge heerka 2aadah. Sidoo kale, ha ka shakiyin in suge heerka 5aad ah lagu qeexi karo suge heerka 4aad ah, sugihii heer 6aad ahna waa lagu qeexi karaa suge heer 5aad ah, sidaas hadday ku socoto, sugihii heer n-aad ahna waa lagu qeexi karaa suge heer 2aad ah!!

Tusaale :

Ku kala bixi sugaha $\begin{vmatrix} 2 & 1 & 0 & 3 \\ 4 & 2 & 5 & 1 \\ 6 & 3 & 4 & 5 \\ 1 & 0 & 0 & 2 \end{vmatrix}$ yarayaalka dhinaxtaxa 1aad.

$$|B| = b_1B_1 + t_1T_1 + j_1J_1 + d_1D_1 ,$$

$$|B| = \begin{vmatrix} 2 & 1 & 0 & 3 \\ 4 & 2 & 5 & 1 \\ 6 & 3 & 4 & 5 \\ 1 & 0 & 0 & 2 \end{vmatrix}$$

$$|B| = (+1)\,2 \begin{vmatrix} 2 & 5 & 1 \\ 3 & 4 & 5 \\ 0 & 0 & 2 \end{vmatrix} + (-1)\,(1) \begin{vmatrix} 4 & 5 & 1 \\ 6 & 4 & 5 \\ 1 & 0 & 2 \end{vmatrix}$$

$$+ (+1)\,0 \begin{vmatrix} 4 & 2 & 1 \\ 6 & 3 & 5 \\ 1 & 0 & 2 \end{vmatrix} + (-1)\,(3) \begin{vmatrix} 4 & 2 & 5 \\ 6 & 3 & 4 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= 2\left\{2\begin{vmatrix} 4 & 5 \\ 0 & 2 \end{vmatrix} - 5\begin{vmatrix} 3 & 5 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 4 \\ 0 & 0 \end{vmatrix}\right. - 1\left\{4\begin{vmatrix} 4 & 5 \\ 0 & 2 \end{vmatrix}\right.$$

$$-5\begin{vmatrix} 6 & 5 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 6 & 4 \\ 1 & 0 \end{vmatrix} + -3\left\{4\begin{vmatrix} 3 & 4 \\ 0 & 0 \end{vmatrix}\right. - 2$$

$$\left.\begin{vmatrix} 6 & 4 \\ 1 & 0 \end{vmatrix} + 5\begin{vmatrix} 6 & 3 \\ 1 & 0 \end{vmatrix}\right\}$$

$$= 2 [2 (8 - 0) - 5 (6 - 0) + 1 (0 - 0)]$$
$$- 1 [4 (8 - 0) - 5 (12 - 5) + 1 (0 - 4)]$$
$$- 3 [0 - 2 (-4) + (-3)]$$
$$= 2(16 - 30) - 1(32 - 35 - 4) - 3(8 - 15)$$
$$= -28 + 7 + 21 = 0$$

Bal ku kala bixi yarayaal dhinactaxa 4aad. Keebaa sahlan? Sabab?.

Layli :

U kala bixi sugayaalka soo socda yarayaalka dhinactaxa ama joogtaxa la isa siiyay.

1. $\begin{vmatrix} 3 & -1 & 0 \\ -2 & -3 & 1 \\ 1 & 6 & 5 \end{vmatrix}$ Dhinactaxa 2. Joogtaxa 3

2. $\begin{vmatrix} -2 & 2 & -3 \\ 4 & 5 & 1 \\ 6 & 7 & 0 \end{vmatrix}$ Dhin. 3 Joog. 1

3. $\begin{vmatrix} 5 & 6 & 1 \\ -2 & -3 & 1 \\ 4 & 5 & 7 \end{vmatrix}$ Dhin. 1 Joog. 3

4. $\begin{vmatrix} 0 & 1 & 5 \\ 2 & -2 & 4 \\ 3 & 1 & 0 \end{vmatrix}$ Dhin. 2 Joog. 1

5. $\begin{vmatrix} 2 & -1 & 0 \\ 3 & -1 & 4 \\ 1 & -2 & 3 \end{vmatrix}$ Dhin. 2 Joog. 3

6. $\begin{vmatrix} 1 & 0 & 1 \\ 3 & 0 & 7 \\ 4 & 0 & 8 \end{vmatrix}$ Dhin. 3 Joog. 2

Ku kala bixi sugayaalka la isa siiyay dhinactax ama joogtax kasta.

7. $\begin{vmatrix} 5 & -3 & 6 \\ 8 & -2 & 5 \\ 1 & 2 & 1 \end{vmatrix}$

8. $\begin{vmatrix} -2 & -1 & -5 \\ 4 & -3 & 3 \\ 6 & 0 & 6 \end{vmatrix}$

9. $\begin{vmatrix} -1 & 8 & -6 \\ -3 & 1 & 7 \\ 6 & 0 & 4 \end{vmatrix}$

10. $\begin{vmatrix} -4 & -1 & 8 \\ 4 & 0 & 4 \\ 4 & 0 & 1 \end{vmatrix}$

U furfur isle'egyadan doorsoomaha:

11. $\begin{vmatrix} 2 & 4 & 2 \\ x & 3 & 4 \\ -1 & -2 & 3 \end{vmatrix} = 2$

12. $\begin{vmatrix} x & 3 & 2 \\ 4 & 6 & 0 \\ 5 & 4 & x \end{vmatrix} = 0$

Qiime :

13. $\begin{vmatrix} 4 & 6 & -1 \\ 3 & 0 & 8 \\ 5 & 0 & -3 \\ 1 & 4 & 2 \end{vmatrix}$

14. $\begin{vmatrix} 3 & 4 & -1 & 6 \\ -2 & 5 & -2 & 2 \\ 5 & 2 & -3 & 0 \\ 1 & 2 & 1 & 4 \end{vmatrix}$

15. $\begin{vmatrix} 1 & -2 & -1 & 4 \\ -1 & -3 & -2 & 3 \\ 7 & 2 & -1 & 1 \\ 0 & 1 & -2 & 6 \end{vmatrix}$

ASTAAMAHA SUGAYAAL

Sugayaalku waxay leeyihiin astaamo, kuwaasoo inaga caawiya xagga fududaynta kala bixintooda. Buuggan, astaamaha oo idili waxay ku tusaalaysan yihiin sugayaal heerka 3aad ah, hase yeeshee astaamuhu waa ku run suge heer kasta ah.

Astaamaha buuggan lagu caddeyn maayo.

Astaan 1. Haddii laba kasta oo dhinactax ama joogtax suge la isku beddelo, marka sugaha soo baxayaa waa tabanaha sugihii hore.

Tusaale :

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix} = -2 - 4 + 12 = 6$$

$$|B| = \begin{vmatrix} 4 & 0 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix} = 4 + 0 - 10 = -6$$

Astaan 2. Haddii laba dhinactax ama laba joogtax suge ay isle'eg yihiin, markaa suguhu waa eber.

Tusaale :

$$|B| = \begin{vmatrix} 1 & 2 & 4 \\ 1 & 2 & 4 \\ 4 & 2 & 1 \end{vmatrix} = -6 + 30 - 24 = 0$$

Astaan 3. Haddii dhinactaxyada iyo joogtaxyada suge oo idil la isugu beddelo si horsan, sugaha soo baxayaa wuxuu la mid yahay kii hore.

Tusaale :

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix} = 6,$$

iyo

$$|B| = \begin{vmatrix} 1 & 4 & 3 \\ 2 & 0 & 1 \\ 3 & 2 & 2 \end{vmatrix} = 6$$

Astaan 4. Haddii kutirsaneyaalka hal dhinactax ama hal joogtax ee suge lagu dhufto tiro maangal K, sugaha soo baxayaa waa kii hore oo K lagu dhuftay.

Tusaale:

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix} = 6$$

$$2\,|B| = \begin{vmatrix} 1 & 2 & 3 \\ 8 & 0 & 4 \\ 3 & 1 & 2 \end{vmatrix} = -4 - 8 + 24 = 12 = 2 \cdot 6$$

OGOW: Astaantan suge, waa ka jaad tii isku dhufashada taxane iyo foolwaa, ayayna iskaga kaa darsamin.

Astaan 5. Haddii hal dhinactax ama hal joogtax kutirsaneyaalkiisu dhammaan eberro yihiin, suguhu waa eber.

Tusaale:

$$|B| = \begin{vmatrix} 0 & 4 & 1 \\ 0 & 1 & 8 \\ 0 & 2 & 10 \end{vmatrix} = 0 + 0 + 0 = 0$$

Sidoo kale

$$|T| = \begin{vmatrix} 8 & 3 & 5 \\ 0 & 0 & 0 \\ 4 & 9 & -1 \end{vmatrix} = 0 + 0 + 0 = 0$$

Astaan 6. Haddii kutirsane kasta oo hal dhinactax suge lagu dhufto tiro maangal K, oo tarannadaa soo baxay loo geeyo kutirsaneyaalka ku beegan ee dhinactax kale, ama haddii kutirsane kasta hal joogtax lagu dhufto tiro maangala, tarannadana loo geeyo kutirsaneyaalka ku beegan ee joogtax kale, markaa labada jeerba sugaha la helayaa wuxuu la midaal yahay kii hore. Astaantani waa midda inta badan loogu isticmaalo qiimeynta sugayaalka.

T u s a a l e :

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix} = 6$$

$$|B| = \begin{vmatrix} 1+3\cdot 2 & 2 & 3 \\ 4+2\cdot 2 & 0 & 2 \\ 3+2\cdot 2 & 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 7 & 2 & 3 \\ 8 & 0 & 2 \\ 7 & 1 & 2 \end{vmatrix} = -14 - 4 + 24 = 6$$

$$\therefore \quad |B| = |B|.$$

Astaan 7. Haddii hal dhinactax ama hal joogtax ee suge uu yahay dhufsane dhinactax ama joogtax kale ee sugahaa, markaa qiimaha sugahaasi waa eber.

$$|B| = \begin{vmatrix} 3 & 2 & 1 \\ -1 & -2 & 4 \\ 6 & 4 & 2 \end{vmatrix} = 0$$

T u s a a l e :

$$\text{Qiimee} \begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 6 \\ -3 & 5 & 6 \end{vmatrix}$$

F u r f u r i s :

1. Ku dhufo -2 dhinactaxa 1aad, una gee taran-nada dhinactaxa 2aad Astaan 6).

$$\begin{vmatrix} 1 & 3 & 4 \\ 2+(-2) & 1+(-6) & 6+(-8) \\ -3 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 4 \\ 0 & -5 & -2 \\ -3 & 5 & 6 \end{vmatrix}$$

2. Ku dhufo $+3$ dhinactaxa 1aad una gee dhinac-taxa 3aad.

$$\begin{vmatrix} 1 & 3 & 4 \\ 0 & -5 & -2 \\ -3+3 & 5+9 & 6+12 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 4 \\ 0 & -5 & -2 \\ 0 & 14 & 18 \end{vmatrix}$$

3. U kala bixi yarayaalka joogtaxa 1aad

$$\begin{vmatrix} 1 & 3 & 4 \\ 0 & -5 & -2 \\ 0 & 14 & 18 \end{vmatrix} = -62$$

Layli:

Qiimee sugayaalka soo socda adoo isticmaalaya Astaamaha 1 — 7 si ay shaqada kuugu fududeeyaan.

1. $\begin{vmatrix} 3 & 4 & 6 \\ 4 & 1 & 3 \\ 5 & 0 & 6 \end{vmatrix}$

2. $\begin{vmatrix} 1 & 2 & 7 \\ -3 & 4 & 6 \\ 7 & 5 & 1 \end{vmatrix}$

3. $\begin{vmatrix} 28 & 30 & 40 \\ 28 & 30 & 40 \\ 31 & 21 & 51 \end{vmatrix}$

4. $\begin{vmatrix} 1 & 0 & 6 \\ 1 & 0 & 3 \\ 1 & 0 & 4 \end{vmatrix}$

5. $\begin{vmatrix} 60 & 30 & 20 \\ 30 & 15 & 10 \\ 70 & 80 & 93 \end{vmatrix}$

6. $\begin{vmatrix} 35 & 45 & 15 \\ 11 & 13 & 12 \\ 0 & 21 & 31 \end{vmatrix}$

7. $\begin{vmatrix} 27 & 36 & 51 \\ 31 & 1 & 2 \\ 21 & 31 & 10 \end{vmatrix}$

8. $\begin{vmatrix} 21 & 3 & 11 \\ 33 & 4 & -2 \\ 3 & 2 & -1 \end{vmatrix}$

9. $\begin{vmatrix} 19 & 14 & 11 \\ 15 & 9 & 8 \\ 7 & 0 & 0 \end{vmatrix}$

10. $\begin{vmatrix} 22 & 8 & 4 \\ 16 & 12 & 5 \\ 11 & 4 & 2 \end{vmatrix}$

XEERKA «GARAMMER»

Marka aynnu furfureynno habdhiska laba isle'eg oo toosan oo laba doorsoome leh, waxaynnu adeegsan karnaa sugayaal.

Tixgeli habdhiska

$$b_1x_1 + t_1y_1 = j_1$$
$$b_2x + t_2y = j_2$$

Haddii aynnu D ka dhigno inay ka joogto sugaha taxanaha weheliyaalka, $D = \begin{vmatrix} b_1 & t_1 \\ b_2 & t_2 \end{vmatrix}$, haddiina (x, y) aynnu ka joojinno furfurista habdhiska, markaa

$$XD = x \begin{vmatrix} b_1 & t_1 \\ b_2 & t_2 \end{vmatrix} = \begin{vmatrix} xb_1 & t_1 \\ xb_2 & t_2 \end{vmatrix}.$$

Markaynnu isticmaalno Astaanta 6:

$$XD = \begin{vmatrix} b_1x + t_1y & t_1 \\ b_2x + t_2y & t_2 \end{vmatrix} = \begin{vmatrix} j_1 & t_1 \\ j_2 & t_2 \end{vmatrix} \longrightarrow X = \frac{\begin{vmatrix} j_1 & t_1 \\ j_2 & t_2 \end{vmatrix}}{D}$$

$$= \frac{\begin{vmatrix} j_1 & t_1 \\ j_2 & t_2 \end{vmatrix}}{\begin{vmatrix} b_1 & t_1 \\ b_2 & t_2 \end{vmatrix}}$$

Bishardi $D \neq 0$. Haddii aynnu doonayno in sugaha sarreeyahu ahi muuqdo DX, waa caddayn karnaa in

$X = \dfrac{Dx}{D}$. Sidoo kale waxaynu caddeyn karnaa in:

$$y = \frac{\begin{vmatrix} b_1 & j_1 \\ b_2 & j_2 \end{vmatrix}}{\begin{vmatrix} b_1 & t_1 \\ b_2 & t_2 \end{vmatrix}} = \frac{Dy}{D}$$

Inagoo isla habkaa raacayna waxaan isticmaali karnaa sugayaal si loo furfuro habdhis kasta oo isle'eg toosan oo doorsoomeyaalkiisu intii la doono yihiin. Haddii $D \neq 0$, markaa x iyo y qiime waan u heli karnaa, qiimeyaalkaasoo haddii lagu beddelo x, y, ... la hubsan karo inay raalligelinayaan isle'egyada iyo in kale. U fiirso in sugeyaalka sarreeyaalka ahi la mid yihiin D oo kutirsaneyaalkeedii weheliye u ahaa doorsoome marba loo furfurayo lagu beddelay j_1 iyo j_2 siday isugu beegan yihiin. Xeerkan loo haysto in lagu furfuro habdhiska isle'egyada toosan waxaa loo yaqaan XEERKII GRAMMER.

Tusaale :

Raadi ururka furfurista habdhiskan soo socda.

$$\begin{aligned} x - y + 2w &= 2 \\ 2x + 3y - w &= 3 \\ 3x + 2y + 3w &= 4 \end{aligned}$$

Ururka furfuristu waa: $x = \dfrac{Dx}{D}$, $y = \dfrac{Dy}{D}$, $w = \dfrac{Dw}{D}$

$$D = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & -1 \\ 3 & 2 & 3 \end{vmatrix} = 11 + 9 - 10 = 10$$

$$Dx = \begin{vmatrix} 2 & -1 & 2 \\ 3 & 3 & -1 \\ 4 & 2 & 3 \end{vmatrix} = 22 + 13 - 12 = 23$$

$$Dy = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & -1 \\ 3 & 4 & 3 \end{vmatrix} = 13 - 18 - 2 = -7$$

$$Dw = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 3 \\ 3 & 2 & 4 \end{vmatrix} = 6 - 1 - 10 = -5$$

$$x = \frac{Dx}{D} = \frac{23}{10} = 2.3$$

$$y = \frac{Dy}{D} = \frac{-7}{10} = -0.7$$

$$w = \frac{Dw}{D} = \frac{-5}{10} = -0.5$$

$\therefore$ Ururka furfurista $F = \{2.3, \ -0.7, \ -0.5\}$.

Layli :

Adoo isticmaalaya xeerka «Grammer» raadi ururka furfurista habdhisyada soo socda.

1. $\begin{aligned} 2x - y &= 0 \\ 3x + 4y &= 5 \end{aligned}$

2. $\begin{aligned} 6x + 4y &= 8 \\ -3 + 7y &= -3 \end{aligned}$

3. $\begin{aligned} 9x - 2y &= -3 \\ -8x - 3y &= 88 \end{aligned}$

4. $\begin{aligned} x + y + w &= 0 \\ 2x - y + 2w &= 1 \\ 3x + 2y - w &= -1 \end{aligned}$

5. $\begin{aligned} 3x + y + 3w &= 0 \\ 2x - 3y + 4w &= 0 \\ 6x + 4y - 5w &= 1 \end{aligned}$

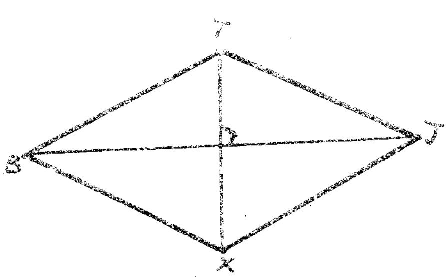
6. $\begin{aligned} 3x - 2w &= 2 \\ 4x + y &= 0 \\ 2y + w &= 4 \end{aligned}$

7. $\begin{aligned} y + y &= 1 \\ 2x + 3y &= 2 \\ 3x + 2y - 5w &= 0 \end{aligned}$

8. $\begin{aligned} x + 3y + w &= 2 \\ y + 2w &= 5 \\ w &= 2x - y + 3 \end{aligned}$

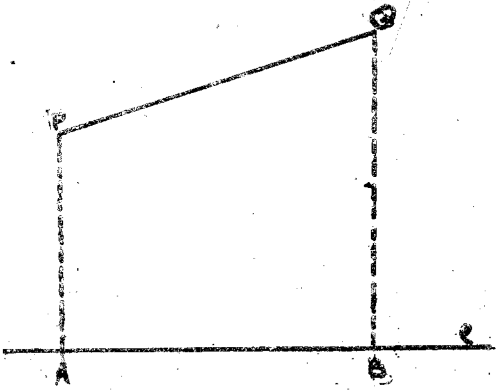
9. $\begin{aligned} x + 2y + 3w &= 0 \\ y - x - 2w &= 5 \\ 5 + 2y &= w \end{aligned}$

10. $\begin{aligned} -4x - 3y + w &= 5 \\ 2x + 4y - 6w &= 6 \\ 5x - 7y + 3w &= -4 \end{aligned}$



JOOMETERI

Ka soo qaad PQ inay tahay xarriijin. Markaa hooska PQ ay ku sameysay xarriiqda L oo jiifta (fiiri shaxan 1) waa harka PQ ay ku sameysay L markii qorraxdu duhur tahay. Haddaba hooska PQ waa AB.

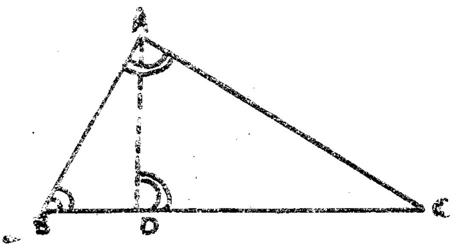


Aragtiinka Koowaad ee «Euclid»

Haddii aan heysanno saddexagal qumman, lug kasta waxay u tahay Tirosin saamigal hooskeeda iyo shakaalka.

Caddeyn :

BD waa hooska lugta AB ay ku sameysan shakaalka BC ee saddexagalka qumman ABC (fiiri Shaxanka 2): waxaa la doonayaa in la caddeeyo: BC : AB = AB : BD.



Labada saddexagal ABC iyo ABD waxay wadaagaan xagasha B, labada xaglood ADB iyo BAC waa isku mid waayo waa qumman yihiin. Marka labada saddexagal waa isu eg yihiin.

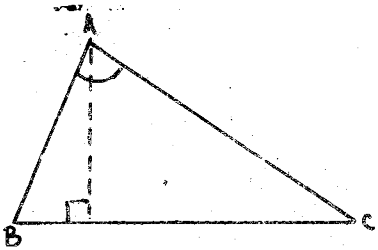
Haddaba BC : AB = AB : BD.

Araagtiinka Labaad ee «Euclid»

Haddii aan heysanno saddexagal qumman, jooga ku qumman shakaalka wuxuu u yahay tirosin saamigal labaad hoos ee lugaha, ku dhacayana shakaalka.

Caddeyn :

Ka dhig BD jooga ku qumman BC ee saddexagalka ABC (Fiiri Shaxanka 3). Waxaa la doonayaa in la caddeeyo.



BD : AD = AD : BC

Labada saddexagal ADB iyo ADC waxay qabaan labada xaglood ADB iyo ADC oo isle'eg waayo waa xaglo qumman: Xaglaha ABD iyo DAC way isle'eg yihiin waayo waxay ku wada sidkan yihiin xagasha BAD. Haddaba labada saddexagal waa isu egyihiin.

∴ BD : AD = AD : BC

Tusaale :

Hel hooska lugta dhererkeedu yahay 12 m. ee saddexagal qumman, haddii shakaalku yahay 18 m.

Furfuris :

Ka dhig x hooska lugta dhererkeedu yahay 12 m. Cusko aragtiinka laad ee «EUCLID».

$$18 : 12 = 12 : x$$

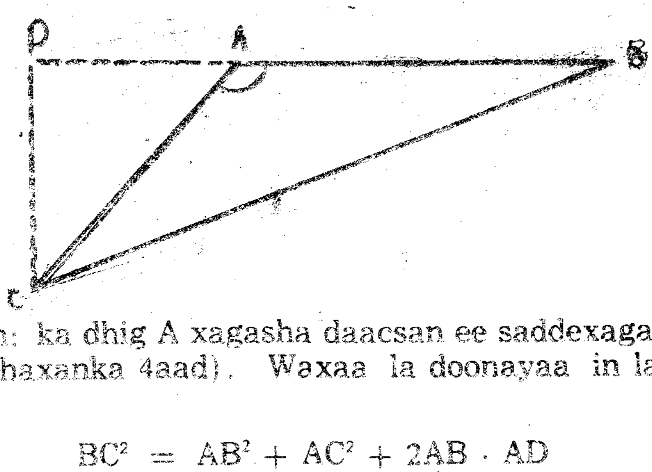
$$x = \frac{12 \cdot 12}{18} = 8 \text{ m.}$$

FIDINTA ARAGTIINKA «PYTHAGORAS»

Waxaan soo aragney, haddii aan heysanno saddexagal qumman, labajibbaaranaha shakaalka wuxuu la mid yahay wadarta labajibbaarranayaasha labada lugood. Haataan waxaan ku fidineynaa aragtiinka Pythagoras saddexagallada fiiqan iyo kuwa daacsan.

Araagtiinka «Pythagoras»

Haddii aan heysanno saddexagal daacsan labajibbaarka dhinaca ka soo horjeeda xagasha daacsan wuxuu le'eg yahay wadarta labajibbaarrada labada dhinac ee kale iyo labalaabka taranka dhinaca kasta oo ah dhinacyadan iyo hooska dhinaca kale uu ku sameeyo isla dhinacaa.



Siin: ka dhig A xagasha daacsan ee saddexagal ABC (Fiiri Shaxanka 4aad). Waxaa la doonayaa in la caddeeyo.

$$BC^2 = AB^2 + AC^2 + 2AB \cdot AD$$

Caddeyn :

$$BC^2 = DC^2 + BD^2 \quad \text{Aragtiinka Pythagoras}$$
$$BC^2 = DC^2 + (BA + AD)^2, \quad BD = BA + AD.$$
$$BC^2 = DC^2 + BA^2 + AD^2 + 2BA \cdot AD$$

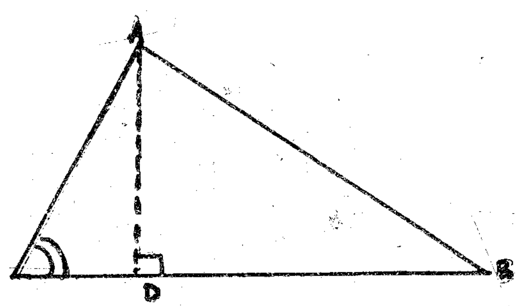
Laakiin :

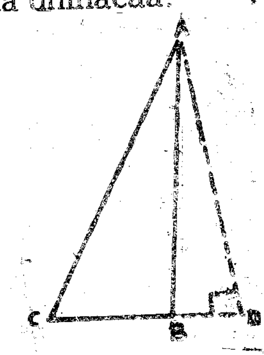
$$DC^2 + AD^2 = AC^2 \quad \text{Aragtiinka Pythagoras.}$$

$$\therefore \quad BC^2 = AB^2 + AC^2 + 2AB \cdot AD$$

Aragtiin

Haddii la haysto saddexagal xagal fiiqan labajibbaarka dhinaca ka soo horjeeda xagasha fiiqan wuxuu le'eg yahay wadarta labajibbaarrada labada dhinac ee kale oo laga jaray labalaabka taranta dhinac kastoo ah dhinacyadatan iyo hooska uu dhinaca kale ku sameeyo isla dhinacaa.





Siin: xagasha ku taal C ee saddexagal ABC. Waxaa la doonayaa in la caddeeyo.

$$AB^2 = AC^2 + BC^2 - 2BC \cdot CD$$

Caddeyn :

1. Fiiri shaxanka 5 (a), haddaba:

$$AB^2 = BD^2 + AD \quad \text{Aragtiinka Pythagoras.}$$
$$AB^2 = (CA - CB)^2 + AD^2, \quad BD = DC - CD$$
$$AB^2 = CD^2 + CB^2 - 2CD \cdot CB + AD^2$$

Laakiin :

$$CD^2 + AD^2 = AC^2 \quad \text{Aragtiinka Pythagoras.}$$
$$AB^2 = AC^2 + BC^2 - 2BC \cdot CD$$

2. Fiiri shaxanka 5 (b), haddaba:

$$AB^2 = AD^2 + BD^2 \quad \text{Aragtiinka Pythagoras.}$$
$$= AD^2 + (CB - CD)^2, \quad BD = CB - CD$$
$$= AD^2 + BC^2 + CD^2 - 2CB \cdot CD$$

Laakiin :

$$AD^2 + CD^2 = AC^2 \quad \text{Aragtiinka Pythagoras.}$$
$$AB^2 = AC^2 + BC^2 - 2BC \cdot CD$$

Tusaale :

Saddexagalayaasha soo socda ku weeba fiiqan.

b) $a = 6$ sm.
$b = 3$ sm.
$c = 4$ sm.

t) $a = 11$ sm.
$b = 13$ sm.
$c = 15$ sm.

Furfuris :

Haddii saddexagalku leeyahay xagal daacsan, dhinaca ugu dheeri waa ihuu ka soo horjeedaa xagasha. Haddaba aragtiinkii aan soo qaadanney wuxuu inoo sheegeyaa in labajibbaarka dhinacaasi uu ka weyn yahay wadarta labajibbaarrada dhinacyada kale.

1. $a^2 = 6^2 = 36, \; b^2 + c^2 = 3^2 + 4^2 = 9 + 16 = 25$

$6^2 > 3^2 + 4^2$

$\therefore \; \triangle$ waa daacsan yahay.

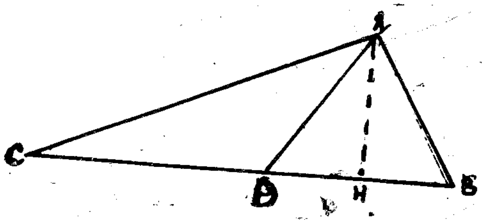
2. $c^2 = 15^2 = 225, \; a^2+b^2 = 13^2+11^2 = 169+121 = 290$

$15^2 < 13^2 + 11^2$

$\therefore \; \triangle$ waa fiiqan yahay.

Aragtiinka «Apollonius»

Saddexagal kasta, wadarta labajibbaarka laba dhinac oo kasta waxay le'eg tahay labajibbaarka dhinaca saddexaad barkii iyo labalaabka labajibbaarka dhinaca dhexfurka saddexaad.



Siin: Saddexagal ABC iyo AD oo ah dhexfur BC. Waxaa la doonayaa in la caddeeyo:

$$AB^2 + AC^2 = \frac{1}{2} BC^2 + 2AD$$

Dhismo: Sawir joogga ah.

Caddeyn:

Ka soo qaad in ADO xagasha daacsan ee saddexagalka ACD, markaa

$$\text{(a)} \quad AC^2 = AD^2 + CD^2 + 2CD \cdot DH$$

Ka soo qaad in ADB tahay xagasha fiiqan ee saddexagalka ABD, markaa

$$\text{(b)} \quad AB^2 = AD^2 + DB^2 - 2BD \cdot DH$$

Haddaba isugee (a) iyo (b).

$$AB^2 + AC^2 = 2BD^2 + 2AD^2, \; BD = CD$$

$$AB^2 + AC^2 = 2\left(\frac{1}{2}BC\right)^2 + 2AD^2, \; BD = \frac{1}{2}BC$$

$$AB^2 + AC^2 = \frac{1}{2}BC^2 + 2AD^2$$

Layli :

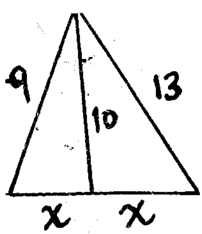
1) Dhererka joogga ku taagan shakaalka saddexagal qumman waa 12 m. hooska lugta yar ay ku sameyso shakaalka waa 9 m. Raadi dhererka saddexagalka iyo bedkiisa.

2) Hoosaska labada lugood ee sameysan shakaalka saddexagal qumman dhererkoodu waa 23, 2 iyo 18.8 m. Raadi dhererka joogga ku taagan shakaalka iyo wareegga saddexagalka.

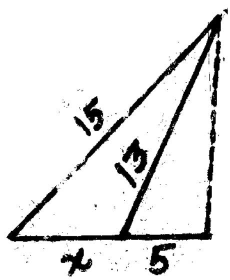
3) Saamiga hoosaska lugaha ku sameysan shakaalka ee saddexagal qumman waa $\frac{9}{16}$, joogga ku taagan shakaalkuna waa 24 m. Raadi dhererka wareegga saddexagalka.

4) Dhererka lugaha saddexagal qumman saamigoodu waa 3 : 4 wareegga saddexagalkuna waa 180 m. Hel joogga ku qumman shakaalka iyo hooska luguhu ku sameyaan shakaalka. Raadi wareegga saddexagal u eeg oo shakaalkiisu yahay 10 m.

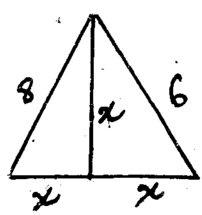
5) Fiiri shaxan 7. Xisaabi dhererrada ku calaamadeysan x.



(i)



(ii)



(iii)

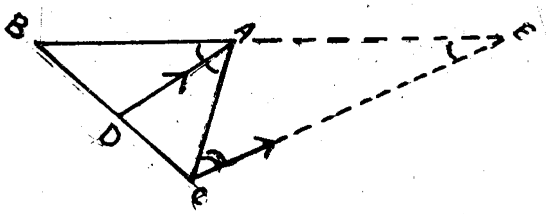
6) Dhererrada labada dhinac ee saddexagal waa 13 m. iyo 11 m. Hooska dhinaca hore uu ku sameeyo ka saddexaad waa labalaabka hooska uu dhinaca labaad ku sameeyo ka saddexaad. Hel dhererka dhinaca saddexaad.

7) △ ABC waa saddexagal labaale ah, AB = AC; CD waa dhexfur. Caddee in:

$$CD^2 = \frac{1}{4} AC^2 + \frac{1}{2} BC^2$$

Aragtiinka Koowaad•ee Kalabaraha

Kalabaraha xagal gudeed ee saddexagal wuxuu u kala qeybshaa dhinaca ku beegan laba xarriijimood oo saamigal u ah labada dhinac ee kale.



Siin: Ka dhig kala baraha xagasha BAC ee saddexagal ABC. (Fiiri shaxanka 8aad). Waxa la doonayaa in la caddeeyo:

$$BD : DC = AB : AC$$

C a d d e y n :

Geeska C ka dhis xarriiq la barbarro ah AD kulana kulmeysa fidinta BA barta E. Xagasha ∠ ACE = ∠ CAD waayo waa xaglo talantaalli gudeed ah.

Xagasha CAB = AEC waayo waa xagallo isku beegan oo ka dhisma barbarrayaasha CE iyo AD uu kala gooyo tikraarka EB; marka astaanta dhexidda daraadeed xagasha AEC = ACE. Haddaba saddexagalka ACE waa labaale. Laakiin saddexagalka EBC waxa ku cad in:

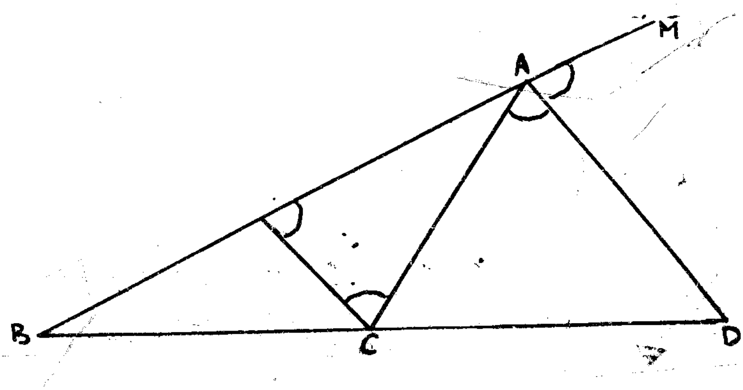
BD : BC = AB : AE

ama BD : DC = AB : AC, AE = AC

Aragtiinka Labaad ee Kalabaraha

Haddii kalabaraha xagal dibadeed ee saddexagal la kulmo fidinta dhinaca ka soo horjeeda, fogaannada cirifyada dhinacaasi ay u jiraan barta kulanka, waxay saamigal u yihiin dhinacyada kale.

Siin: Ka dhig AD kalabaraha xagal dibadeedka CAM ee △ABC ee kula kulma fidinta dhinaca ka soo hor-



jeeda BC barta D. (Fiiri shaxanka 9aad). Waxa la doonayaa in la caddeeyo:

DB : DC = AB : AC

Caddeyn :

Geeska C ka dhis xarriiqda CN oo la barbarro ah kalabaraha AD. Waxa la haystaa: ANC = MAD waa xaglo isku beegan kana dhisma barbarrayaasha NC iyo AD uu kala gooyo tikraarka BM. MAD = DAC, dhisma ahaan.

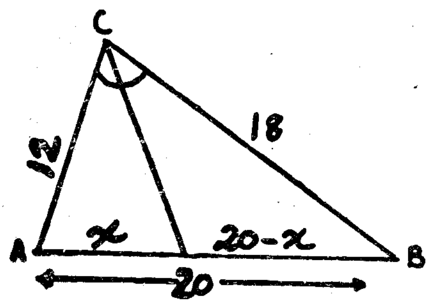
DAC = ACN, waa xaglo talantaalli gudeed ah oo ka dhisma barbarrayaasha NC iyo AD uu kala gooyo tikraarka AC. ∠ ANC = ∠ ACN Astaanta dhexidda. Markaa saddexagalka ANC waa labaale. Laakiin saddexagalka waxaa ku cad in

$$DB : BC = AB : AN$$

ama $DB : AB : AC, \quad AN = AC.$

T u s a a l e :

Hel dhererka xarriijimmaha ku yaalla dhinaca AB ee saddexagalka ABC, haddii CD uu yahay kalbaraha xagasha gudeed C, AB = 20, AC = 12, BC = 18.



F u r f u r i s :

Ka dhig AD = x (fiiri shaxanka 10aad).
Haddaba

$$\frac{AD}{DB} = \frac{AC}{BC}$$

ama $$\frac{x}{20 - x} = \frac{12}{18}, \quad \frac{x}{20 - x} = \frac{2}{3}$$

$3x = 40 - 2x; \quad 5x = 40$

$x = 8$, ama. $AD = 8$, $DB = 12$.

Layli :

Layliyada 1—3 raadi dhererka xarriijimmada ku yaalla dhinaca AB oo uu kala qaybiyey kalabaraha xagasha gudeed C ee saddexagalka ABC (fiiri shaxanka 10aad).

1) Siin: AB = 4.5, AC = 4, BC = 5

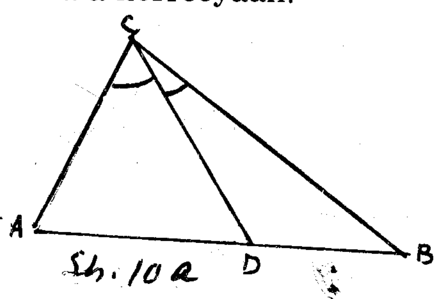
2) Siin: AB = 10, AC = 6, BC = 8

3) Siin: AB = 7, AC = 16, BC = 12

4) Kalabaraha xagasha dibadeed iyo tan gudeed ee $\angle BAC$ waxay ka gooyaan BC iyo BC oo la fidiyay Q iyo P siday u kala horreeyaan.

Caddee :

$$\frac{BP}{PC} + \frac{BQ}{CQ}$$



5) Kalabarayaasha gudeed iyo dibadeed ee xagasha BCA, waxay ka gooyaan BC iyo BC oo la fidiyay P iyo Q siday u kala horreeyaan, BP = 5, PC = 5. Raadi CQ.

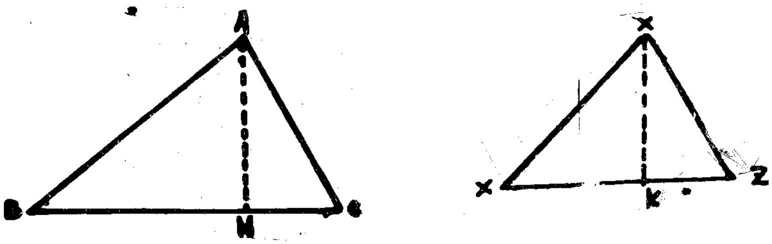
6) Saddexagalka ABC, AB = 6 sm., BC = 5 sm., CA = 4 sm. Kalabarayaasha gudeed iyo dibadeed ee xagasha BAC waxay ka gooyaan BC iyo BC oo la fidiyey P iyo Q siday u kala horreeyaan. Raadi PB iyo BQ.

Tus in :

$$\frac{1}{BP} + \frac{1}{BQ} = \frac{1}{BC}$$

Aragtiin

Saamiga bededka laba saddexagal oo isu'eg wuxuu le'eg yahay saamiga labajibbaarka dhinacyada isku beegan.



Siin: Labada saddexagal ABC iyo XYZ way isu'eg yihiin waxaa la doonayaa in la caddeeyo:

$$\frac{\text{Bedka ABC}}{\text{Bedka XYZ}} = \frac{BC^2}{TZ^2}$$

Caddeyn :

Sawir joogyada AH, XK. Saddexagallada AHB iyo XKY waxay ina siinayaan:

$\angle ABH = \angle XYK$, waayo ABC iyo XYZ waa isu-eg yihiin.

$\angle AHB = \angle XKY$, xaglo qumman dhisma ahaan.

$\therefore \quad \angle BAH = \angle YXK.$

$\therefore \quad \triangle$ AHB iyo $\triangle$ XKY waa isu-egyihiin.

$$\therefore \quad \frac{AH}{XK} = \frac{AB}{XY}$$

Laakiin $\frac{AB}{XY} = \frac{BC}{YZ}$, waayo ABC iyo XYZ waa isu-egyihiin.

$$\frac{AH}{XK} = \frac{BC}{YZ}$$

Laakiin bedka $\triangle$ ABC = $\frac{1}{2}$ AH · BC
bedka $\triangle$ XYZ = $\frac{1}{2}$ YK · YZ

Marka, $\frac{\text{Bedka ABC}}{\text{Bedka XYZ}} = \frac{\frac{1}{2} AH \cdot BC}{\frac{1}{2} XK \cdot YZ}$

Laakiin $\frac{AH}{XK} = \frac{BC}{YZ}$

$$\therefore \frac{\text{Bedka ABC}}{\text{Bedka XYZ}} = \frac{BC^2}{YZ^2}$$

OGOW: Haddii laba geesooleyaal ay isu-egyihiin, waxaa loo qaybin karaa tiro isle'eg oo saddexagallayaal isu-eg. Markaa saamiga bededka laba geesoole oo isu-eg wuxuu le'eg yahay saamiga labajibbaarrada dhinacyada isku beegan, arrimaha soo socdana waa kuwa lagama maarmaan ah.

b) Saamiga bededka dulaha malaasyo isu-eg wuxuu le'eg yahay labajibbaarka addimahooda toosan.

t) Saamiga Muggaaga ee malaasyo isu-eg wuxuu le'eg yahay saamiga saddexjibbaaarka addimahooda toosan.

Tusaale :

Bededka laba geesoole oo isu-eg waa 11.56 m^2. iyo 44.89 m^2. Hel saamiga dhinacyadooda isku beegan.

Furfuris :

Ka dhig P iyo P^1 laba dhinac oo isku beegan: marka

$$\frac{11.56}{44.89} = \frac{P^2}{P'^2}$$

$$\therefore \frac{34}{67} = \frac{P}{P'}$$

Layli :

1) Bedka hal geesoole waa 169 sm^2. dhinaciisa ugu yarina waa 4 sm. Hel bedka geesoole u-eg haddii dhinaciisa ugu yari yahay 8 sm.

2) Bededka laba geesoole oo isu-eg waa 648 mm^2 iyo 392 mm^2. Haddii dhinaca geesoolaha hore yahay 36 mm. hel dhinaca ku beegan ee geesoolaha dambe.

3) Haddii saamiga bededka laba geesoole oo isu-eg yahay 16 : 9, dhinaca geesoolaha horena yahay 8 m. Hel dhinaca ku beegan ee geesoolaha danbe.

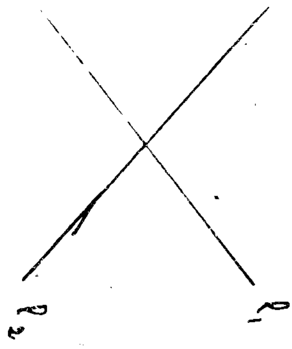
4) Wadarta bededka laba saddexagal oo labaale ah waa 6000 m^2. Haddii labada sal ay yihiin 30 m^2. iyo 195 m^2. Hel dhererrada joog iyo dhinacyada kale.

5) Wadarta bededka laba saddexagal oo labaale ah waa 195 m^2. Haddii labada sal ay yihiin 10 iyo 15 m. Raadi labada bed iyo dhererka wareeggooda.

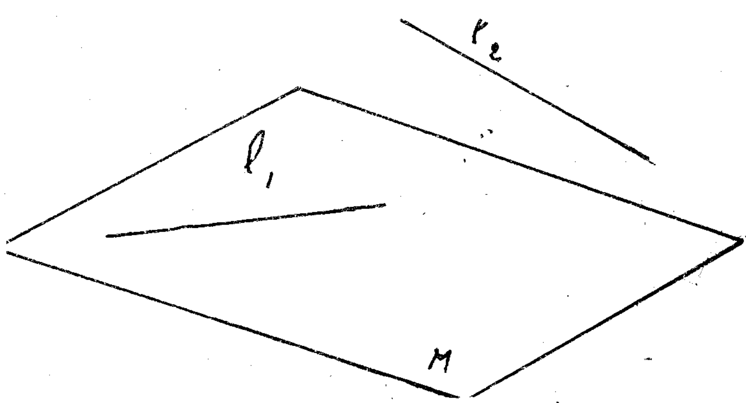
XARRIIQO IYO SALLAXYO BARBARRO AH

Xarriiqo barbarro ahi waa laba xarriiq oo toosan oo aan weligood kulmeyn kuna wadajira isku sallax. Laba xarriiq ee isku sallax ahi waa is-gooyaan ama waa barbarro. Laakiin, haddii ay ku wada jiraan hal dulalaati waxaa la heli karaa laba xarriiq oo aan isgoyn, barbarrona ahayn. Xarriiqahaas oo kale waxaa la yiraa **Jilladan.**

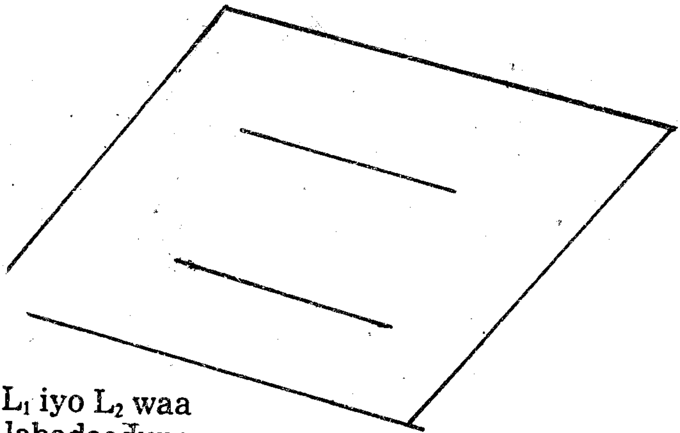
Tusaale :



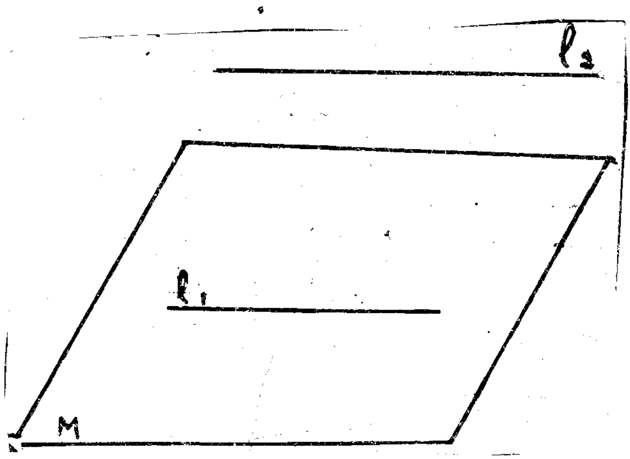
Xarriiqaha L_1 iyo L_2 waxay ku kulman bar,



Xarriiqaha L_1 iyo L_2 ma kulmaan barbarrona ma aha, laakiin waa jilladan.

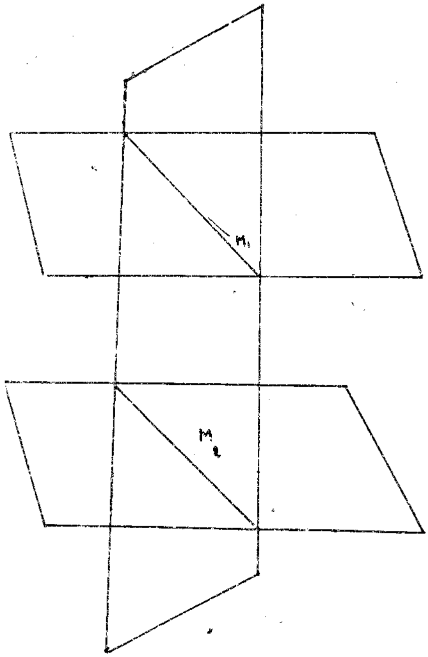


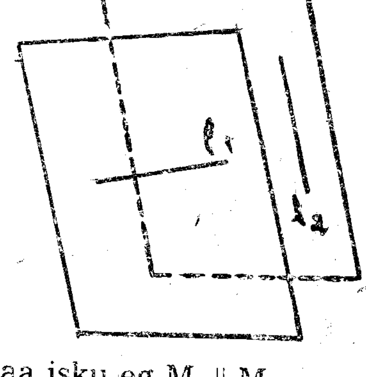
Xarriiqaha L_1 iyo L_2 waa barbarro labadooduna waa isku sallax.



Xarriiqaha L_1 iyo L_2 waa barbarro. Xarriiqda L_1 waxay ku jirtaa sallaxa M. Xarriiqda L_2 kuma jirto sallaxa M.

Laba xarriiq oo barbarro ah waxay sameeyaan hal sallax, xarriiq iyo sallaxna waa barbarro haddii aanay kulmin si kastoo loo fidiyo. Sidoo kale, sallaxyo barbarro ahi waa sallaxyo aan weligood kulmin si kastoo loo fidiyo. Laba xarriiqood oo toosan oo ku kala jira laba sallax waa barbarro ama jilladan.

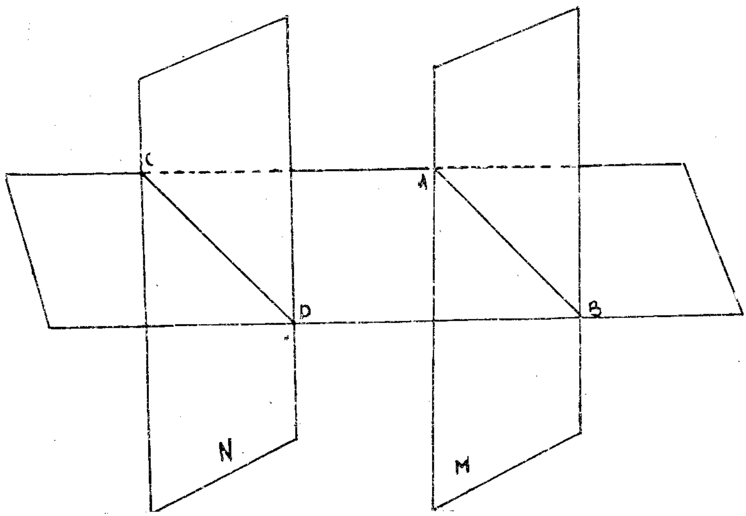




L_1 iyo L_2 waa isku eg $M_1 \parallel M_2$.

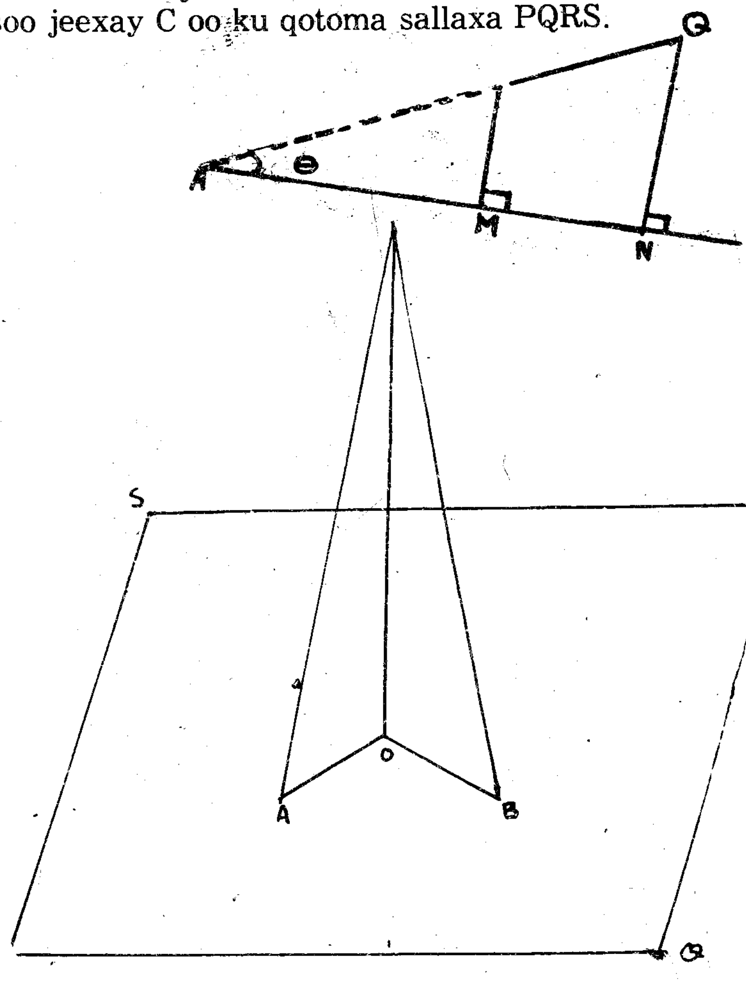
Isgoyska laba sallax oo barbarro ah iyo mid sad-dexaad waxay dhalisaa xarriiqyo barbarro ah. Shaxanka 13: sallax M || Sallax N, sallax P wuxuu M ka gooyaa AB, Nna wuxuu ka gooyaa CD.

Markaa, waxa la caddeyn karaa ni AB||CD.



LIGANE SALLAX

Xarriiqi waxay noqotaa Ligane sallax marka ay la sameyso xaglo qumman xarriiq kasta oo ku jirta sallaxa oo ay la kulanto. Matalan: xarriiq taagani waxay ku ligan tahay sallax jiifa, shaxan 14aad wuxuu ina tusayaa sallax PQRS iyo barta C oo ka sarreysa sallaxa. Xarriiq ayaa laga soo jiiday C oo kula kulantay sallaxa barta O. OA iyo OB waa xarriiqyo ku jira sallaxa PQRS. Marka, haddii ∠COA iyo ∠COB ay yihiin xaglo qumman, xarriiqda CO waxay u noqonaysaa ligane xarriiq kasta oo ku jirta sallaxa PQRS. CO waa liganaha laga soo jeexay C oo ku qotoma sallaxa PQRS.

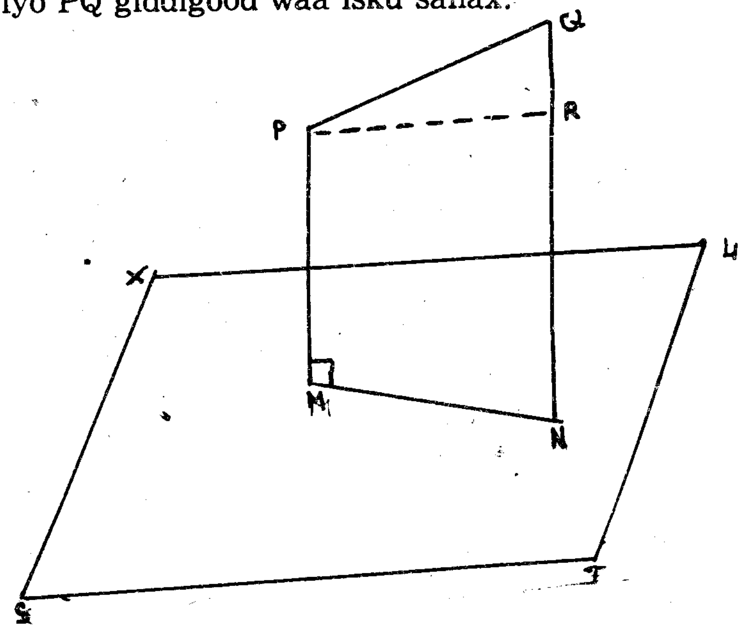


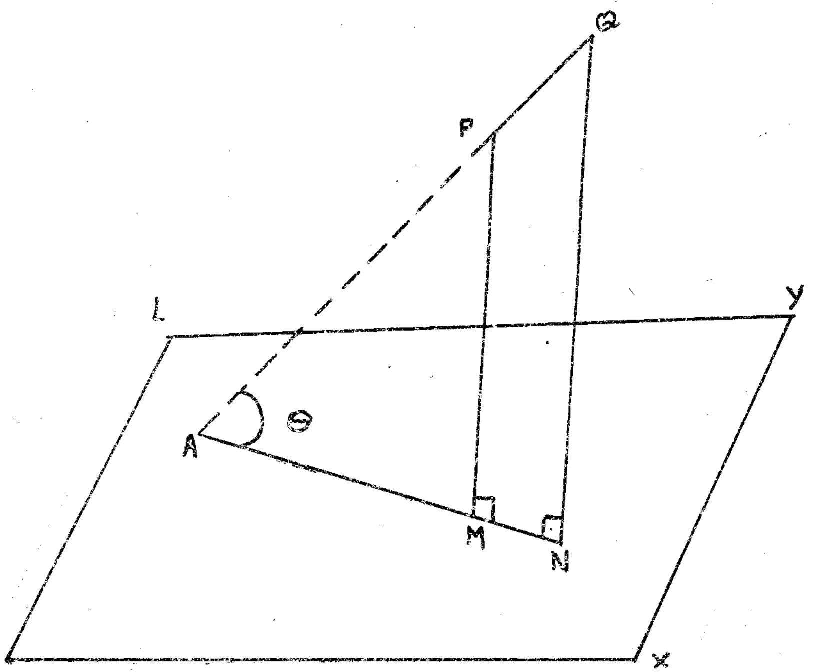
HOOS KU DHACAY XARRIIQ

Shaxan 15 wuxuu ina tusayaa xarriiq PQ iyo AB, haddii PM ⊥ AB, markaa M waa hooska P ee ku dhacay AB. Haddii PQ ama QP la fidiyo waxay la kulantaa AB iyadoo la sameysa xagasha Θ. $\cos \Theta = \frac{MN}{PQ}$.

HOOS KU DHACAY SALLAX

Ka dhig p bar ka baxsan sallax, PMna qotome (Ligane) laga soo jeexay P oo ku qumman Sallaxa. Haddii M yahay Cagta Qotomaha (Liganaha) laga soo jeexay P ee ku qumman sallax, markaa M waa hooska P ee ku sameysan sallaxa, Shaxan 16 wuxuu inna tusayaa xarraaqda PQ ee ku dul taal sallaxa STUY. PM, QN waa qotomayaal laga soo jeexay P, Q sidey u kala horreeyaan oo ku wada qumman sallaxa. Marka MN waa hooska PQ ee ku dul yaal sallaxa. PM || QN, xarriiqaha PM, MN iyo PQ giddigood waa iskú sallax.



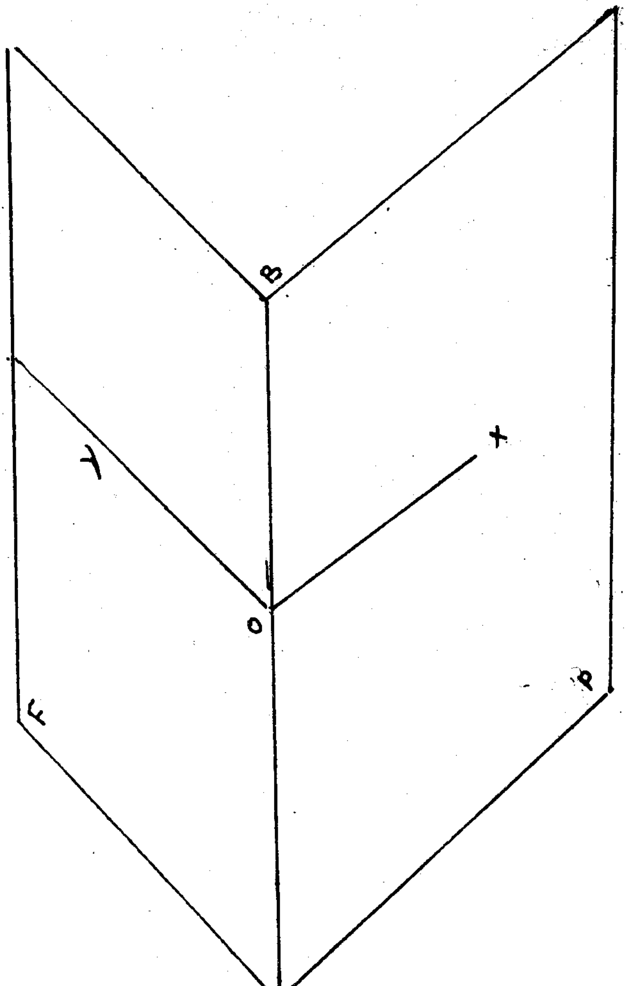


XAGAL U DHEXAYSA XARRIIQ IYO SALLAX

Xagasha u dhexaysa xarriiq iyo sallax waa xagasha u dhexaysa xarriiqda iyo hooskeeda ku dul yaal sallaxa. Shaxan 18, O wuxuu ku dul yaal sallaxa ABCD, PM-na waa liganaha P ee sallaxa. Markaa OM waa hooska OP ee ku dul yaal sallaxa. Xagasha MOP waa xagasha u dhexaysa xarriiqda OP iyo sallaxa ABCD.

Si loo helo xagasha u dhexaysa PQ iyo MN ee shaxan 16, sawir PR || MN, oo R kula kulmeysa QN; markaa ∠ QPR waa xagashii la baadi goobayey. Shaxanak 17 ∠ MAP waa xagasha u dhexaysa PQ iyo sallaxa WXYZ

$$\text{markaa Cos } \angle MAP = \frac{MN}{PQ}.$$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